

## **Application of Different Control Techniques to Railway Semi-Active Suspension Systems**

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### **Abstract**

High speed technology may benefit from the acquisition of smart know-how from the automotive and aerospace sectors. In particular in the automotive sector the application of semi-active magneto-rheological (M-R) dampers has been successfully tested. Application of M-R technology and more generally of semi-active system on rail systems is documented and it is still confined to a limited number of known examples. However there is a growing interest for these kinds of solution which represents a good trade-off between the performance of a fully active system and the reliability assured by passive ones.

In this paper some benchmark cases are proposed and used to verify how different control approaches lead to different results both in terms of robustness and performance.

**Keywords:** magneto-rheological dampers, semi-active suspension system, gradient projection method

## **1 Introduction and state of the art**

This work summarizes various research activities [1],[2],[3] performed by researchers of university of Florence as direct and un-direct products of a project named Train-New and proposed and financed by Regione Toscana.

Typical applications of active and semi-active suspension systems are mainly related to high speed railways or to tramways (metros, light rail etc.):

In the first case the research of extreme performances and the remuneration of the railway sector justifies the application of more expensive solutions to the railway sectors.

In the second case, tramways, the particular design of the line, design constraints concerning internal volume distribution and/or geometry of the train and in

particular of bogies and car-body may justify the choice of unconventional solutions and conversely the use of more sophisticated suspension systems.

Currently one of the most diffused and commercially common solution, as an active expedient is the so-called “tilting”, successfully applied on Italian Pendolino Trains and on Swedish X2000 from the 80’. This solution can be only marginally considered as an active suspension system since the correction of the roll angle of the carbody is used to compensate a quasi-static disturbance due to cant deficiency on curves as visible in Figure 1. More recent solution as the one proposed by Sumitomo Heavy industries also introduce the use of pneumatic actuators for a limited tilting[7].

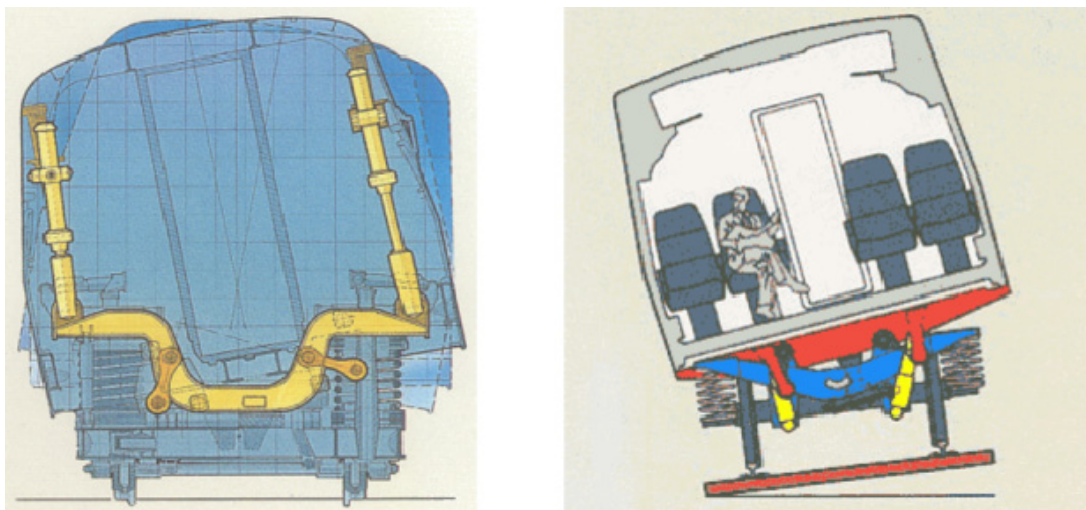


Figure 1: an example of tilting ERT401 (on left) and ETR460 (on right) [4],[5],[6]

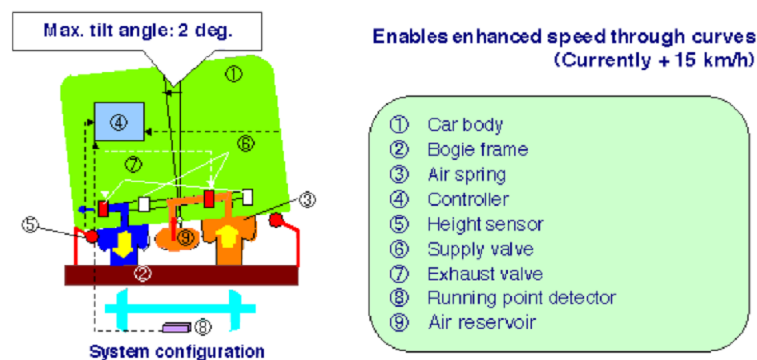


Figure 2: pneumatic tilting proposed by Sumitomo for Shinkansen Trains [7] ,[9]

Pneumatic active solutions as the one proposed by [7] are often integrated with hybrid or semi-active suspension systems based on pneumatic or hydraulic mechanisms [9].

