Proceedings of the Sixth International Conference on
Soft Computing, Machine Learning and Optimisation in
Civil, Structural and Environmental Engineering
Edited by: P. Iványi, J. Lógó and B.H.V. Topping
Civil-Comp Conferences, Volume 5, Paper 1.2
Civil-Comp Press, Edinburgh, United Kingdom, 2023
doi: 10.4203/ccc.5.1.2

A Tractable Robust Topology Optimization for Anomalous Non-Symmetric Cases

A. Csébfalvi[1] and J. Lógó[2]

[1]Department of Civil Engineering, University of Pécs
Pécs, Hungary
[2]Budapest University of Technology and Economics
Budapest, Hungary

Abstract

In the practice of robust optimality design, robust and deterministic optimal configurations are usually expected to differ. Therefore, most research in this area tries to illustrate the effect of uncertainty by comparing robust and deterministic designs, which is not generalizable because it is easy to define a non-symmetric case in which the robust expected-compliance minimum and the nominal-compliance minimum design will be the same. This anomaly requires new techniques that allow for deeper insights. In this paper, an anomaly-resolving strategy is presented for such cases when the nominal and robust compliance are the same in the optimization of a volume-constrained continuous topology with directionally uncertain loads.

Keywords: robust, topology, uncertainty, load direction, non-symmetric, anomaly.

1 Introduction

Taking uncertainty into account is important for robust and reliable design. There are several ways to take uncertainty into account in the topology optimisation of continuous structures, which can be distinguished according to the structural and design characteristics that introduce uncertainty. Most models that consider the uncertainty in the direction of the load use parametric statistical tools to describe the directional uncertainty of the applied loads (e.g., Chen et.al [1], Dunning et.al [2], Alvarez and Carrasco [3], Guest and Igusa [4], Conti et.al [5], and Schuëller and Jensen [6], where the preferred measure of robustness is the expected compliance with a normal distribution. The goal is to create robust structures that are as insensitive as possible to directional uncertainty. The preferred measure of robustness in the most popular parametric statistical approaches is the expected correspondence to the normal distribution.

In a recent study of the authors [7], it was shown that the expected compliance is not a universally applicable robustness measure, because it can give misleading results in some non-symmetrical cases. The aim of this study to propose a strategy to tackle generally the effects of uncertainty, which could be apply even in anomaly cases pointed out above.

In this paper, an anomaly-resolving strategy is presented for such cases when the nominal and robust compliance are the same in the optimization of a volume-constrained continuous topology with directionally uncertain loads. It will be presented with experimental evidence that in the anomalous cases, the results of minimizing nominal compliance and minimizing the robustness measures under consideration form a Pareto-optimal point, and therefore increasing the volume-percentage is the only way to improve the design robustness.

2 Methods

There are several deterministic and stochastic approaches that attempt to cope with uncertainty problem in structural design to produce robust structures that are as insensitive as possible to changes in applied load directions. The problem is that without a universally applicable and "easy to understand" measure of robustness, there is no correct answer to the question "how robust is the robust design given by a given robust optimization approach". Recently an exact algorithm was presented by Csébfalvi [8] for the volume-constrained expected-compliance-minimization problem with normally distributed loading directions using exact objective and gradient functions. The algorithm is based upon the finding that for a particular set of statistical parameters the integration in the expected compliance function can be done symbolically and automatically using symbolic manipulation software.

The mathematical formulation of the applied worst-case model to determine the deterministic topology optimization problem:

$$c(\boldsymbol{x}) = \boldsymbol{U}'\boldsymbol{K}\boldsymbol{U} \rightarrow min \tag{1}$$

$$V(\boldsymbol{x}) = \varphi V_0 \tag{2}$$

$$\boldsymbol{K}\boldsymbol{U} = \boldsymbol{F} \tag{3}$$

$$0 \leq \boldsymbol{x} \leq 1 \tag{4}$$

where $\boldsymbol{x}$ is the vector of design variables (the element densities), $c(\boldsymbol{x})$ is the compliance, $\boldsymbol{U}$ and $\boldsymbol{F}$ are the global displacement and load vectors, respectively, $\boldsymbol{K}$

is the global stiffness matrix, $V(\boldsymbol{x})$ and V_0 are the material volume and design domain volume, respectively, and φ is the prescribed volume fraction.

