Proceedings of the Sixth International Conference on
Soft Computing, Machine Learning and Optimisation in
Civil, Structural and Environmental Engineering
Edited by: P. Iványi, J. Lógó and B.H.V. Topping
Civil-Comp Conferences, Volume 5, Paper 1.9
Civil-Comp Press, Edinburgh, United Kingdom, 2023
doi: 10.4203/ccc.5.1.9


# Optimization of bowstring tied-arch concrete bridges

**A.M.B. Martins[1], L.M.C. Simões[1] and J.H.J.O. Negrão[2]**

**[1]University of Coimbra, ADAI, Civil Engineering Department, Portugal**
**[2]University of Coimbra, Civil Engineering Department, Portugal**

## Abstract

This paper presents an optimization-based approach for the design of bowstring tied-arch concrete bridges. This is composed by a convex optimization algorithm combined with a multi-start procedure to obtain local optimum solutions and the best of which is selected as the optimum design. The finite element method is used for the three-dimensional analysis considering dead and road traffic live loads, geometrical nonlinearities and time-dependent effects. The design is formulated as a multi-objective optimization problem with objectives of minimum cost, deflections and stresses considering service and strength criteria defined according to the Eurocodes provisions. This minimax problem is solved indirectly by the minimization of a convex scalar function obtained through an entropy-based approach. The discrete direct method is used for sensitivity analysis. The design variables are the arch and deck sizes, the hangers and tendons cross-sectional areas and prestressing forces, and the arch rise. The optimization of a 120 m span bridge illustrates the features and applicability of the proposed approach. Minimum cost solutions are obtained featuring a balance between the arch and deck stiffness, and the suspension effect provided by the hangers. The optimum solution features a deck slenderness of 1/120 and an arch rise-to-span ratio of 1/5.3.

**Keywords:** optimization, bowstring, tied-arch, bridges, concrete, prestressing.

## 1 Introduction

Bowstring tied-arch bridges are a type of arch bridges with the deck working from below. The acting vertical loads on the deck are transmitted to the arch by hangers which provide the deck with a continuous support. Unlike other arch bridges, in which the horizontal component of the support reaction of the arch is transmitted to the foundations, in these bridges this horizontal force is taken by the deck, working as a tie member together with its beam behaviour. Therefore, the structure becomes self-equilibrated and the arch transmits only vertical support reactions to the foundations. These bridges constitute an efficient and aesthetically appealing solution to spans up to 300 m. This structural solution was adopted by several designers for road bridges, railway bridges and footbridges using prestressed concrete, steel or steel-concrete composite solutions for the deck. The arches are usually made up of concrete or steel [1, 2, 3].

The design of bowstring tied-arch bridges may be a challenging task aiming at an appropriate balance between the stiffness of the arch and deck, and the suspension effect provided by the hangers. An efficient design depends on a large number of parameters, such as, cross-sectional sizes, prestressing forces, number and shape of the arches with central or lateral suspension, number of hangers and hanger arrangement (vertical, inclined or network). Moreover, several load cases, geometrical nonlinearities need to be considered. In concrete bridges, the time-dependent effects are of major relevance and should be also considered. Structural optimization is not usually employed in civil engineering design practice. However, due to the complexity and the large amount of information involved in the design of these structures, optimization techniques are especially suited to help designers achieving structurally efficient, economic and sustainable solutions.

A literature review shows some research on the optimization of these type of bridges with a major focus on network arch bridges [4, 5, 6, 7]. These bridges are characterized by a large number of inclined hangers crossing each other at least twice. Concerning this particular bridge typology, it is worth referring the extensive research by Per Tveit [8, 9, 10, 11]. Previous works about bowstring tied-arch bridges featuring vertical or inclined hangers addressed the optimization of the arrangement of hangers [12,13], the optimal arch shape [14], the hangers' pretension forces [15, 16] and the optimum design of steel bridges [17]. From the literature review in can be stated that, from the best of the authors' knowledge, the optimization of bowstring tied-arch concrete bridges was not yet reported.