In particular it's possible to cite as the examples the solutions proposed for the Italian ETR500 [8] and for Shinkansen trains [9].

In Figure 3, some details concerning the solution proposed for Shinkansen [9],[7] are shown: a pneumatic actuator and a controlled hydraulic damper are coupled trying to compensate limitations of different technologies:

- Pneumatic actuator is the best solution to assure a controllable contact forces and can be used to compensate low frequency disturbances. Bandwidth and involved energy consumptions are unacceptable for the rejection of higher frequency disturbances (over 1Hz). Also a failsafe behaviour can be easily assured.
- Controllable Hydraulic Damper: incompressibility of oil assures an high frequency behaviour, since the element is passive, a failsafe response and more generally, stability and reliability of the control system are easier to be assured using safety by-pass system which can be obtained with different quite known design criteria.

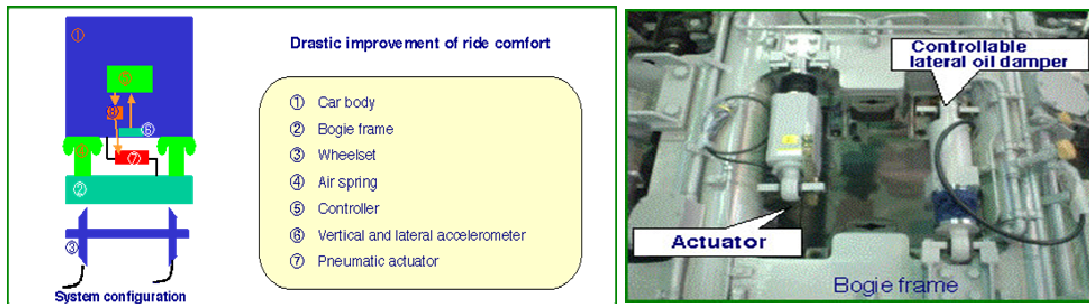


Figure 3: lateral semi-active suspension proposed by Sumitomo for Shinkansen [9]

On the other hand advanced research works from the end of 90' propose very complex control systems with active and semi-active solutions with multiple actuators able to perform the coordinate control of several actuators passive and/or active and controlled degree of freedom aiming to regulate different indexes of performances of bogies concerning steering, comfort and stability, such as the example many studies carried by the UK groups of Loughborough[10],[11] and RTU of Manchester Metropolitan [12] Universities. Also there are examples of very complex prototypes of mechatronic bogies tested in the prospective of the development of reliable industrial products [13],[14].

In this work authors concentrate their attention on the modelling of simpler systems based on the idea that for the short to medium term development of innovative industrial product, attention has to be focused on semi-active systems and in particular on damping/impedance control, which are quite promising technologies in terms of stability reliability and robustness. Also this kind of solutions may be applied on existing or conventionally designed bogies to upgrade or improve their performances. This choice may be winning considering that all the system affecting expensive experimental activities which can be unaffordable considering the specific features of the railway market.

## 2 Proposed benchmark: The secondary vertical suspension stage of a passenger coach

In order to verify the performances of different control strategies, a classical benchmark[15] is proposed for the optimization of the secondary vertical suspension stage of a passenger coach with a 2-2(trailer axles) or B-B(motorized axles) wheelsets as defined by UIC regulations. Main features of the benchmark coach are shown in Table 1 and Table 2 whose data are referred to the bogie scheme of Figure 4.

Dampers of the secondary vertical suspension stage are supposed to be controllable Magneto-Rheological Dampers according the scheme of Figure 4.

Body	Mass (kg)	$I_{xx}$ (kgm <sup>2</sup> )	$I_{yy}$ (kgm <sup>2</sup> )	$I_{zz}$ (kgm <sup>2</sup> )
Carbody	52000	130000	2600000	2500000
Bogie	2900	2500	2000	4000
Wheelset	1600	800	160	800
Axlebox	210	5	15	15
Bolster	1020	650	15	680

Table 1: inertial properties of the bodies

Element	Transl. Stiffness $X(N/m)$	Transl. Stiffness $Y(N/m)$	Transl. Stiffness $Z(N/m)$	Rot. Stiffness $X(N/rad)$	Rot. Stiffness $y(N/rad)$	Rot. Stiffness $z(N/rad)$
Primary Suspension	850000	850000	800000	10500	10500	0
Secondary Suspension	120000	120000	350000	-	-	-
Axlebox sutuco	4000000	6500000	4000000	45000	10000	45000