As it was demonstrated by Andreassen et al. [9], it is very easy to extend the algorithm to account for multiple load cases. In fact, this can be done by adding only a few additional lines and making minor changes to another few lines. In the case of m load cases, the force and displacement vectors can be defined as m column vectors and the objective function will be the sum of m compliances:

$$c(\boldsymbol{x}) = \sum_{i=1}^{m} \boldsymbol{U}_i' \boldsymbol{K} \boldsymbol{U}_i \rightarrow min \tag{5}$$

$$V(\boldsymbol{x}) = \varphi V_0 \tag{6}$$

$$\boldsymbol{K}\boldsymbol{U}_i = \boldsymbol{F}_i \qquad i \in \{1, 2, \ldots, m\} \tag{7}$$

$$0 \leq \boldsymbol{x} \leq 1 \tag{8}$$

Further steps of the exact solution are described in paper Csébfalvi [8], where is shown that the multiple load compliance-minimization models after simple modifications can be used to solve this directional uncertainty problem. In order to avoid dealing with infinite number of constraints we have to replace the original problem with a more tractable equivalent algorithm based on a finite number of constraints. The result is a robust volume-constrained compliance-minimal design which is invariant to the feasible load perturbations. The conception is independent from the applied modelling frame. The problem with this approach is that it is easy to define such a non-symmetrical case (pointed out by Rozvany [10]) in which the robust expected-compliance minimum and the nominal-compliance minimum design will be the same.

In this study, a normally distributed expected compliance ($\boldsymbol{expc}$), the rarely investigated uniformly distributed expected compliance ($\boldsymbol{unic}$), and the currently developed by Csébfalvi [8] non-parametric total-compliance-variance ($\boldsymbol{tvrc}$) are used as robustness measures.

It will be shown, that in the anomalous cases the result of the nominal-compliance minimization and the results of the minimization of the investigated robustness measures form a so-called Pareto-point, therefore the volume-percentage increasing is the only way of the design robustness improving. The SIMP-type algorithms were implemented in MATLAB in a common frame. In the case of parametric-measures ($\boldsymbol{expc}, \boldsymbol{unic}$) the Optimality Method (OC) was used to solve the measure-minimization problems. The total-compliance-variance minimization problem with constrained volume-percentage increase was solved by a constrained-nonlinear-minimization algorithm (**`fmincon`**).

In Figure 1 and Figure 2 it is shown that what are the fundamental differences in the robust design searching process in the popular symmetric and the practically non-investigated non-symmetric cases. Because in the non-symmetric cases the robustness measures and the nominal-compliance form a pareto optimal point there is only one way to get a more robust design and it is the material fraction increasing.

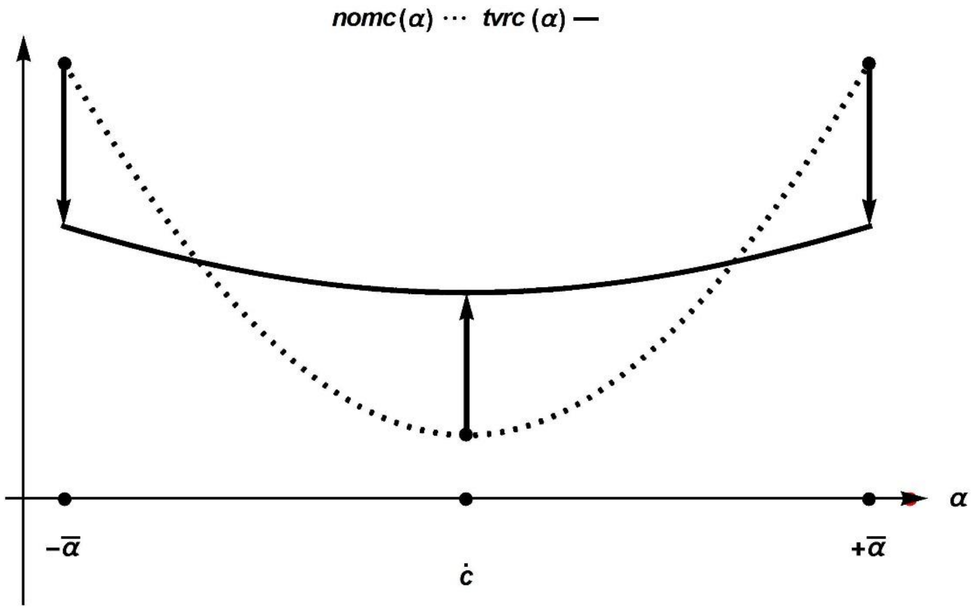

Figure 1: Solution strategy in normal case.

