Mesh Partitioning Techniques and Domain Decomposition Methods

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Mesh Partitioning Techniques
and
Domain Decomposition Methods

Edited by
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Preface

Domain decomposition methods are very efficient for computing the solution of large scale problems in parallel. These methods mainly consist of splitting the global domain into several sub-domains and computing the solution on the global domain through the resolution of the problem associated with each sub-domains. Because the aspect ratio of the subdomains and the shape of the interface between the subdomains have a strong influence on the convergence of domain decomposition methods, it is interesting to collect mesh partitioning techniques and domain decomposition methods in the same book. In this manuscript, mesh partitioning techniques and domain decomposition methods are presented in thirteen chapters. Each chapter, written by different authors, presents a state-of-the-art review of some well known methods, techniques and algorithms. A bibliography is included at the end of each chapter.

The first chapter authored by R. Baños Navarro and C. Gil focuses on describing the main methods for mesh and graph partitioning. The goal of mesh partitioning is to divide the set of elements of a mesh into a certain number of parts such that the imbalance between domains and the number of elements sharing common faces are both minimised. Graph partitioning algorithms can be used to satisfy these conditions by first modelling the mesh by a graph, and then partitioning it into several sub-graphs which represent the mesh decomposition. This chapter provides a description of this problem and the current state-of-the-art in mesh and graph partitioning, including new approaches based on multi-constraint and multi-objective optimisation.

The second chapter by C. Walshaw and M. Cross gives an overview of the research into graph-partitioning, with particular reference to JOSTLE, the parallel, multilevel software package, available from the University of Greenwich. Recent years, particularly since 1995, have seen major advances in this field and one of the key innovations, the multilevel paradigm, which proved to be so successful for partitioning, has been extended to many other areas as a metaheuristic. JOSTLE has been an important part of this development and so, after discussing its core algorithms in the context of multilevel refinement, the authors go on to highlight some of the key research issues it has been used to address. They also demonstrate the flexibility of the multilevel paradigm by outlining some enhancements such as multiphase mesh-partitioning, heterogeneous mapping and partitioning to optimise subdomain shape.

The third chapter by F. Magoulès and R. Putanowicz consists of an introduction to the visualisation of distributed finite element data and graph partitioning with VTK - the Visualisation ToolKit library. VTK is a software system for computer graphics, visualisation and image processing. The chapter presents complex data file visualization with VTK and explains how to program important functionalities such as drawing a mesh, labelling points, labelling cells, colour bar, saving to a file, and so on. The authors investigate the case of unstructured finite element mesh partitioning and provide programming methodology to the reader. Detailed examples written in the scripting language Tcl with calls to the VTK library are clearly described.

The chapter of F. Magoulès and F.-X. Roux discusses the basis of substructuring methods and the most classical domain decomposition methods in an homogenous formulation. Theory, algorithms and implementation details of each method are presented to the reader. The parallel finite element matrix forming based on substructuring is first introduced. Then the parallel iterative solution of the associated linear system is presented. Direct methods with parallel matrix factorisation based on substructuring are then detailed. Finally, several domain decomposition methods including the Schur complement method, the Dual Schur complement method, the FETI method and the FETI-H method are successively described.

The chapter authored by G.P. Nikishkov discusses basic issues of the domain decomposition method for parallel finite element analysis. An algorithm of domain partitioning based on a graph labelling scheme is presented. First, the domain decomposition method with a direct LDU equation solver is considered. It is shown that load balancing for subdomain assembly and condensation can be achieved by expressing subdomain operation counts through their element numbers and by solving the nonlinear equation system. Next, a domain decomposition method with an iterative solver is described. It is then demonstrated that efficiency of the parallel preconditioned conjugate gradient solver for decomposed finite element problems can be improved by overlapping communication and computation.

The chapter of J. Kruis reviews the dual-primal finite element tearing and interconnecting (FETI-DP) method, a very powerful and general non-overlapping domain decomposition method. Firstly, the basic ideas and assumptions are summarised and discussed. Then, all necessary equations, matrices and vectors are defined. The chapter also contains a simple one-dimensional example of heat transfer solved by the FETI-DP method which clarifies the reviewed topic. Finally, numerical examples solved in parallel on a cluster of PCs are summarised.