Table 2: elastic characteristics of the elements

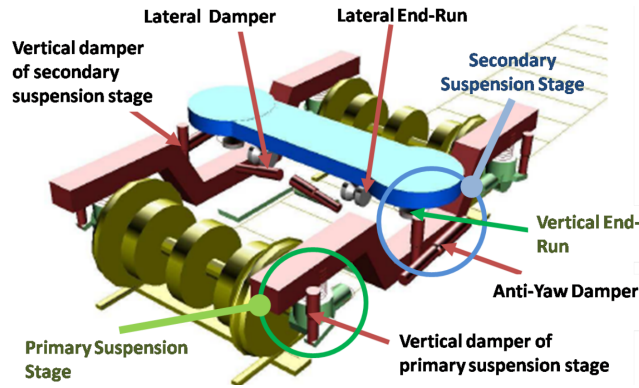


Figure 4: scheme of the benchmark bogie

The coach model is implemented in Matlab-Simulink™ using the multibody library sim-mechanics™[1]. Wheel-rail contact model has been developed in Matlab-Simulink™ during previous research activities [16]. The response of the simulated vehicle is affected by several non-linear effects due to contact geometry, bump stops etc. As a consequence to verify the performances of a regulator is important to apply

to the rail vehicle model a pattern of irregularities able to excite different frequencies and amplitudes in order to verify the response in different operating points corresponding to different system behaviour. For this reason a line with the length of 700m and the disturbance pattern visible in Figure 5 is applied.

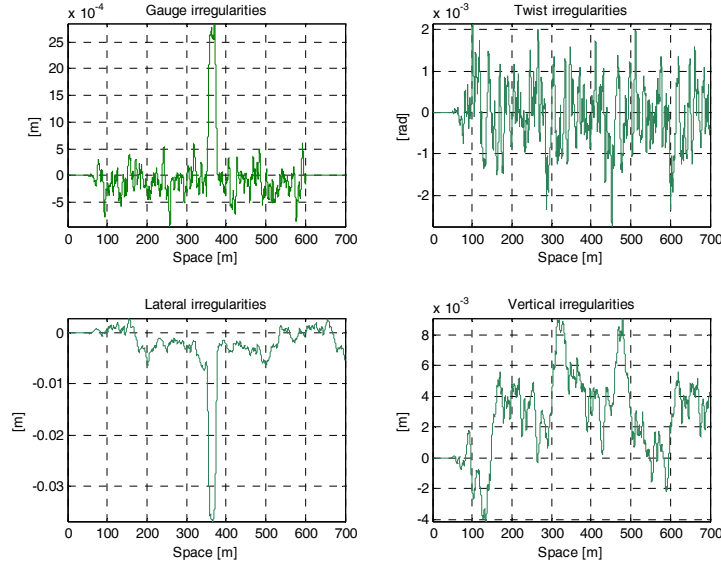


Figure 5: applied irregularity pattern

Simple and easy computable performances indexes are needed to evaluate vehicle response especially when iterative parameter optimization procedures of proposed control layouts are used. For this reason authors propose for their research activities a set of six optimization indexes described in and derived as simplified from criteria partially or totally embraced by known standards/regulations [17].

$$\begin{aligned}
 I_1 &= \sqrt{\text{rms}(a_{1x})^2 + \text{rms}(a_{1y})^2 + \text{rms}(a_{1z})^2} & I_x &= \sqrt{\text{rms}(a_{1x})^2 + \text{rms}(a_{2x})^2 + \text{rms}(a_{3x})^2} \\
 I_2 &= \sqrt{\text{rms}(a_{2x})^2 + \text{rms}(a_{2y})^2 + \text{rms}(a_{2z})^2} & I_y &= \sqrt{\text{rms}(a_{1y})^2 + \text{rms}(a_{2y})^2 + \text{rms}(a_{3y})^2} \\
 I_3 &= \sqrt{\text{rms}(a_{3x})^2 + \text{rms}(a_{3y})^2 + \text{rms}(a_{3z})^2} & I_z &= \sqrt{\text{rms}(a_{1z})^2 + \text{rms}(a_{2z})^2 + \text{rms}(a_{3z})^2}
 \end{aligned} \tag{1}$$

Acceleration measurements  $a_{ij}$  are measured along on three different points defined by the sub-index  $i$ ; the corresponding scalar components along  $x$  (longitudinal),  $y$ (transversal), and  $z$  (vertical) axes are indicated by the sub-index  $j$ . Position of the measurement points is described by the scheme of Figure 6. Measured accelerations are filtered using the same filters proposed by a known standard whose frequency behaviour is described by the filters of Figure 7, Figure 8.

