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# Meshless simulation of infinitesimal elastoplastic deformation of a 3D body

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## Abstract

Modelling elastoplasticity is a non-linear problem which requires multi-step iterative solving. Traditionally such problems are solved with the finite elements method (FEM), which is also often featured in commercially available software for modelling solid mechanics. The accuracy of FEM solutions depends on the quality of the mesh, which can become computationally expensive when discretizing complex domains. Meshless methods do not require meshing, and can alternatively be used to solve diverse solid-mechanics problems. In this work we present our implementation of elastoplasticity into our C++ library for solving partial differential equations, which is based on the use of meshless methods – Medusa. The implementation is tested on a simple 3D von Mises elastoplasticity case, and results are compared against the solution of the same case in Abaqus. Results of both approaches are in good agreement with each other.

**Keywords:** elastoplasticity, meshless, WLS, von Mises plasticity model.

## 1 Introduction

Modelling solid mechanics is of interest in structural, mechanical, and other engineering branches. Without dipping into rheology, we can roughly categorize the solid materials into three groups, depending on the behaviour after the external load applied had been released [1,2]. If the body returns to its original shape, it is considered to be elastic. However, under sufficiently large loads, the body may not return to its original shape. In such cases it is considered to be plastic or elastoplastic if only part of the domain had undergone plastic deformation. Commonly, solid mechanics models are solved with the finite element method (FEM) [3]. However, meshing the computational domain, as is required by the FEM, can represent a significant burden when complex 3D domains are of interest. Meshless computational methods, however, do not suffer from this drawback. Some applications of meshless methods solving elasticity problems can be found in the literature [4,5]. Plasticity problems have also been solved with meshless methods previously [6,7].

In this work we use the meshless weighed least squares (WLS) approach to solve a 3D case of elastoplastic deformation. WLS is utilized through our in-house developed Medusa C++ library [8].

## 2 Methods

Modelling the deformation of solids follows the Navier-Cauchy equation [1,2]. However, as it is characteristic of plastic materials to not return to their original shape, this must also be accounted for in the physical model, i.e., when the stresses present in the solid body are higher than the yield stress, the solution of the Navier-Cauchy needs to be corrected, as the original solution violates the material properties. The threshold for violation of physical properties, also known as yielding criterion, can be determined by different empirical models [1,2]. In this work we utilize the von Mises plasticity model, as it is described in [1].

To obtain a solution, we apply the external load to the object in several steps – we apply partial loads. The solution procedure is then as follows [1]:

1. Solve the Navier-Cauchy equation with the partial load applied, assuming elastic behaviour.
2. Check the von Mises stress at each point. If it exceeds the yield stress, perform the local iterative correction.
3. The local correction is done by solving the return-mapping of the von Mises model by e.g., the Newton-Raphson method [9].
4. Compute force residual in every point of the object. If it exceeds the prescribed tolerance, perform the global iterative correction.
5. The global correction is done by adding a force residual in place of external forces to the Navier-Cauchy equation and solving it.
6. Continue at 2, until force residual is sufficiently small, and the von Mises yield criterion is not violated anywhere in the domain.

The above solution procedure has been implemented in the C++ environment where meshless methods have been employed to solve the governing system of partial differential equations. In the context of partial differential equations, the linear differential operators are approximated over a set of neighbouring nodes also referred to as stencil nodes [10]. The form of the approximation is very desirable as it can be obtained with a simple dot product between the yet to be defined weights and nodal values. The weights are obtained for a chosen set of basis functions. We used Gaussian basis functions and Gaussian WLS weights with $\sigma=2$ to increase the importance of stencil nodes further away from the central node. The stencil consisted of 200 nodes as this option proved most stable in our testing.

## 3 Results

The von Mises model was applied to a case of a 10 mm × 10 mm × 10 mm cube, consisting of a material with arbitrarily chosen properties: Young's modulus 10 GPa, Poisson's ration 0.4, and the yield function consisting of the following points (equivalent plastic strain, yield stress): (0, 20 MPa), (0.001, 25 MPa), (0.005, 30 MPa), (0.02, 40 MPa) [7]. This cube was fixed in place (displacement is 0) at z = 0, and a displacement of 0.05 mm in x-direction was applied to it at z = 10 mm, with all other faces traction free. A sketch of the domain, and its displaced solution is shown in Fig. 1.

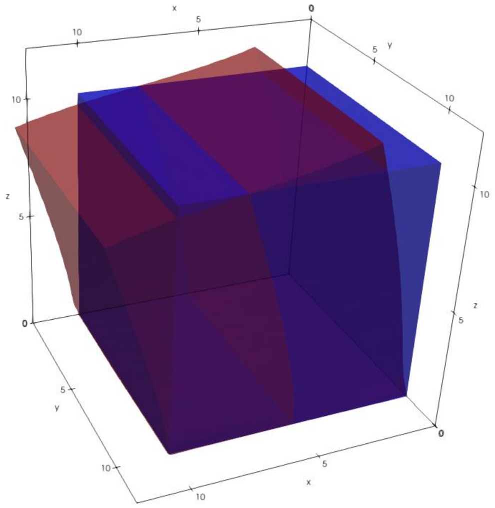


Figure 1: Sketch of the domain (blue), and the solution (red). The solution's displacement is multiplied by factor 50 for clarity.