Therefore, the main objective of this work is to develop an optimization-based computational model to help designing bowstring tied-arch concrete bridges under dead load and road traffic live load. To this aim, a computer program previously developed for the optimization of other concrete cable-supported bridges [18, 19] was adapted for the optimization of bowstring tied-arch concrete bridges. A convex optimization strategy with multiple starting points is proposed to solve the original nonconvex optimization problem. A multi-start approach is used to obtain local optimum solutions and the minimum cost solution is selected as the optimum design.

## 2 Analysis and optimization algorithm

The proposed approach was implemented in a computational model developed in MATLAB environment comprising a structural analysis module, and a sensitivity analysis and optimization module. The flowchart of the proposed approach is depicted in Figure 1.

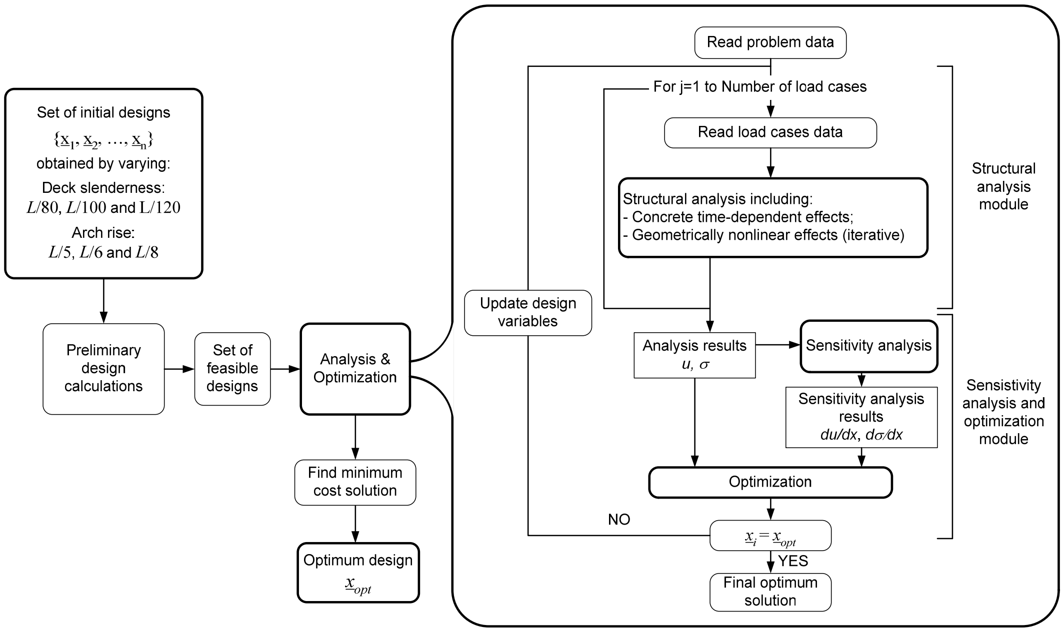


Figure 1: Flowchart of the optimization strategy.

The finite element method was used for the three-dimensional analysis under static loading (dead load and road traffic live load), including geometrical nonlinearities and time-dependent effects. The arch and deck were modelled with 2-node and 12-degrees of freedom Euler-Bernoulli beam elements, and 2-node bar elements were used to model the hangers. To consider the second-order effects, the stiffness matrix of the beam elements includes the elastic and geometric contributions and the structural analysis was conducted iteratively to perform a second-order elastic analysis. The deck internal bonded prestressing was modelled using 2-node tendon elements with linear profile. These elements are defined connected to the 2-node and 12 degrees of freedom Euler Bernoulli beam elements, sharing the same nodal displacements.

Structural concrete was modelled as a linear viscoelastic material and the time-dependent effects of ageing, creep and shrinkage of concrete, and relaxation of prestressing steel were evaluated according to NP EN 1992-1-1 [20] formulation. The time-dependent effects were modelled by equivalent nodal forces. These forces are computed from the creep and shrinkage deformations from a given time interval, and produce the same displacements field as the time-dependent effects. Thus, stresses are calculated using only the elastic constitutive relationship between stresses and mechanical origin deformations. Detailed information about the time-dependent effects' modelling can be found in a previous work by the authors [21].