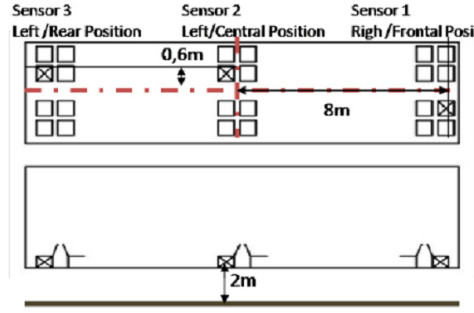


Figure 6: Position of the measurement points used for the evaluation of the comfort indexes (1)

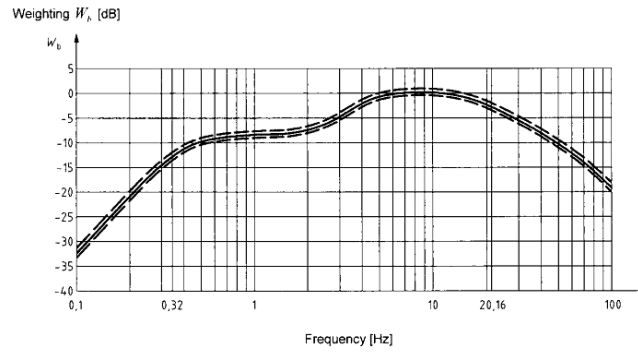


Figure 7: Weighting curve  $W_b$  for vertical acceleration

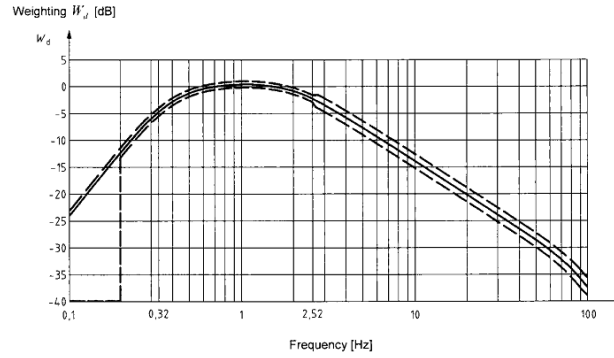


Figure 8: Weighting curve  $W_d$  for longitudinal and lateral accelerations

### 3 Modelling of the actuator

The behaviour of Magneto-Rheological actuators used for secondary suspension systems is simulated using relation (2)

$$F = F_0 I \text{sign}(v) + c_0 v \quad (2)$$

$F$  represents the force exerted by the actuator which is roughly proportional ( $F_0$  is the constant of proportionality) to the current  $I$  used to feed the solenoids which generate the magnetic field that it's used to modify/control the properties of the M-R fluid inside the actuator. A residual viscosity coefficient  $c_0$  is used to model damping which produce a damping force proportional to the actuator speed  $v$  even if no current is applied.

Dynamical Behaviour of the actuator and the saturation of the magnetic field is modelled introducing the transfer function (3) between the current  $I$  and the corresponding reference/desired value  $I_{ref}$ .

$$I = \frac{sat(I_{ref})}{\tau s + 1} \quad (3)$$

$sat$  is the saturation operator ( $I_{ref}$  is saturated to a value  $I_{max}$ )

Since the aim of the model described by (2) and (3) is to describe in a real to realistic way the behaviour of a M-R damper, coefficients are calculated with a best fitting procedure from the simulated behaviour of a complete FEM model of the actuator using Comsol Multiphysics™[1],[2]. Dimensions of the simulated actuator are designed in order to be compatible with the encumbrances available on a railway bogie.

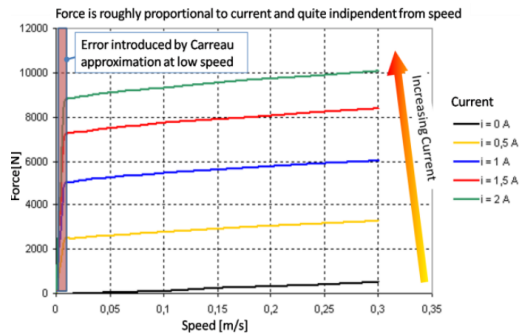


Figure 9: Simulated behaviour of a MR damper through a FEM model[1],[2]

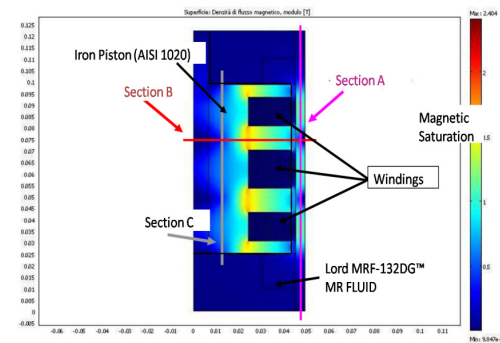


Figure 10: FEM model of a M-R damper[1],[2]