The domain was discretized using a dedicated node positioning algorithm [11,12] implemented in Medusa. We used discretizations of different densities, producing systems with 657, 1333, 2339, 3787, 5669, 8155, and 11283 computational points. An example of the discretization is shown in Fig. 2.

The computations were performed by applying 10 partial loads to the body.

In addition to the meshless solution, we solve the same problem with a commercial FEM solver Abaqus. This traditionally used numerical solver allows us to evaluate the quality of the numerical solution obtained by mesh-free method. The model implemented in Abaqus was meshed with 8000 linear hexahedral elements of type C3D8R.

Results of computations are shown in Figs. 3, and 4. The pictured results represent the values of variables along the undeformed body diagonal (0,0,0) → (10,10,10).

The lines representing meshless solutions in Figs. 3, and 4 were obtained by Sheppard interpolation of values of nearest 9 points. Note that this interpolation introduces an additional source of error when comparing the results. WLS results converge towards the Abaqus solution, but even at higher discretization densities there are significant differences in stress tensor components at the boundary points. As the results are extracted from corner node, to corner node, this is somewhat expected, as singularities in corners were not given any special treatment in the WLS approximation method.

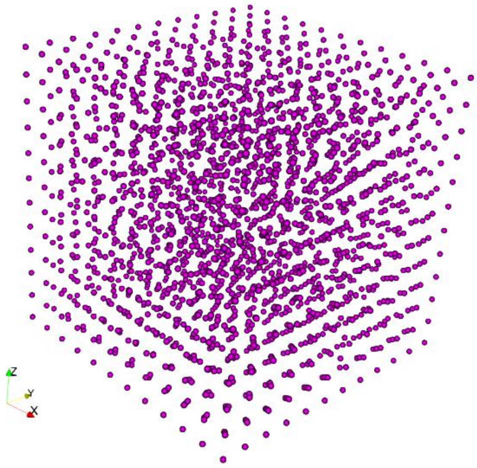


Figure 2: Discretized cubic domain with 2339 points.

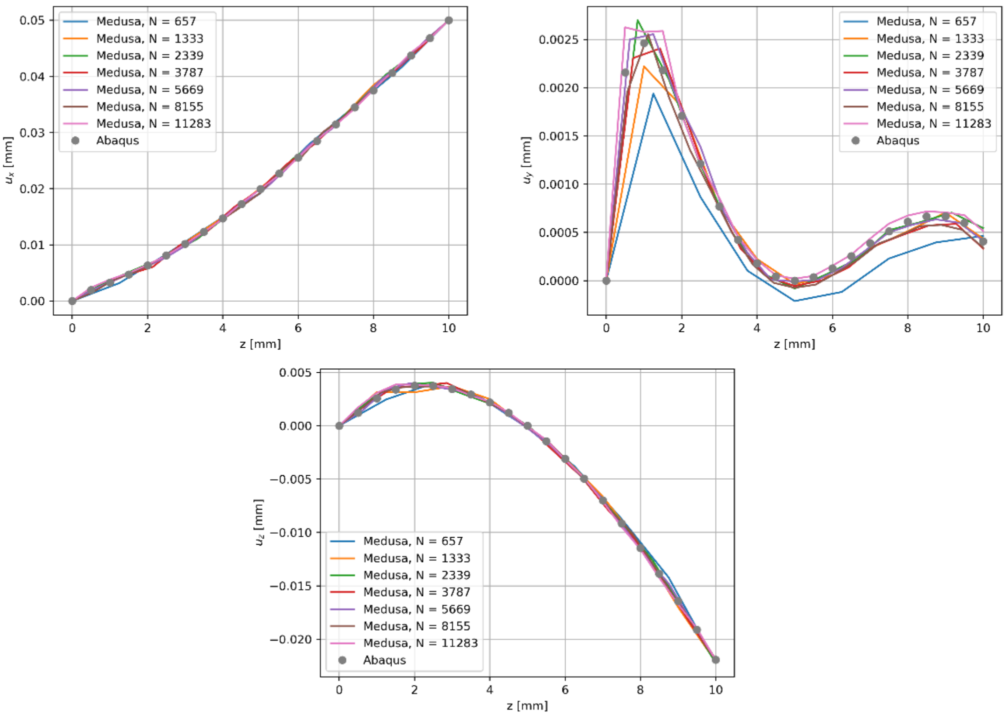


Figure 3: Results of meshless WLS (Medusa) computations (solid lines) vs. Abaqus (grey dots). $N$ is the number of discrete points in the domain. Displayed are the three components of displacement $\boldsymbol{u}$, clockwise starting from top left: $u_x$, $u_y$, $u_z$.

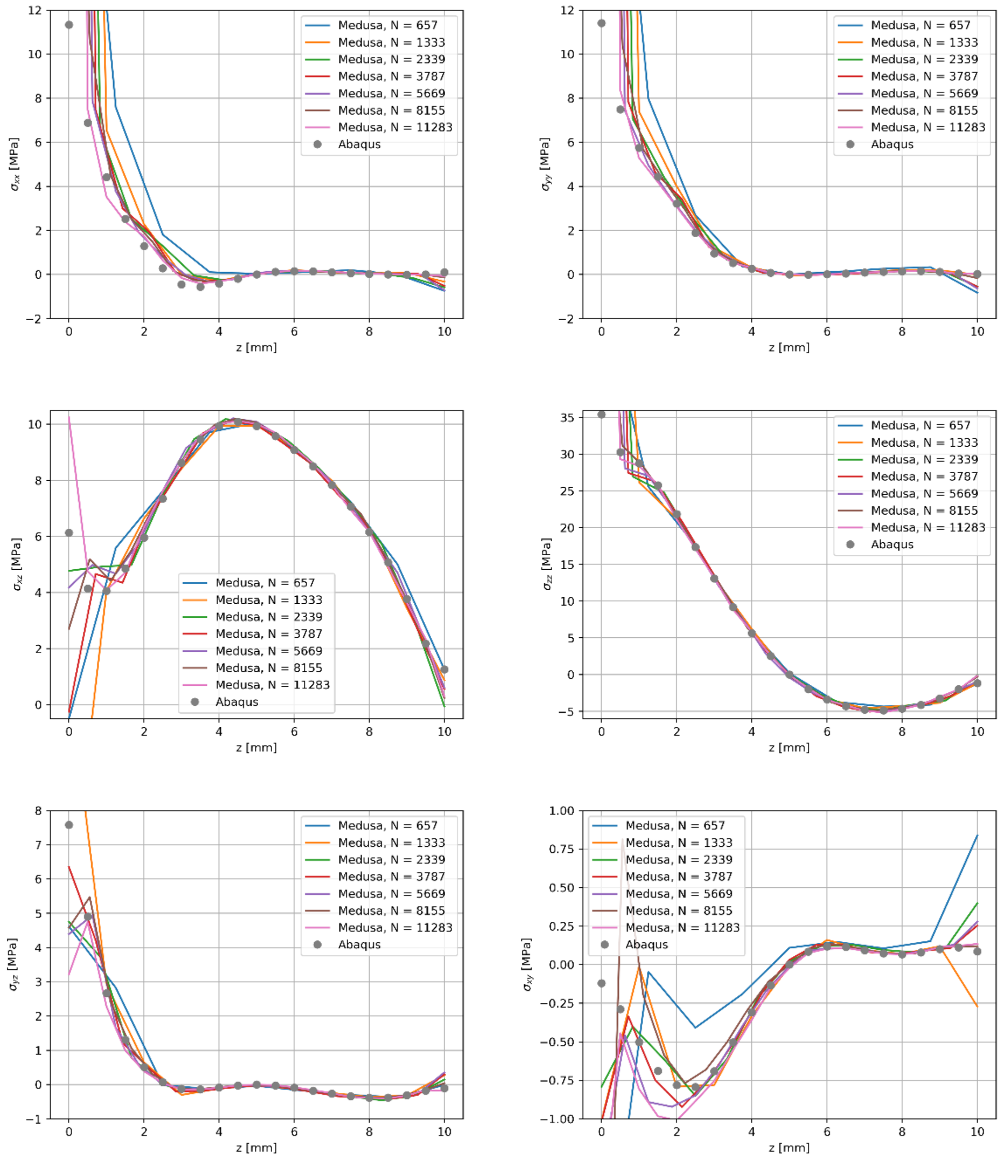


Figure 4: Results of meshless WLS (Medusa) computations (solid lines) vs. Abaqus (grey dots). $N$ is the number of discrete points in the domain. Displayed are the six components of the stress tensor σ, clockwise starting from top left: $\sigma_{xx}$, $\sigma_{yy}$, $\sigma_{zz}$, $\sigma_{xy}$, $\sigma_{yz}$, $\sigma_{xz}$.

## 4 Conclusions and Contributions

In this paper we have shown the application of meshless methods for plasticity problems using a WLS scheme. For comparison we have also provided a solution of the identical case obtained with a commercially available FEM solver Abaqus. We show that the meshless solution converges towards the FEM solution, albeit with some discrepancies in stress tensor at domain corners where the stress tensor components diverge. This simple application of WLS for solving plasticity problems serves merely as a demonstration of the possibilities of our Medusa library, and meshless methods, although their use becomes especially advantageous when dealing with complex domains.

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