A linear elastic behaviour of the materials (concrete, reinforcing steel and prestressing steel) was adopted in the analysis and the materials nonlinearities were considered in the strength design goals of the optimization problem. Homogeneous concrete cross-sections were assumed and the steel reinforcement was considered only for design purposes.

The design of bowstring tied-arch concrete bridges was formulated as a multi-objective optimization problem from which an optimum solution (in the Pareto sense) is obtained for each starting design. Using an entropy-based approach [22] the original minimax optimization problem is solved indirectly by the minimization of an unconstrained convex scalar function (Equation (1)). This function creates an inside convex approximation of the original nonconvex domain. Given that the design goals, $g_j(\underline{x})$, do not have an explicit algebraic form, the problem is solved using an explicit approximation given by the Taylor series expansion of all the goals, around the current design variable vector, truncated after the linear term

$$\min F(\underline{x}) = \min \frac{1}{\rho} \ln \left[ \sum_{j=1}^{M} e^{\rho \left( g_j(\underline{x}) + \sum_{i=1}^{N} \frac{dg_j(\underline{x})}{dx_i} \Delta x_i \right)} \right] \tag{1}$$

where $\underline{x}$ is the vector of design variables, $M$ is the number of objectives, $N$ is the number of design variables, $g_j(\underline{x})$ is the *j-th* design objective, $dg_j(\underline{x})/dx_i$ is the sensitivity of the *j-th* design objective with respect to *i-th* design variable. The control parameter $\rho$ must not be decreased during the optimization process and its value should be tuned for each problem. However, values in the range 100 – 2000 lead to similar results. Bound constraints with move limits were used to ensure the accuracy of the explicit approximation. The MATLAB function *fmincon*, which minimizes a scalar function of several variables subjected to bound constraints using a sequence of quadratic problems, was adopted to minimize the objective function. Figure 2 shows the design variables considered.