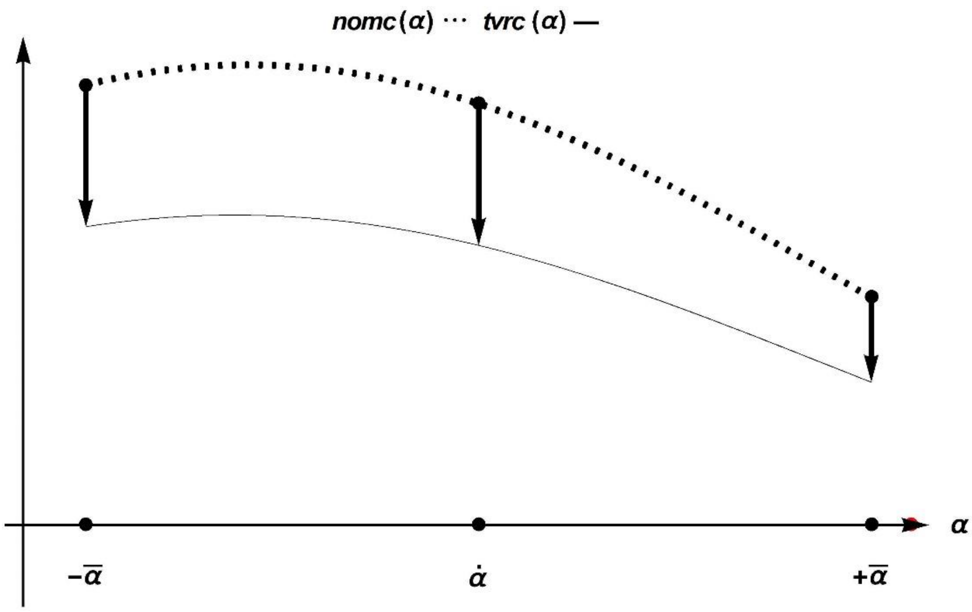

Figure 2: Anomalous solution strategy.

In Figure 3 and Figure 4 the robust $\boldsymbol{tvrc}$ solution of a popular symmetric problem is shown with the compliance function shapes given by the minimization of the nominal- compliance ($\boldsymbol{nomc}$) and the ($\boldsymbol{expc}, \boldsymbol{unic}, \boldsymbol{tvrc}$) robustness measures.

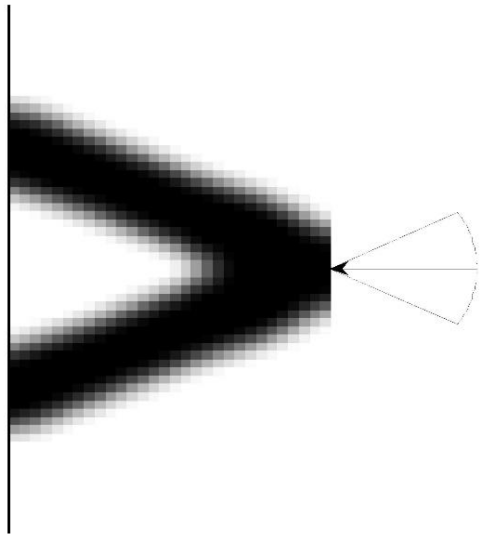

Figure 3: A robust symmetric design.

