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Quasi-Periodic Galloping Response under Combined Random and Harmonic Excitations at a Large Frequency Detuning

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Abstract

Aero-elastic processes at large slender engineering structures are closely related with nonlinear interaction of a stream and a vibrating structure. Although more sophisticated models can be adopted, a commonly used single-degree-of-freedom (SDOF) system represents a reasonable compromise between accuracy and simplicity. Experiments in a wind tunnel show that the regime of the vortex shedding is typical by quasi-periodic beatings that are encountered in the lock-in regimes. Here the vortex shedding frequency ω_s becomes close to the SDOF eigen-frequency ω_0 with a small positive or negative detuning $\Delta \approx |\omega_0 - \omega_s|$.
Experimental and also theoretical investigations indicate three regimes can be encountered that in the lock-in area, provided a combined deterministic (harmonic) and random excitation is applied: (i) $0 < \Delta < \Delta_l$ (small detuning): the response falls into synchronization and no beating effect occurs; (ii) $\Delta_l < \Delta < \Delta_u$: a quasi-periodic response of the SDOF system emerges consisting of self-excited and forced components; (iii) $\Delta > \Delta_u$: self-excited oscillations do not occur and only (nearly) mono-harmonic forced vibration can be observed.
The work proved an existence of a frequency detuning interval where the system response has a quasi-periodic character. The main difference from conventional approaches consists in a possibility that the random excitation component can change qualitatively the response portrait. The detuning interval, where the quasi-periodic response occurs, seems to be larger than in the deterministic case. On the other hand, the variability of the response amplitude within one quasi-period is not so dramatic as in the purely deterministic case. The particular form of the excitation spectral density

is important and depends predominantly on its value in frequency which coincides with the eigen-frequency of the adjacent linear system and its integer multiples.

Keywords: auto-parametric response, flow-induced vibration, van der Pol equation, vortex shedding, random load, combination of deterministic and stochastic excitation, Galerkin-Petrov method, Fokker-Planck equation.

1 Introduction

Aero-elastic processes at large slender engineering structures, e.g., bridge decks, towers, masts, high rise buildings, ropes, are closely related with nonlinear interaction of a stream and a vibrating structure, see e.g. Pirner, [1]. Hence, the motivation of the paper originates from effects related with vortex shedding, see for instance Pospíšil et al., [2]. Although more sophisticated models can be adopted, a commonly used SDOF system represents a reasonable compromise between accuracy and simplicity, see Figure 1.

Experiments in a wind tunnel show that the regime of the vortex shedding is typical by a quasi-periodic beatings that are encountered in the lock-in regimes. Here the vortex frequency ω_s becomes close to the SDOF eigen-frequency ω_0 with a small positive or negative detuning $\Delta \approx |\omega_0 - \omega_s|$. The deterministic problem for various values of the detuning was investigated for special settings in [3, 4, 5] and later an investigation of a general formulation was published by the authors, [6].

Experimental and also theoretical investigations indicate that in the lock-in area three regimes can be encountered, provided a combined deterministic (harmonic) and random excitation is applied:

(i) for a very small detuning $0 < \Delta < \Delta_l$, the response falls into synchronization and no beating effect occurs; this case was discussed by the autors the last year at the CSEEC 2019, [7];

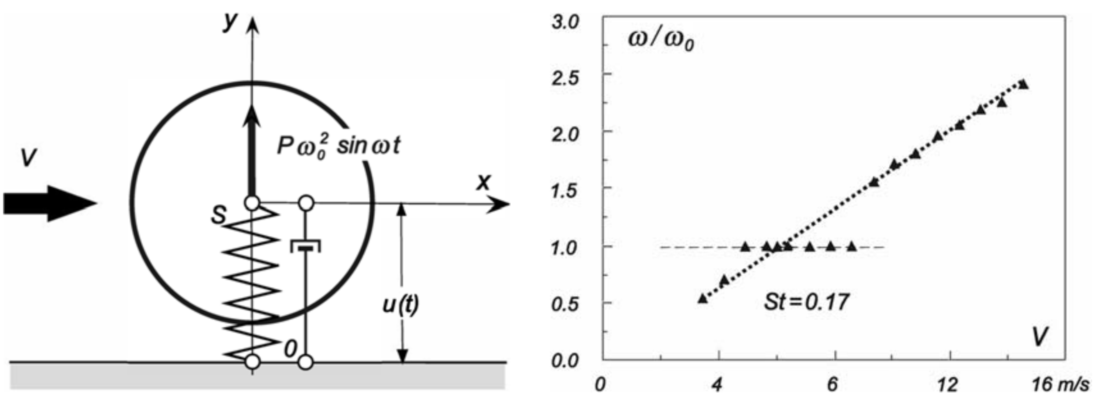


Figure 1: SDOF system outline and vortex shedding frequency in the lock-in domain as a function of the flow velocity.