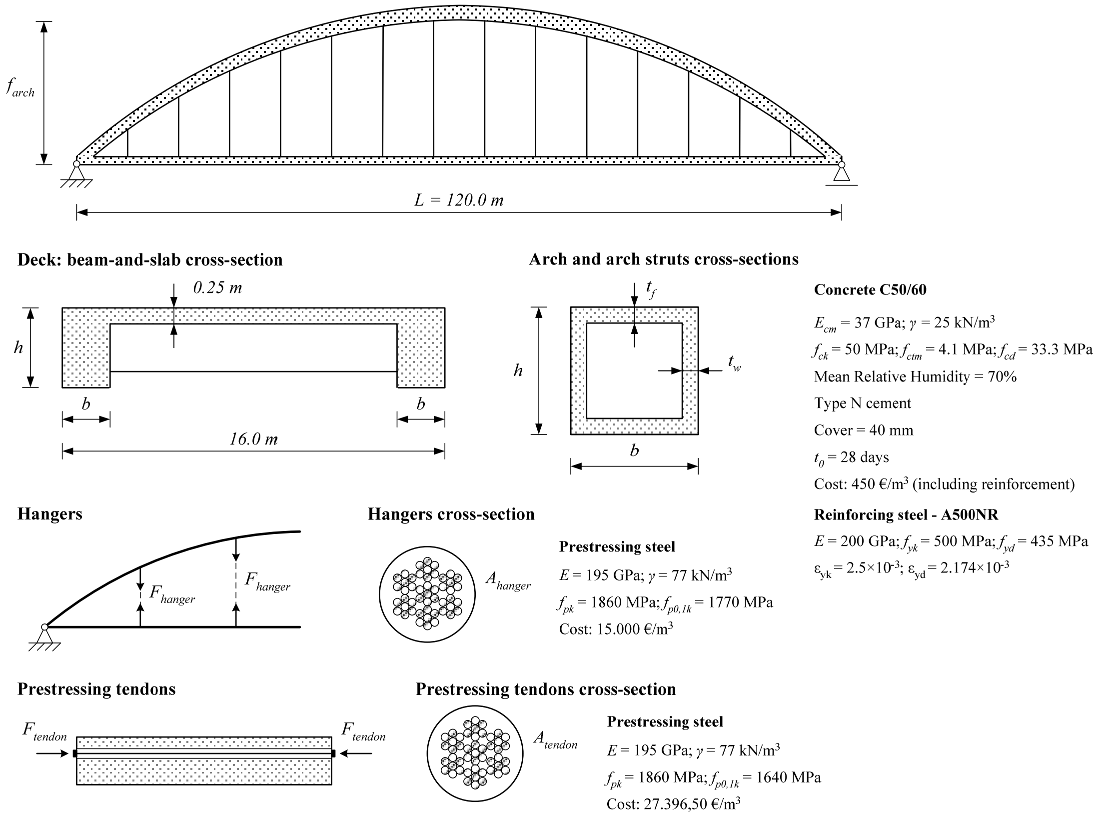

Figure 2: Bridge example, material properties and unit costs, and design variables.

Design goals of minimum cost, deflections and stresses related to strength and service criteria defined according to NP EN 1992-1-1 [20] provisions were considered. The first design goal concerns the cost minimization and can be expressed as

$$g_1(\underline{x}) = \frac{C}{C_0} - 1 \leq 0 \tag{2}$$

where C is the current cost of the structure and $C_0$ represents the initial cost of each analysis and optimization cycle. This approach makes the cost always one of the main objectives for the optimization algorithm. A second set of objectives refer to limiting the deck vertical displacements under service conditions and considering the time-dependent effects

$$g_2(\underline{x}) = \frac{|\delta|}{\delta_0} - 1 \leq 0 \tag{3}$$

where $\delta$ and $\delta_0$ are the displacement value and the limit value for the displacement under control, respectively. A value of $L/1000$ was considered for $\delta_0$, being a usual value for road bridges [1].

The stress goals for the deck and arch members were defined based on the NP EN 1992-1-1 [20] provisions. In general, these goals can be expressed by

$$g_3(\underline{x}) = \frac{\sigma}{\sigma_{allow}} - 1 \leq 0 \tag{4}$$

where $\sigma$ and $\sigma_{allow}$ are the acting stress and the corresponding allowable stress, respectively. For concrete members, different values of the allowable stress were considered for service conditions and for strength verification. For service conditions, values of 4.1 MPa and 22.5 MPa were considered for the concrete in tension and compression, respectively. For strength verification, the allowable value represents the structural concrete member's resistance, including reinforcement, evaluated according to acting internal forces, such as, bending and axial force or shear force. Another set of goals refers to the stresses in the hangers and internal prestressing tendons which can be written as

$$g_4(\underline{x}) = \frac{\sigma}{k \cdot f_{pk}} - 1 \leq 0 \tag{5}$$

$$g_5(\underline{x}) = 1 - \frac{\sigma}{0.10 \cdot f_{pk}} \leq 0 \tag{6}$$

where $\sigma$ and $f_{pk}$ are the acting stress and the characteristic value of the prestressing steel tensile strength, respectively. The value of $k$ in Equation 5 was considered equal to 0.50 for service conditions and 0.74 for strength verification for the hangers. For the prestressing tendons $k$ assumes values of 0.75 for service conditions and 0.88 for strength verification. Equation 6 refers to a lower limit for tension in the hangers to ensure their structural efficiency.

The discrete direct method, with analytical and semi-analytical derivatives, was used for sensitivity analysis. This approach was adopted due to the computational efficiency, accuracy, availability of the source code and because the number of design goals is far larger than the number of design variables. The sensitivity analysis provides the gradients of the objective function and all the design goals with respect to the design variables. This information is needed by the convex optimization algorithm used.