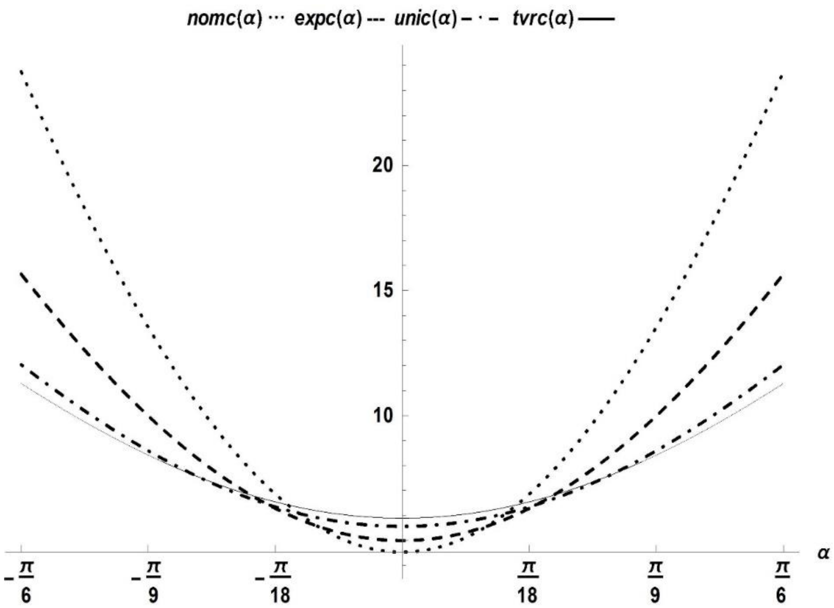

Figure 4: Symmetric measure shapes.

During the $\boldsymbol{tvrc}$ minimisation 30% nominal-compliance increment was allowed as a design parameter. In the case of the $\boldsymbol{unic}$ ($\boldsymbol{expc}$) minimisation the corresponding response variable was 20% (10%) with clearly detectable larger variability.

3 Results

The results obtained by the proposed method are presented for two load variants, one-dimensional and two-dimensional cases.

In the first case a concentrated force was applied to the end joint of the structure, in the second case two downward concentrated forces were applied.

In Figure 5 and Figure 6, the nominal compliance (**nomc**) minimal designs are presented for two non-symmetric designs with one and two directionally uncertain loads and $vol = 0.25$ volume setting.

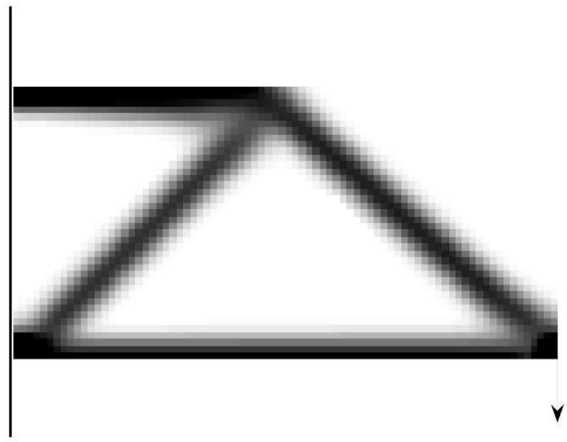

Figure 5: $\boldsymbol{nomc}$ minimal design with $vol = 0.25\%$ volume percent.

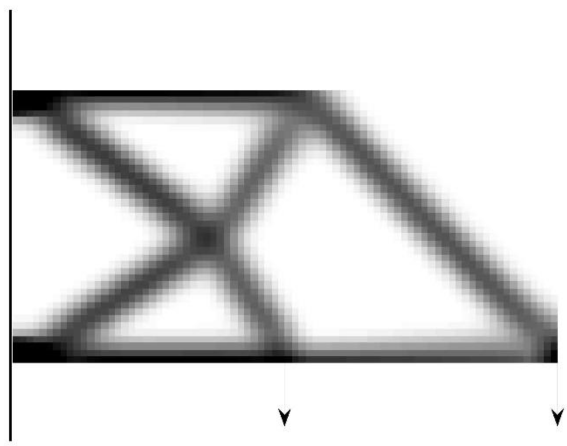

Figure 6: $\boldsymbol{nomc}$ minimal design with $vol = 0.25\%$ volume percent.

In Figure 7 and Figure 8, the nominal compliance ($\boldsymbol{nomc}$) minimal designs of the same examples are presented with $vol = 0.25 + 0.05 = 0.30$ setting to get more balanced designs characterized by the total variation ($\boldsymbol{tvrc}$) measure.

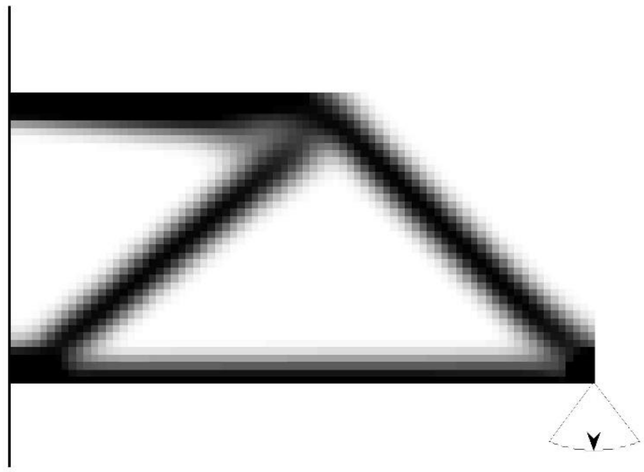

Figure 7: $\boldsymbol{nomc}$ minimal design with $vol = 0.30\%$ volume percent.