## 4 Tested Control Strategies

In this work, the performances of three different control strategies are evaluated:

- Sky-Hook[18] : each damper works independently on the other one, trying to keep steady the position of the controlled body;
- Dynamic compensation: the dynamical behavior of vehicle car body is estimated using a simplified linear model, namely a simple plate connected to a movable ground by means of spring and M-R dampers that are actuated in order to minimize the dynamic behavior of the system.
- Dynamic Compensation with acceleration feedback: the dynamic compensation is coupled with a PID controller in order to reject the contribution of non-linear

components, such as bump-stops, which cannot be simulated by the online model used by the Dynamic Compensation Strategy [19][20].

## 4.1 SISO Sky-Hook Control

Four M-R dampers are supposed to be applied in parallel to the pneumatic spring of the secondary vertical stage.

Each active vertical damper is controlled independently on the other ones, by exerting a force  $F$  that is directly proportional to the absolute velocity of the carbody measured on the corresponding damper link (4):

$$F = -c_x \dot{x}_c \quad (4)$$

$\dot{x}_c$  carbody speed measured on the corresponding damper attachment

Since a M-R damper is a semi-active element, the force is applied only if the corresponding behaviour is dissipative(5):

$$P = Fv = -c_x \dot{x}_c v < 0 \leftrightarrow \dot{x}_c v > 0; \quad (5)$$

The application of eq.(4-5) involve the measurements of the speed of both the ends of the actuator. In particular in this work it's supposed that speed are estimated by integration from corresponding inertial measurements as follows (6),(7):

$$\dot{x}_c = \int \ddot{x}_c dt \quad (6)$$

$$v = \int (\ddot{x}_c - \ddot{x}_b) dt \quad (7)$$

The sub-indexes  $b$  and  $c$  are used to indicate measurements referred to bogie and carbody frames.

Implementation of the Sky-Hook control, corresponding to equations (4),(5), is quite simple in terms of computational resources, calibration complexity and required hardware.

Also the calibration is quite fast as only the Sky-Hook constant  $c_s$  has to be chosen. Finally, this kind of control strategy ensures a fail-safe behaviour: each damper is controlled independently so a single point failure affect only one of the four actuators. Also the failing actuator works as a damper with a constant viscous constant  $c_0$ .

## 4.2 Dynamic Compensation

In order to improve the comfort of the carbody three degree of freedom should be controlled: vertical motion ( $z_g$  vertical position of carbody center of mass and corresponding derivatives) and the corresponding pitch and yaw rotations corresponding to angles  $\phi$  and  $\theta$ .

The reference performance that an ideal controller would achieve is the minimization of the norm of the acceleration vector described by (8):

$$\ddot{x} = [\ddot{z}_g, \ddot{\phi}, \ddot{\theta}]^T \quad (8)$$



The control output vector  $u=[u_1, u_2, u_3, u_4]^T$  is defined as the vector of the four forces  $u_i$  exerted by the  $i$ -th damper. calculated in order cancel the dynamic behaviour of the carbody.

The vector of the reduced forces  $F_{dc}=[F_g; M_x; M_y]$  applied to the carbody can be calculated from  $u$  through the matrix  $H$  as described by (9).

$$F_{dc} = Hu; \quad H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ b & b & -b & -b \\ a & -a & -a & a \end{bmatrix} \quad (9)$$

Constants  $a$  and  $b$  are the scalar components along longitudinal and transversal distances between the centre of mass of the carbody and the point of application of damper forces.

In order to reject the disturbances  $F_{dc}$  has to compensate the dynamical behaviour due to disturbances which can be calculated by an estimator which have to run in real time.

In this work the estimator is supposed to be a very simple model: the carbody is considered as a rigid body suspended on four lumped springs of constant stiffness  $k$  corresponding to the secondary suspension stage. Also two rotational spring which stiffness  $k_t$  are introduced to simulate the presence of the roll-bars usually installed on coaches to reduce the “*souplesse*” on curves. Disturbances are modelled as a vector of imposed motions  $w$  applied the end of the springs which represents the suspension system. Disturbance vector  $w$  is composed by four components  $w_i$  corresponding to the four imposed displacements imposed to secondary suspension system. From a practical point of view vector  $w$  can be estimated from inertial measurements.