## 3 Numerical example

The numerical example concerns the optimization of a real-sized bowstring tied-arch concrete bridge (Figure 2). The deck is simply supported at the abutments featuring a beam-and-slab cross-section. Two parallel arches with parabolic shape and a rise represented by the design variable, $f_{arch}$, were considered. A vertical hanger arrangement with lateral suspension and a total of 28 hangers was considered. Five horizontal struts connecting the two arches were adopted to ensure the out of plane stability. The deck was modelled with longitudinal and transverse beams, and each arch was discretized in 30 beam elements. The bridge finite element model is depicted in Figure 3 and has a total of 120 nodes and 186 finite elements.

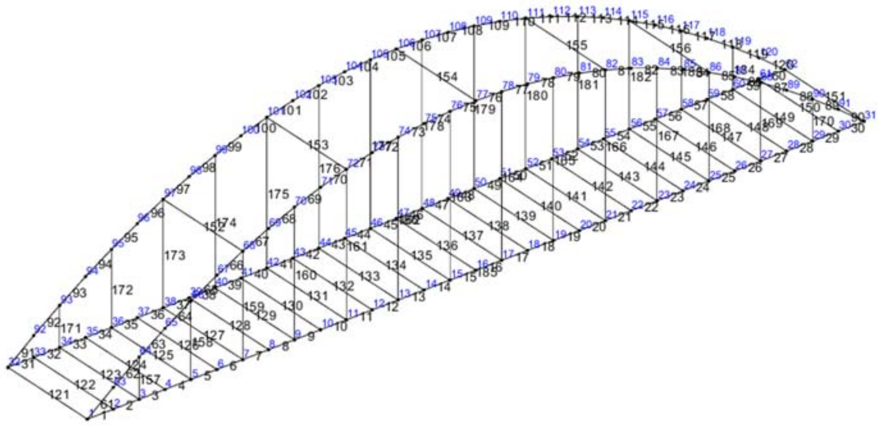

Figure 3: Finite element mesh of the bridge example.

Five load cases were defined to check the relevant service and strength design goals. The first case corresponds to the bridge under dead load (self-weight and an additional dead load of 2.5 kN/m$^2$ corresponding to flooring, walkways, safety barriers and guardrails) at the end of construction. The second case refers to the bridge under the quasi-permanent load combination (dead load plus 20% of road traffic live load) and a long-term analysis (18,250 days) is conducted. The remaining load cases refer to strength verifications. Therefore, the road traffic live load (5 kN/m$^2$) was placed on the entire deck length, or only on half length, or in the central third of the span, to produce the most unfavourable effects. The erection stages may be relevant in the design of these structures. However, the current paper focuses in the static response of the complete bridge and thus, the erection stages were not directly considered. The longitudinal reinforcement and the shear reinforcement were considered constant design parameters with usual practical values defined as percentages of the concrete cross-sectional area. A total of 15 design variables and almost 950 design goals for the 5 load cases were considered. In the multi-start procedure, three values of the arch rise, $f_{arch}$ ($L/5$, $L/6$ and $L/8$) and three values of deck slenderness ($L/80$, $L/100$ and $L/120$) were considered. Therefore, a total number of 9 initial designs were generated and optimized. Considering the multi-start approach used, the results presented in Table 1 correspond only to the initial and final values of the optimum solution.

Figure 4 presents the evolution of the bridge cost throughout the optimization process. The optimized solutions are obtained after a relatively small number of iterations, around 50 iterations. Similar optimized solutions are obtained with the different initial designs. The optimum solution is obtained with the starting design characterized by $f_{arch}$ = 15 m and $h$ = 1.50 m (f15_h150 in Figure 4). The optimum solution presents a cost reduction of 32.6% compared with the initial solution due to a reduction in the sizing design variables of the deck, arch and prestressing tendons (Table 1).