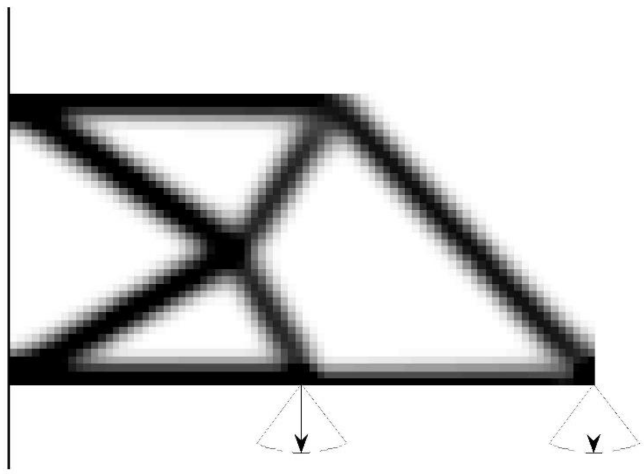

Figure 8: $\boldsymbol{nomc}$ minimal design with $vol = 0.30\%$ volume percent.

According to the behaviour of the investigated $\boldsymbol{nomc} = \boldsymbol{tvrc}$ "anomaly", the 0.05% volume percentage increment drastically changes the compliance function shape in each case (see Figure 9 and Figure 10).

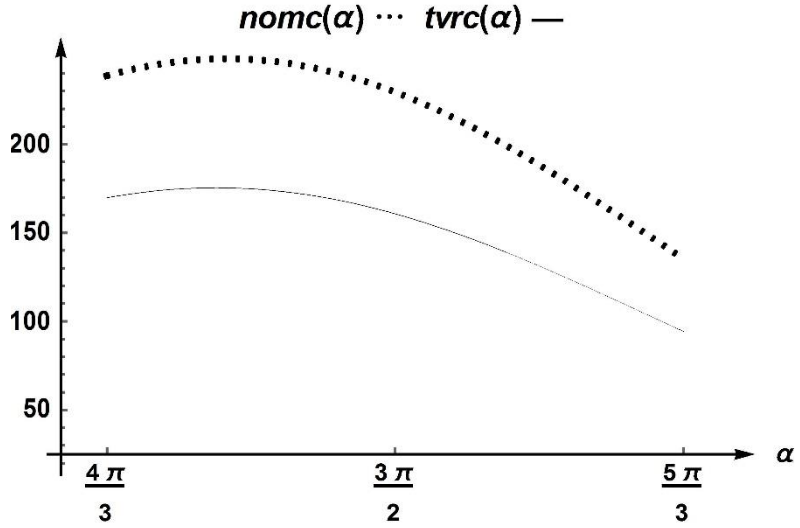

Figure 9: Common plot of $\{\boldsymbol{nomc}, \boldsymbol{tvrc}\}$ compliance curves.