(ii) the detuning lies in the interval $\Delta_l < \Delta < \Delta_u$ and a quasi-periodic response of the SDOF system emerges. The response consists of two components: self- excited vibrations in post-critical nonlinear state with the frequency ω_0 and forced vibration due to vortex shedding in a slightly different frequency ω_s. Both response components combine together which results in a beating effect. Some inspiration comes from the deterministic case see, e.g., Stankevich [5] or paper by the authors, [6].

(iii) The detuning is higher than the upper limit of the lock-in interval $\Delta > \Delta_u$. Self-excited oscillation does not occur and only (nearly)mono-harmonic forced vibration can be observed.

In general, the response in all three regimes includes deterministic and random components due to the fact that also the external excitation consists basically of a harmonic component (vortex shedding) and a random part originating from a turbulence generated by an interaction of the stream and the moving body.

2 Methods

From the theoretical viewpoint is the problem defined by a strongly nonlinear SDOF oscillator with an additive excitation combining deterministic and random components. Despite of simplification by an SDOF system, the process of a nonlinear aeroelastic interaction is still handled by a number of parameters which make it non-transparent for the first view.

$$\begin{aligned} \dot{u} &= v, \\ \dot{v} &= (\eta - \nu u^2)v - \omega_0^2 u - P\omega^2 \cos\omega t + h \cdot \xi(t). \end{aligned} \quad (1)$$

u, v — displacement *[m]* and velocity, *[ms⁻¹]*;

η, ν — parameters of the damping: *[s⁻¹,s⁻¹m⁻²]*; functions of the stream velocity;

ω_0, ω_s — eigen-frequency of the adjoint linear SDOF system, frequency of the vortex shedding *[s⁻¹]*;

$P\omega^2, \xi(t)$ — amplitude of the harmonic excitation force *[ms⁻²]*, broadband Gaussian random process *[ms⁻²]*;

h — multiplicative constant [*ms⁻²*].

The physical specification of the problem enables to formulate solution in the form:

$$\begin{aligned} u(t) &= \quad a_c \cdot \cos\omega t + a_s \cdot \sin\omega t, \\ \dot{u}(t) &= -a_c\omega \sin\omega t + a_s\omega \cos\omega t, \end{aligned} \quad (2)$$

which provides the SDE system for slowly variable amplitudes a_c, a_s

$$
\begin{aligned}
\dot{a}_c =& \frac{\omega_0^2 - \omega^2}{\omega} \sin\omega t(a_c \cos\omega t + a_s \sin\omega t) \\
& - P\omega \sin\omega t \cos\omega t - \frac{h}{\omega} \sin\omega t \cdot \xi(t) \\
& - \sin\omega t \left(\eta - \nu(a_c \cos\omega t + a_s \sin\omega t)^2\right) (-a_c \sin\omega t + a_s \cos\omega t), \\
\dot{a}_s =& - \frac{\omega_0^2 - \omega^2}{\omega} \cos\omega t(a_c \cos\omega t + a_s \sin\omega t) \\
& + P\omega \cos\omega t \cos\omega t + \frac{h}{\omega} \cos\omega t \cdot \xi(t) \\
& + \cos\omega t \left(\eta - \nu(a_c \cos\omega t + a_s \sin\omega t)^2\right) (-a_c \sin\omega t + a_s \cos\omega t).
\end{aligned}
\tag{3}
$$

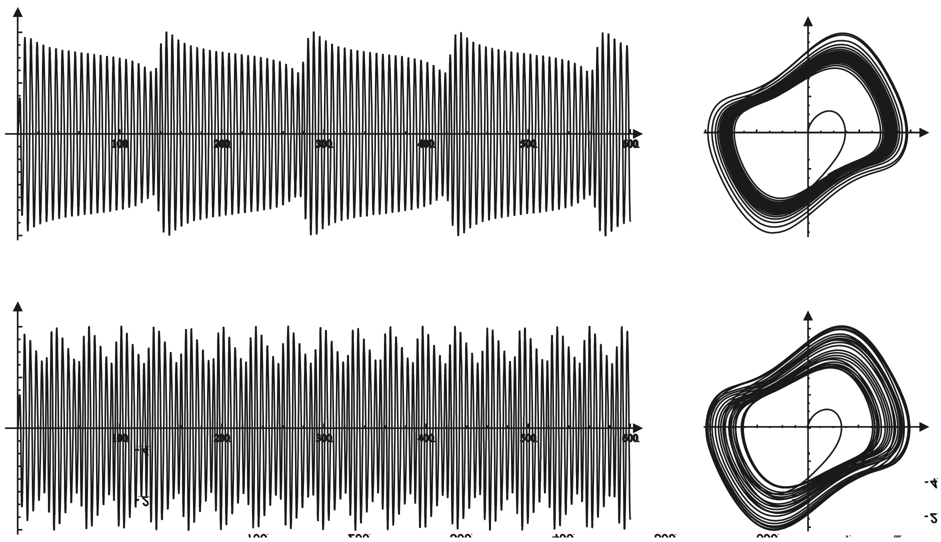
SDE (3) was investigated by the authors using the Fokker-Planck equation by means of stochastic averaging method and results were submitted for publication, [7]. This procedure enabled to identify many new phenomena ruling in the regime of a small detuning when $0 < \Delta < \Delta_l$.

However, the stochastic averaging is not applicable in the interval $\Delta_l < \Delta < \Delta_u$. The reason is that the averaging operation eliminates the time variable and FPE loses dependence on time in a "macroscopic" meaning. Amplitudes ac; as in further analysis represent constants. In order to describe the beating effect and the fully time dependent history within one quasi-period, it is necessary to keep the time dependence of FPE. Hence, the FPE solution should be conducted by means of the Galerkin-Petrov method applied on the series of stochastic moments.

3 Results

Performing this transformation, one obtains a system of ordinary differential equations (ODE) in the deterministic form keeping the time dependence, which follows from the deterministic part of excitation. It is included in the drift coefficient of the FPE. The ODE system for stochastic moments is non-linear and it is written in the normal form. The excitation consists of two parts, which pass to the FPE from the right-hand side of Equations (3) and subsequently to the ODE system. They are: (i) the random part of excitation (influence of a turbulence component) which appears as a certain constant resulting from the stochastic characteristics of the input process, in particular from the value of its spectral density in points (ω, 2ω, ... etc.); and (ii) the deterministic part of excitation (vortex shedding itself), which passes into the ODE system in the form of a parametric excitation.

This ODE structure and excitation setting indicates that a solution with a strong periodical (or quasi-periodical) mean value (zero stochastic moment) representing the deterministic part of the response can be expected and, moreover, that the dynamic stability in probability (DSP) of the response is worthy to be examined. The DSP can be tested in a certain limited meaning of the term because only limited number of stochastic moments are available. However, if moments reveal to be convergent, the response stability in probability can be adopted.



(a) (b)

Figure 2: Sample of a system response: (a) time history, (b) trajectory in Poincaré diagram; detuning: upper row Δ = 0.20, lower row Δ = 0.30

To investigate a quasi-periodic response and limits of detuning Δ_l, Δ_u, which determine presence of quasi-periodic beatings, the numerical simulation of the ODE system was multiply performed for various parameter settings. The subsequent evaluation of numerical results shows the existence of a quasi-periodic response for combined excitation consisting of deterministic and random (mono-harmonic) components. It is obvious that the quasi-period length approaches an infinite value for $\Delta = \Delta_l$. For increasing detuning $\Delta_l < \Delta < \Delta_u$ the quasi-period length is getting shorter, see Figure 2 demonstrating the response shape presented by the zero moment (mathematical mean value).

In the next step, a more objective way of the stability testing, period length, its internal shape and other parameters will be attempted by means of the Floquet formalism applied to an ODE system with periodic coefficients.

4 Conclusions and Contributions

In general, it seems that the basic tendencies determined for the deterministic problem remain qualitatively in force. The work proved the existence of a frequency detuning interval in which the system response has a quasi-periodic character. The main difference from conventional approaches consists in a possibility that the random excitation component can change qualitatively the response portrait. The detuning interval, where the quasi-periodic response occurs, seems to be larger than in the case when only a deterministic excitation is assumed. While the value of Δ_l is basically the same as in the deterministic case, Δ_u is much higher. On the other hand, the variability of the response amplitude within a single quasi-period is not as dramatic as in the purely deterministic case. The particular form of the excitation spectral density is important, which depends predominantly on its value in frequency that coincides with the eigen-frequency of the adjacent linear system and its integer multiples. At the boundary points of the detuning interval (Δ_l, Δ_u), where the quasi-periodic response character vanishes, a good continuity with stationary solution types was observed.

Some more effects and properties of the system have remained hidden so far, because the ODE system has only been investigated numerically. Semi-analytical investigation of this system by means of the Floquet theoretical background and other strategies appears very promising for the recognition of additional properties. These steps are planned in the near future.

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