| Design variable | Initial value | Final vale |
|---|---|---|
| $f_{arch}$ [m] | 15.0 | 22.780 |
| $h$ [m] | 1.50 | 1.00 |
| $b$ [m] | 0.70 | 0.70 |
| $h_{arch}$ [m] | 2.00 | 1.973 |
| $b_{arch}$ [m] | 2.00 | 1.001 |
| $t_{w\_arch}$ [m] | 0.30 | 0.20 |
| $t_{f\_arch}$ [m] | 0.30 | 0.20 |
| $h_{strut}$ [m] | 2.00 | 1.061 |
| $b_{strut}$ [m] | 2.00 | 1.002 |
| $t_{w\_strut}$ [m] | 0.30 | 0.20 |
| $t_{f\_strut}$ [m] | 0.30 | 0.20 |
| $F_{hangers}$ [kN] | 1020 | 1125 |
| $A_{hangers}$ [m$^2$] | $1.20\times10^{-3}$ | $9.69\times10^{-4}$ |
| $F_{tendons}$ [kN] | 29500 | 16199 |
| $A_{tendons}$ [m$^2$] | $2.37\times10^{-2}$ | $1.13\times10^{-2}$ |
| Cost | Initial value | Final vale |
| Deck | 381,240 € | 343,496 € |
| Arch | 302,614 € | 145,166 € |
| Hangers | 5,376 € | 6,591 € |
| Tendons | 155,568 € | 74,374 € |
| Total cost | 844,798 € | 569,627 € |

Table 1: Initial and final values of the cost and design variables – optimum solution.

The least cost solution presents a maximum value of 5.55 cm for the deck vertical displacements considering the time-dependent effects (Figure 5). The active design goals at the optimum are the hangers, deck and arches resistance for load cases 3 and 4, and the tendons and arch struts stresses for service conditions.

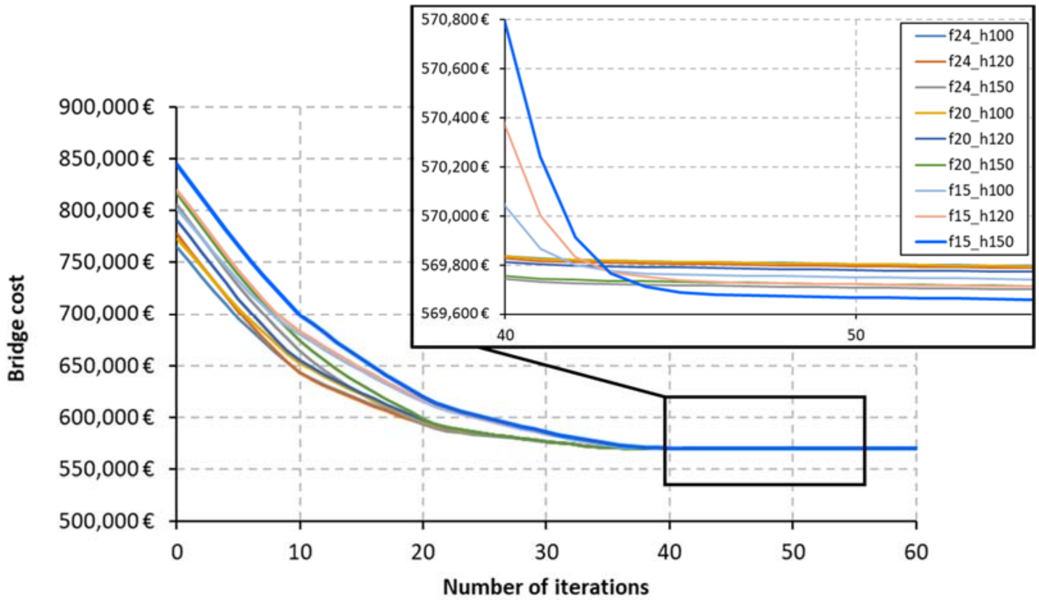


Figure 4: Bridge cost *vs.* number of iterations – multiple starting points.