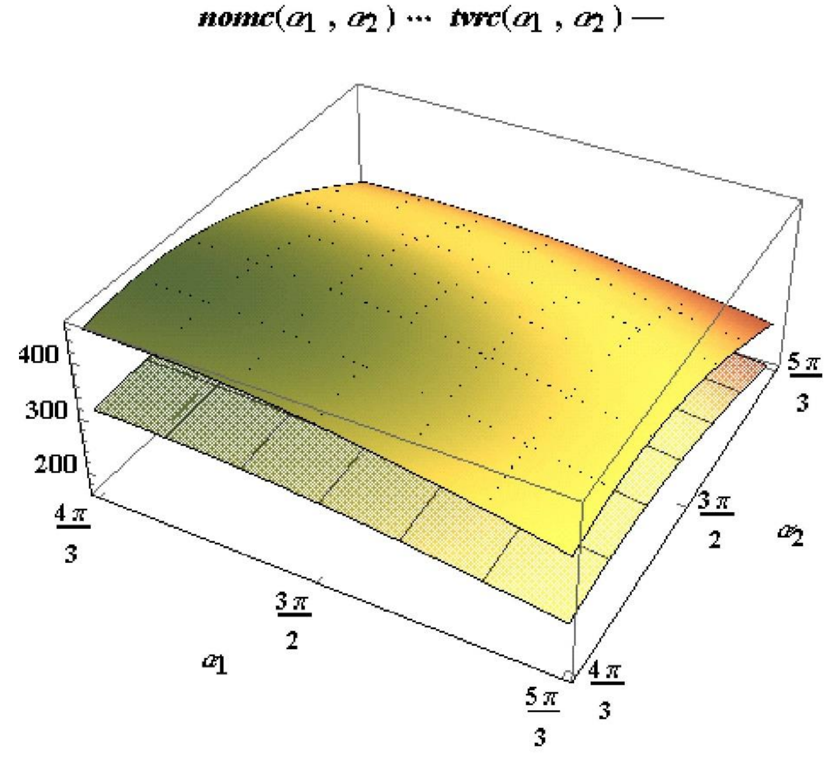

Figure 10: Common plot of $\{\boldsymbol{nomc}, \boldsymbol{tvrc}\}$ compliance areas.

In figures above it was demonstrated that using the terminology of the traditional variational analysis, the essence of the non-parametric robustness measure minimization is very simple: in the case of one directionally uncertain load it is a curve-length minimization problem, and in the case of two directionally uncertain loads a surface-area minimization problem must be solved.

4 Conclusions and Contributions

In this paper, experimental evidence are presented to resolve the anomalous behaviour of the robust design searching process when solutions of the investigated robust approaches and the nominal approach form a pareto optimal point in the design space in the volume-constrained continuous topology optimization with directionally uncertain loads. The proposed methodology is based in a very simple and natural finding: in the "abnormal" case there is only way to increase the robustness of the design in a controllable form which is the volume fraction increasing. It is clear, that higher the allowed volume fraction increments the higher the chance to get a more balanced (more robust) design. The computational efficiency of the proposed approach in the anomalous case is extremely good, because to get a more robust design only the nominal compliance minimization process can be used. In this case, the different robustness measures (normally or uniformly distributed expected compliance and the total compliance variation) are only potential diagnostic tools which may help in the deeper understanding of the anomalous case. It is important to note, that the currently developed total compliance variation measure is the only one which has a controllable design parameter in the symmetric cases. It must note, that this feature is very important one from engineering point of view. Changing the allowed maximum nominal compliance increment we can select the best robust design. Using the terminology of the traditional variational analysis, the essence of the non-parametric robustness measure minimization is very simple: in the case of one directionally uncertain load it is a curve-length minimization problem, and in the case of two directionally uncertain loads a surface-area minimization problem must be solved. Contrarily, in the cases of parametric normally or uniformly distributed stochastic measures the nominal compliance increment will be an uncontrollable response parameter which depends only on the fictitious statistical assumptions without statistically correct validation (for example: normality validation) and parameters (for example: normally distributed standard deviation estimation). We hope that this work will contribute to a better understanding of what probabilistic and non-parametric robust optimization really means and it may be an inspirational starting point of further investigations in the future

Acknowledgements

The present study was supported by the Hungarian National Scientific and Research Foundation (OTKA) (grant K 138615).

References

[1] S. Chen, W. Chen, S. Lee, "Level Set Based Robust Shape and Topology Optimization Under Random Field Uncertainties", Struct Multidisc Optim, 41 (4), 507–524, 2010. doi: 10.1007/s00158-014-1119-6

[2] P. Dunning, H.A. Kim, and G. Mullinuex, "Introducing Loading Uncertainty in Topology Optimization", AIAA J, 49 (4), 760–768, 2011. doi: 10.2514/1.J050670

[3] F. Alvarez, and M. Carrasco, "Minimization of the Expected Compliances an Alternative Approach to Multiload Truss Optimization", Struct Multidisc Optim, 29(6), 470–476, 2005. doi: 10.1016/j.cma.2015.02.003

[4] J. K. Guest, T. Igusa, "Structural Optimization Under Uncertain Loads and Nodal Locations, " Comput Methods Appl Mech Eng, 198(1), 116–124, 2008. doi: 10.1016/j.cma.2008.04.009

[5] S. Conti, H. Held, M. Pach, M. Rumpf, R. Schultz, "Shape Optimization Under Uncertainty: A Stochastic Programming Perspective", SIAM J Optimiz, 19(4), 1610–1632, 2009. doi: 10.1137/070702059

[6] G. I. