Computational Methods
for
Acoustics Problems
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for
Acoustics Problems

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8 Advances in the On-Surface Radiation Condition Method: Theory, Numerics and Applications

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9 Fast Frequency Response Function Computation by Model Reduction Methods

K. Meerbergen

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The computation of acoustic phenomena is regarded today as one of the most challenging in scientific computation. This field has been an area of active research for decades. A major current difficulty lies in the effective treatment of unbounded domains by standard domain-based methods. Finite element methods cannot directly handle such configurations in an effective way. An artificial boundary that truncates the unbounded domain is used to form a bounded computational domain. Special techniques are then required to reduce the spurious reflection of waves that impinge on this artificial boundary, including infinite elements, boundary element, etc. Additional techniques enable an increase in the accuracy of the numerical schemes and the computational efficiency as well as the robustness of the methods. In this manuscript, computational methods for acoustics problems are presented in eleven chapters. Each chapter, written by different authors, presents a state of the art of well established or innovative methods, techniques or algorithms. A large bibliography is included at the end of each chapter.

The chapter authored by A. Bendali and M. Fares presents some new techniques, specially adapted to high performance computing, for solving acoustic scattering problems involving a bounded zone filled by a heterogeneous medium. The discretization in this zone is carried out by using a standard finite element method. The rest of the computational domain is dealt with by using a boundary integral equation. The final linear system resulting from the coupling of two solution procedures is partly sparse and partly dense. Serious difficulties arise when the solution has to be tackled on a parallel platform. The authors start by presenting a condensed but systematic way to obtain the most used boundary integral equations for acoustic scattering problems. Then, they show how the non overlapping domain decomposition methods, usually used for the Helmholtz equation, can be used to overcome the difficulties induced by the coupling. They finally present an efficient procedure, which requires the solution of only a small linear system of finite element equations at each iteration, to solve the scattering problem. It is mainly based on the utilization of a non overlapping domain decomposition method, but this is in the context of a nodal finite element method with appropriate treatment of the nodes being shared by more than two sub-domains. Numerical experiments illustrate the methods presented.

The chapter authored by J. Astley presents the infinite element concept for acoustic problems. Infinite elements were first proposed in the 1970s and 80s for the propagation of surface waves on water, but were not developed specifically for acoustics
until the 1990s. They are now widely used and are particularly attractive for inclusion in commercial finite element codes since they largely preserve the structure of conventional finite element models. They are able to act as a high order non-reflecting boundary condition at the outer boundary of a truncated finite element model for an exterior problem, while also predicting directly the far field solution. The method is presented for mapped and for separable elements and for symmetric (unconjugated) and non-symmetric (conjugated) formulations. Spherical and spheroidal formulations are discussed. The accuracy and conditioning of the various formulations is investigated in some detail and the extension to transient problems is outlined.

The chapter of M.N. Guddati, K.-W. Lim and A. Zahid presents Perfectly Matched Discrete Layers (PMDL), a new method that combines the advantages of various existing methods for modeling unbounded domains. PMDL links the seemingly disparate techniques of PML and local absorbing boundary conditions (ABCs). PMDL is a special discretization of PML, which preserves the property of perfect impedance matching even after discretization. Thus, PMDL is as flexible as PML and is in fact an improvement. PMDL is also shown to be equivalent to rational approximation of the exact impedance, thus inheriting the desirable accuracy properties of local ABCs. Furthermore, unlike the existing methods, PMDL is applicable not only to rectangular computational domains, but also to polygonal domains. The chapter contains the basic derivation of PMDL as well as the illustration of its superior performance for acoustic as well as elastic wave propagation problems.

The chapter authored by L.L. Thompson, P. Kunthong and S. Subbarayalu discusses original Time-Discontinuous Galerkin (TDG) methods which provide high-order accuracy and stability for second-order hyperbolic systems including those governing structural dynamics and acoustics. Generalized gradients of residuals of the governing equations are added to the standard TDG variational equation to provide high-order accuracy and stability properties of the parent TDG method while gaining significant reductions in computational cost comparable with standard second-order accurate single-step/single-solve (SS/SS) time-stepping algorithms. Using this stabilized framework, together with optimal design of temporal approximations and time-scales, efficient multi-pass iterative solution algorithms are developed which: maintain C- and L-stability; provide high-order accuracy in only two or three iterative passes; and can easily be implemented in standard finite element codes. Alternative decoupling strategies are also developed using spectral decomposition of the time arrays to provide fast solves of space-time matrix equations resulting from the TDG method which are ideal for parallel implementation. In addition, new space-time finite element strategies based on the TDG method are presented including high-order accurate non-reflecting boundary conditions. The time-discontinuous Galerkin (TDG) variational method is used to divide the time-interval into space-time slabs, the solution advances from one slab to the next. Within each global space-time slab subdivisions of multiple local space-time elements are allowed. This gives the flexibility required to change local time-step size for different elements in the spatial mesh in a truly local self-adaptive space-time methodology. By maintaining orthogonality of
the space-time mesh and pre-integrating analytically in the time-dimension through each local element in the time-slab, an efficient yet robust adaptive method is obtained which accommodates any standard spatial element without modifications. The resulting technologies provide significant advances in accuracy, efficiency, and reliability over standard time-stepping methods, especially for long-time simulations which track response over large distances and long time intervals.