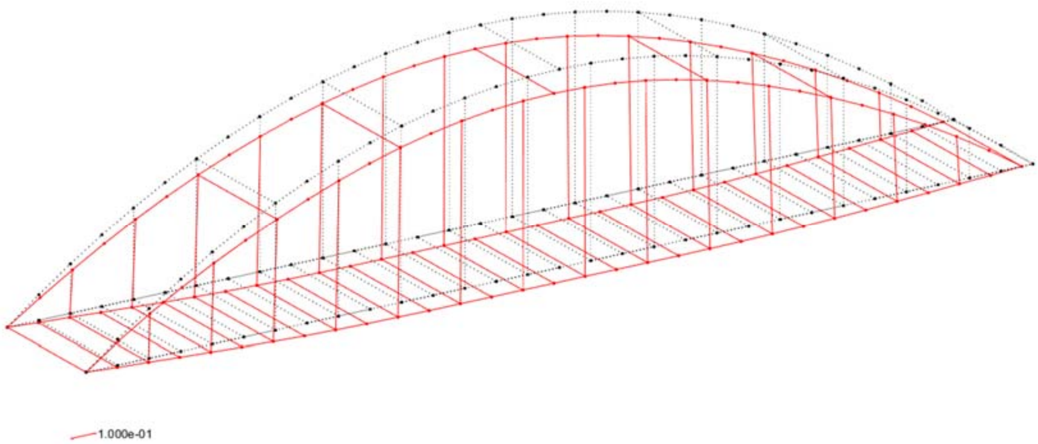


Figure 5: Deformed configuration of the bridge for load case 2 – optimum solution.

## 4 Conclusions and Contributions

The following conclusions can be drawn:

- The design of bowstring tied-arch concrete bridges can be formulated and solved as a multi-objective optimization problem with objectives of minimum cost, and service and strength criteria.
- The original nonconvex optimization problem is solved through a multi-start convex optimization strategy. Local optimum solutions are obtained and the least cost solution is selected as the optimum design. This is an efficient procedure to obtain optimised solutions for the design of bowstring tied-arch concrete bridges

under static loading and considering the most relevant service and strength design goals.

- The optimization algorithm finds solutions that balance the stiffness of the arch and deck, and the suspension effect provided by the vertical hangers to improve the structural behaviour and reduce the overall cost. The optimum solutions satisfy all the design goals and present cost reduction due to a decrease in the values of the sizing design variables of the deck, arches and prestressing tendons.
- In the optimum solution the deck, arches, hangers and prestressing tendons represent 60.3%, 25.5%, 1.2% and 13.0% of the total cost, respectively.
- The design is governed by the hangers, deck and arches resistance. The optimum solution features a deck slenderness of 1/120 and an arch rise-to-span ratio of 1/5.3.
- Future developments should consider additional geometrical and topological design variables describing the arch shape (parallel, convergent or divergent), the number of arches (thus, central or lateral suspension), the number of hangers and their arrangement (vertical or inclined). The optimization considering different types of cross-sections (solid or voided slab, box girder) and solutions (prestressed concrete, steel, steel-concrete composite) for the bridge deck should be also considered.
- The optimum design of network arch bridges with concrete deck and arch will be addressed in upcoming research. Furthermore, the optimization considering dynamic actions in road bridges and footbridges should be also considered in upcoming research.

# References


[1] A.J. Reis, J.J.O. Pedro, "Bridge design: Concepts and analysis", First edition, Hoboken, NJ, John Wiley & Sons, Ltd, 2019.

[2] J. Manterola, "Puentes apuntes para su diseño, cálculo y construcción", Primera Edición, Colegio de Ingenieros de Caminos, Canales y Puertos, Madrid, 2006.

[3] G.A.R. Parke, N. Hewson, Institution of Civil Engineers (Eds.), "ICE manual of bridge engineering", Third edition, ICE Publishing, London, 2022.

[4] N. Islam, S. Rana, R. Ahsan, S.N. Ghani, "An Optimized Design of Network Arch Bridge using Global Optimization Algorithm", Advances in Structural Engineering, 17(2), 197-210, 2014. doi: 10.1260/1369-4332.17.2.197

[5] D. Bruno, P. Lonetti, A. Pascuzzo, "An optimization model for the design of network arch bridges", Computers & Structures, 170, 13-25, 2016. doi: 10.1016/j.compstruc.2016.03.011

[6] R. Belevičius, A. Juozapaitis, D. Rusakevičius, "Parameter study on weight minimization of network arch bridges", Periodica Polytechnica Civil Engineering, 62(1), 48-55, 2018. doi: 10.3311/PPci.10036

[7] R. Belevičius, D. Rusakevičius, S. Valentinavičius, "Insights into Optimal Schemes of Radial Network Arch Pedestrian Bridge, Advances in Civil Engineering, 2022. doi: 10.1155/2022/6951775

[8] P. Tveit, "Optimal network arches save 50 to 75% of the steel". IABSE Conference New Delhi, India 2005: Role of Structural Engineers Towards Reduction of Poverty, 401-408, 2005.