System (10) describes the carbody dynamics and it is used by the estimator to evaluate the dynamical behaviour of the car and consequently the scalar components  $F_g, M_x, M_y$  of the  $F_{dc}$  vector that have to be compensated by the control forces  $u$ .

$$\begin{aligned} m\ddot{z}_g + 4kz_g - k(w_1 + w_2 + w_3 + w_4) &= F_g; \\ I_{xx}\ddot{\phi} + 4kb^2\phi - \left(kb + \frac{k_t}{2b}\right)(w_1 + w_2 - w_3 - w_4) + k_t 2\phi &= M_x; \\ I_{yy}\ddot{\theta} + 4ka^2\theta - ka(w_1 - w_2 - w_3 + w_4) &= M_y; \end{aligned} \quad (10)$$

In order to reject the disturbances estimated by (10) the difference (11) have to be minimized: the set of  $u$  values which satisfies (11) represent the optimal control solutions that have to be applied by actuators.

$$\min_u \|F_{dc} - Hu\|^2 \quad (11)$$

The chosen actuator can only dissipate power and the exerted forces are then limited.

As a consequence the solution of (11) have to be obtained using an algorithm able to solve a constrained minimization problem. In particular in this work, to solve the constrained minimization problem authors propose to use the Gradient Projection Method (GPM) [20] already applied by the research group for different applications. Numerical solutions obtained with algorithms like GPM may calculate a set of  $u$  forces which may compromise the reliability or the life of actuators

(chattering/excessive switching of actuators).

As a consequence to limit an excessive wear of actuators an additional weighting matrix  $\Lambda$  :  $\Lambda$  matrix is diagonal and its elements  $\lambda_{ii}$  represents the weights individually applied to each element of the vector  $u$ .

As a consequence GPM minimization algorithm is used to solve the system (12) instead of the (11) in order to introduce the possibility of limiting  $u$  :

$$\min_u \left[ \|F_{dc} - Hu\|^2 + \|\Lambda u\|^2 \right] \quad (12)$$

### 4.3 Dynamic Compensation with Acceleration Feedback

The dynamic compensation algorithm proposed in the previous section is affected by robustness problem since the simplified model used by the estimator (10) is affected by errors:

- Estimation of disturbance vector  $w$  are affected noise and disturbances that necessary produce errors in the estimator (10)
- Estimation (10) is performed using a simplified model of the vehicle which is necessary approximated respect both to the real one or to the complete multibody model used to simulate the vehicle.

As frequently done on sliding mode regulators a feedback term evaluated through a PI (Proportional Integrator) regulator is introduced. A null acceleration vector of the three Lagrangian coordinates is assumed to be the optimal performance, so a feedback force  $F_{re}$  (14) aiming at the minimization of the error vector  $e$  defined according (13) is introduced:

$$e = \ddot{x} = [\ddot{z}_g, \ddot{\phi}, \ddot{\theta}]^T \quad (13)$$

$$F_{re} = K_p^T e + K_i^T \int e = \begin{bmatrix} k_{pz} \ddot{z}_g + k_{iz} \dot{z}_g \\ k_{p\phi} \ddot{\phi} + k_{i\phi} \dot{\phi} \\ k_{p\theta} \ddot{\theta} + k_{i\theta} \dot{\theta} \end{bmatrix} \quad (14)$$

The calculated control force  $F_{tot}$  is evaluated as the sum (15) of two contribution  $F_{re}$  and  $F_{dc}$  which is calculated according the estimator (10):

$$F_{tot} = F_{re} + F_{dc} \quad (15)$$

The GPM algorithm is then used to find a solution  $u$  which minimize the system (16):

$$\min_u \left[ \|F_{tot} - Hu\|^2 + \|\Lambda u\|^2 \right] \quad (16)$$

The corresponding control scheme is shown in Figure 11.

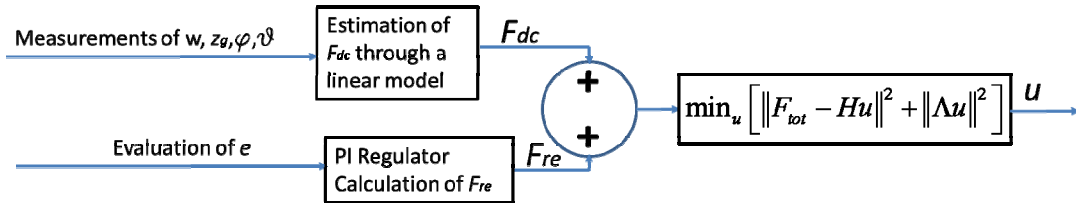


Figure 11: control scheme corresponding to the proposed dynamic compensation with acceleration feedback

## 5 Simulation Results

In order to compare the results of the control strategies described in the section 4 of this work simulation are performed using the multibody model previously described in section 2. In particular a reference performance is calculated performing the simulation of the benchmark vehicle equipped with passive dampers without any control system applied. Tests are then repeated on the same track with irregularity patterns at different speeds.