In the chapter authored by M. Sarkis and D. Szyld nonsymmetric and indefinite linear systems arising from differential equations are studied. The interplay between minimum residual iterative methods and a general theoretical framework for constructing and analysing two-level overlapping additive Schwarz techniques for these systems is presented in a step by step manner. The chapter discusses the relation between Euclidean and energy norm minimum residual methods, with right and left preconditioners, and it culminates in the theoretical justification for the practical use of Euclidean norm minimisation methods. These theoretical developments are illustrated with numerical experiments.

The next chapter authored by L. Giraud and R.S. Tuminaro is devoted to an algebraic description of most well-known domain decomposition preconditioners for the parallel solution of large linear systems arising from the discretisation of partial differential equations. These preconditioners are presented for approaches based on mesh-splitting or matrix partitioning. Computer science concerns for their implementations on parallel computers are discussed and numerical performance is reported.

The chapter authored by F. Magoulès investigates three dimensional optimised Schwarz methods without overlap for predicting transmission loss in mufflers and silencers. Optimised Schwarz methods are very efficient iterative algorithms for the parallel solution of computational mechanics problems. These methods are similar to the classical Schwarz methods, but they use absorbing interface conditions between the sub-domains. These interface conditions are then optimized for efficiency and lead to a fast and robust convergence behaviour of the iterative algorithm. A wide range of optimisation techniques has been proposed and developed for the literature in the two dimensional case. This chapter presents the extension of the optimised Schwarz methods to the three-dimensional case and applies these techniques to realistic industrial problems for the prediction of transmission loss in mufflers and silencers.

The chapter authored by T. Knopp, G. Lube and G. Rapin discusses recent developments of domain decomposition methods for linearised incompressible flow problems based on the Stokes or Oseen model. First, a critical review of the literature on these techniques is presented. Then the authors describe a non-overlapping method with interface conditions of Robin type in more detail. They present available convergence results. Moreover, they discuss the design of parameters in the interface conditions between adjacent subdomains which allow a remarkable acceleration of convergence of the method. Finally, a typical application of the latter method to a complex model is presented. It is shown that domain decomposition techniques are mandatory for the numerical simulation of large scale problems like indoor air flows.

The chapter of F. Hülsemann introduces the Aitken-Schwarz algorithm, which is an acceleration technique for overlapping Schwarz domain decomposition methods that was introduced by M. Garbey and D. Tromeur-Dervout at the beginning of the decade. Combining, under certain circumstances, low communication requirements with fast convergence, the Aitken-Schwarz method is particularly well suited for metacomputing settings in which communication between different computing sites has to be kept to a minimum. This chapter covers the derivation of the method, an assessment of its computational complexity, an extension to grids with a certain type of refinement and finally provides numerical examples.

The chapter of L. Champaney and D. Dureisseix discusses a mixed domain decomposition in the field of structural mechanics. This method, initiated by P. Ladevèze, was originally developed within the framework of the LArge Time INcrement method. For structural simulations, such an approach is particularly useful for non univoque constitutive relations on interfaces, such as contact with or without friction. A multilevel version with a strong relationship to homogenisation is also described, and significant application examples are presented and discussed for different situations.

The chapter of C. Lacour is dedicated to the study, in the context of domain decomposition, of the mortar element method approach for solving fourth-order elliptic problems discretised by a non conforming finite element, discrete Kirchhoff triangles. To reduce the computation cost, a decomposition of the whole domain into non overlapping subdomains is introduced. The discretisation relies on the variational formulation of the continuous problem, and is, in general, nonconforming. There are some mathematical difficulties for the resolution of these kinds of problems when finite element discretisation is used. In the beginning of the chapter, the spaces of approximation and the decomposed discrete problem are defined. Since the main difficulty in this formulation is to verify some properties of the associated discrete bilinear form, one part of the chapter deals with this difficulty. The second part of the chapter deals with the uniform continuity (independent from the discretisation parameter) of the discrete bilinear form. The remaining part of the chapter contains a mathematical analysis of the method with optimal error estimates.

Naturally, the present book cannot provide a complete record of the many approaches, applications, features, and schemes related to mesh partitioning techniques and domain decomposition methods. However, it does provide an excellent tutorial of the most well known methods used in academia and in industry. This book will be of interest to engineers, computer scientists and applied mathematicians.

The editor wishes to thank the authors for their willingness to contribute to this edited book dedicated to mesh partitioning techniques and domain decomposition methods.

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