[9] P. Tveit, “An introduction to the optimal network arch”, Structural Engineering International: Journal of the International Association for Bridge and Structural Engineering, 17(2), 184-187, 2007. doi: 10.2749/101686607780680727
[10] P. Tveit, “Concrete in the optimal network arch”, Proceedings of the International FIB Symposium 2008 – Tailor Made Concrete Structures: New Solutions for Our Society, 164, 2008. doi: 10.1201/9781439828410.ch120
[11] P. Tveit, “Optimal network arches for rail and road bridges”, Proceedings of the 4th International Conference on New Dimensions in Bridges, Flyovers, Overpasses and Elevated Structures, 281-288, 2017.
[12] Y. Tan, Y. Yao, “Optimization of hanger arrangement in pedestrian tied arch bridge with sparse hanger system”, Advances in Structural Engineering, 22(12), 2594-2604, 2019. doi: 10.1177/1369433219849842
[13] Y. Tan, Y. Yao, “Optimization of hanger arrangement in tied arch bridge using genetic algorithm”, IOP Conference Series: Earth and Environmental Science, 189(2), 2018. doi: 10.1088/1755-1315/189/2/022016
[14] E. Järvenpää, R. Heikkilä, M.-E. Järvenpää, “Geometric Non-linear Form-Finding Design for Optimal Tied Arch Bridge. Structural Integrity, 11, 230-237, 2020. doi: 10.1007/978-3-030-29227-0_22
[15] X. Sun, Y. Wang, F. Chen, “Design application in suspender tensile force of tied-arch bridges based on inversion optimization method”, Applied Mechanics and Materials, 226-228, 1586-1590, 2012. doi: 10.4028/www.scientific.net/AMM.226-228.1586
[16] H. Cao, Y. Chen, L. Yang, L. Feng, “Hanger pre-tensioning force optimization of steel tied-arch bridges considering operational loads”, Structural and Multidisciplinary Optimization, 63(2), 867-880, 2021. doi: 10.1007/s00158-020-02717-x
[17] M.A. Latif, M.P. Saka, “Optimum design of tied-arch bridges under code requirements using enhanced artificial bee colony algorithm”, Advances in Engineering Software, 135, 2019. doi: 10.1016/j.advengsoft.2019.102685
[18] A.M.B. Martins, L.M.C. Simões, J.H.J.O. Negrão, “Optimization of concrete cable-stayed bridges under seismic action”, Computers & Structures, 222, 36-47, 2019. doi: 10.1016/j.compstruc.2019.06.008
[19] A.M.B. Martins, L.M.C. Simões, J.H.J.O. Negrão, “Optimization of extradosed concrete bridges subjected to seismic action”. Computers & Structures, 245, 106460, 2021. doi: 10.1016/j.compstruc.2020.106460
[20] NP EN 1992-1-1, “NP EN 1992-1-1 Eurocódigo 2 – Projecto de estruturas de betão, Parte 1-1: Regras gerais e regras para edifícios”, IPQ – Instituto Português da Qualidade, 2010.
[21] A.M.B. Martins, L.M.C. Simões, J.H.J.O. Negrão, “Cable stretching force optimization of concrete cable-stayed bridges including construction stages and time-dependent effects”, Structural and Multidisciplinary Optimization, 51(3), 757-772, 2015. doi: 10.1007/s00158-014-1153-4
[22] L.M.C. Simões, A.B. Templeman, “Entropy-based synthesis of pretensioned cable net structures”, Engineering Optimization, 15(2), 121-140, 1989. doi: 10.1080/03052158908941147