In particular the index  $I_z$  obtained are considered the most important results in order to evaluate the performances in terms of vertical comfort.

In Table 3 and in Figure 12, same results concerning the index  $I_z$  defined in (1) are shown: test are repeated considering different control configurations and traveling speed: all the proposed control techniques improve the comfort if compared with the passive configuration; however, the dynamic compensation-based approach is negatively affected by the robustness of plant estimator since the modified version with an added feedback terms shows much better performances.

Performance improvement is higher for the fastest test run, since higher speed involves higher amplitude/higher spectral content disturbances that can be more easily rejected by a damping-based actuation system.

<i>Speed km/h</i>	<i>Passive</i>	<i>skyhook</i>	<i>Dyn. Comp.</i>	<i>Dyn.Comp. with feedback</i>
140	0.136 (100%)	0.122 (89.7%)	0.12 (88.7%)	0.079 (58.1%)
200	0.204 (100%)	0.187 (91.7%)	0.190 (93.2%)	0.104 (50.8%)
250	0.276 (100%)	0.230 (83.4%)	0.221 (80.2%)	139 (50,2%)

Table 3: performances in terms of  $I_z$

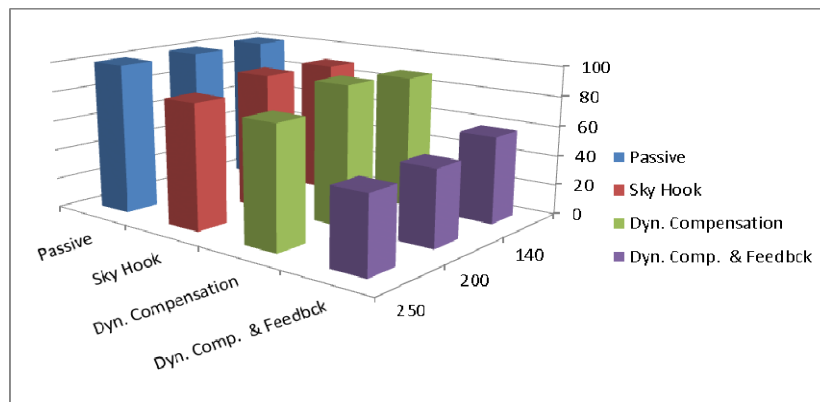


Figure 12: calculated  $I_z$  values (relative values respect to the passive solution) with different speed and control layout

Calculated performance indexes defined by (1) considering different control configuration and traveling speed are shown in Table 4, Table 5, Table 6, and Table 7: all the proposed control strategies involve comfort benefit in all the three sensor locations, damping all the three comfort-related vibrations of the carbody, such as vertical translation and roll and pitch rotations. Lateral and longitudinal motions (indexes  $I_y$  and  $I_x$ ) are less affected by different control strategies since only the vertical dampers of the secondary suspension stage are actuated.

<i>Speed km/h</i>	$I_1$	$I_2$	$I_3$	$I_x$	$I_y$	$I_z$
140	0.177	0.105	0.221	0.0144	0.108	0.136
200	0.281	0.171	0.314	0.0229	0.164	0.204
250	0.296	0.251	0.385	0.0262	0.150	0.276

Table 4: perf. indexes for passive reference configuration

<i>Speed km/h</i>	$I_1$	$I_2$	$I_3$	$I_x$	$I_y$	$I_z$
140	0.167	0.102	0.206	0.0139	0.108	0.122
200	0.260	0.174	0.290	0.0224	0.158	0.187
250	0.256	0.221	0.326	0.0256	0.141	0.230

Table 5: perf. indexes calculated for skyhook control

<i>Speed km/h</i>	$I_1$	$I_2$	$I_3$	$I_x$	$I_y$	$I_z$
140	0.170	0.0984	0.197	0.0139	0.105	0.121
200	0.260	0.179	0.290	0.0222	0.156	0.190
250	0.250	0.214	0.325	0.0260	0.142	0.222

Table 6: perf. indexes calculated for Dynamic Compensation

<i>Speed km/h</i>	$I_1$	$I_2$	$I_3$	$I_x$	$I_y$	$I_z$
140	0.113	0.0745	0.143	0.0127	0.0811	0.079
200	0.134	0.0960	0.151	0.0170	0.0750	0.104
250	0.179	0.141	0.199	0.0230	0.104	0.139

Table 7: perf. indexes for Mod. Dyn. Comp. (feedback added)

Performances have been also compared in terms of spectral composition of car-body accelerations: in Figure 13, the spectral composition of the vertical accelerations measured on the center-left sensor according the layout Figure 6.