The chapter of E. Turkel surveys absorbing boundary conditions and iterative methods for the numerical solution of the Helmholtz equation in unbounded regions. For acoustic scattering around a body it is necessary to enclose the infinite region by an artificial surface. One needs to set a boundary condition that absorbs outgoing rays. The chapter discusses several approaches for such boundary conditions for various shapes of the artificial surface. After discretizing the total system one needs to solve a large but sparse system that is neither positive nor Hermitian. For three dimensional problems or high frequencies one must use an iterative method, usually a Krylov space method. To converge within a reasonable number of steps it is necessary to precondition the system. A survey of different preconditioners is also presented in this chapter.

The chapter authored by F. Magoulès and F.-X. Roux presents a general methodology to solve coupled fluid-structure problems with non-matching grids arising from vibro-acoustics problems. The coupling is ensured through the boundary conditions defined along the fluid-structure interface. Here the coupled quantities are integrated over a set of quadrature points defined on the fluid-structure interface. This integration involves the computation of nodal values and interpolation at given Gauss points which allows any $h/p$-refinement. Several preconditioned sub-structuring methods and domain decomposition preconditioning techniques are proposed. Implementation aspects of these methods are provided in details and validated on two vibro-acoustic problems. Numerical results illustrate the efficiency, robustness and performance of the proposed preconditioning techniques applied to the global coupled problem.

The chapter by K. Meerbergen on theory and numerical methods for eigenvalue problems reviews the eigenvalue problems that arise in the analysis of vibrations. The best known problem is the definite generalized eigenvalue problem of the finite element discretization of the Helmholtz equation, which produces the eigenfrequencies and eigenmodes. For this problem the Lanczos method is a success story and this chapter discusses some aspects of the method without dwelling on implementation details. When damping is present, the Helmholtz equation often becomes a quadratic eigenvalue problem. The solution of this problem is still under investigation but good progress has been made over the last decade. The author shows some particular features of this problem and some differences with the definite generalized eigenvalue problem.

The chapter of X. Antoine proposes a complete overview of the theoretical and numerical developments related to the On-Surface Radiation Condition (OSRC) method in computational acoustic scattering since its introduction in the middle of the 1980s. This asymptotic technique tends to produce some approximate and fast numerical
computations of scattered fields and far-field patterns for large wavenumbers and various boundary conditions. The proposed discussion follows the chronological developments and explains the strengths and future applications of the OSRC for prospecting large scale problems. Numerical treatments are discussed in details for three-dimensional problems using surface finite element methods. Finally, several applications are pointed out such as the construction of artificial boundary conditions and generalized impedance boundary conditions, the development of well-posed and well-conditioned new integral equations for scattering problems as well as preconditioning techniques for integral equations by open structures.

Frequency response computations over a frequency range are relatively expensive because a large number of linear systems has to be solved. The chapter of K. Meerbergen on fast frequency response function computation by model reduction presents the application of Krylov methods for this purpose. They build rational approximations to the frequency response as a function of the frequency. In general, modal superposition often is the fastest method, but the method is not always applicable, for example the case of damped problems or in the presence of infinite elements. These Krylov methods are relatively new to the community concerned with computational acoustics, but they are very reliable and show excellent speed-ups to the direct approach.

The chapter authored by R. Djellouli discusses an original computational methodology for solving inverse acoustic scattering problems. The problem considered here is an inverse obstacle problem where the objective is to determine the shape of an obstacle, or a part of this shape, from the knowledge of some scattered far-field patterns, and assuming certain characteristics of the surface of the obstacle. Although this problem is one of the simplest problem arising in the inverse scattering field, the issues and ideas discussed here are relevant to many applications such as sonar, radar, geophysical exploration, medical imaging and nondestructive testing. The proposed solution methodology is a regularized iterative-based method, that distinguishes itself from similar Newton-type procedures by a sensitivity-based and frequency-aware multi-stage solution strategy, a computationally efficient usage of the exact sensitivities of the far-field pattern to the specified shape parameters, and a numerically scalable domain decomposition method for the fast solution in a frequency band of three-dimensional direct acoustic scattering problems.

The chapter authored by O. von Estorff, M. Markiewicz, and O. Zaleski discusses a number of representative examples where results obtained using the finite element and boundary element computations are compared to measured values. The considered systems are of different complexity levels and include academic as well as rather sophisticated industrial examples. The contribution covers coupled and uncoupled radiation problems, followed by two transmission loss investigations. The major objective is the validation of the models and to show their limitations and accuracy. Moreover, the authors try to share parts of their wide experience with measurements and computation in the fields of acoustics and vibro-acoustics. The chapter is concluded with a number of essential hints and remarks to provide some help for possible
validation activities planned by the reader.

Naturally, the present book cannot provide a complete record of the many approaches, applications, features, and schemes related to computational methods for acoustics problems. However, it does provide a good tutorial on the most important methods used for academic and industrial research. This book will be of interest to engineers, computer scientists and applied mathematicians. The editor wishes to thank the authors for their willingness to contribute to this edited book dedicated to computational methods for acoustics problems.

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