The analysis range is extended up to 30Hz, over all the observed range the spectral component of measured accelerations are always far below the corresponding performance of the reference passive system.

In terms of robustness and reliability of the control, this behaviour is very attractive since there is no-noise regeneration over a wide range of frequencies. This feature is quite useful in terms of system stability, as it involves a lower sensitivity to undamped mechanical modes of the system which may be excited by the action of actuators.

Higher performances are reached by the “modified dynamic compensation with acceleration feedback” since the added PI acceleration feedback is able to reject the disturbances due to modelling errors between the real plant and the simplified estimation model used to estimate the dynamical behaviour of the system.

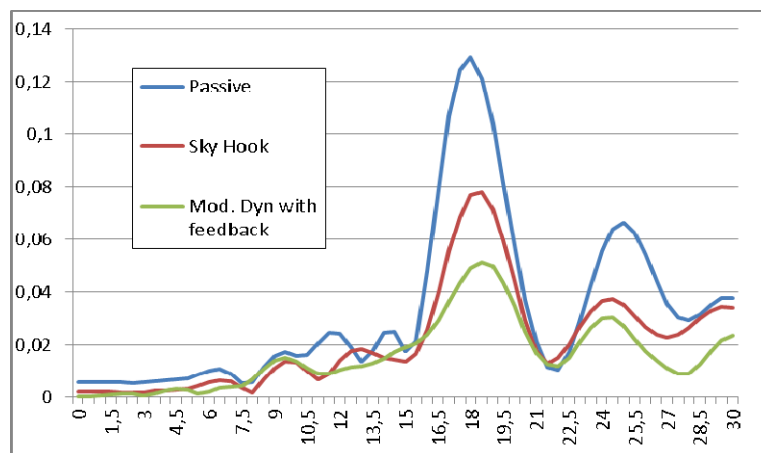


Figure 13: spectral composition of vertical acceleration [ $\text{m/s}^2$ ] measured on center left sensor (according the layout of Figure 6) as a function of frequency [Hz]

Performance of Sky-Hook control are lower compared respect to centralized control strategies. However the implementation is much simpler.

Also the robustness of the Sky-hook controller is much higher respect to centralized control strategies especially when the modelling errors between estimator and real controlled plant (the vehicle) are higher.

Also the behaviour of the Sky-Hook control is very robust in case of failure for example of one of the actuators. The failure is reproduced by cutting out the corresponding command signal. In Figure 14 some results are shown: in particular in Figure 14 the spectral composition of vertical acceleration measured on center-left sensor (according the layout of Figure 6) is shown: the observed performances are degraded exhibiting an higher rolling motion of carbody; In particular the unbalanced application of forces due to the unavailability of a damper (simulated fault) causes a cross/mutual excitation of roll and pitch modes of the secondary stage suspensions; however the behavior is still smooth and the performances are better than in the passive reference simulation.

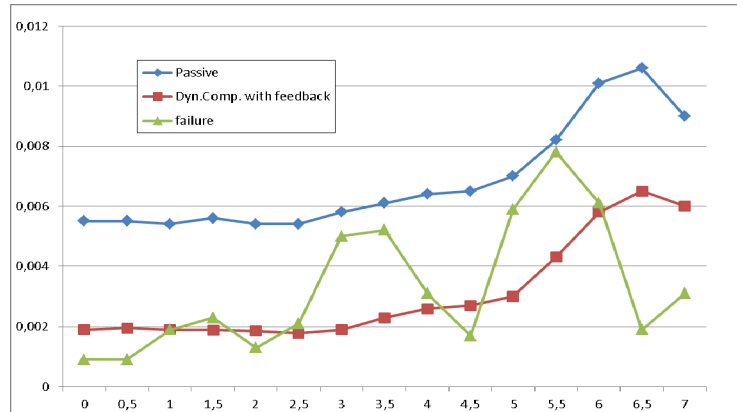


Figure 14: spectral composition of vertical acceleration[m/s<sup>2</sup>] measured on center-left sensor (according the layout of Figure 6) considering different working conditions as a function of frequency[Hz]

## 6 Conclusions

In this paper, the application of controlled dampers to the secondary suspension stage of a railway vehicle has been used as benchmark to verify the performances of different control techniques. Centralized control system seems to be more performing especially when different control approaches are applied in a synergic way. On the other hand SISO controllers such as the Sky-Hook damper are much simpler to implement and exhibit a very robust behaviour.. In both cases the achieved results are quite attractive.

Currently authors are evaluating to further tests robustness and performance both with more sophisticated models (considering vehicle car-body flexibility and more sophisticated models of actuators and sensors) and experimental activities in the MDM laboratories of Pistoia, Italy.

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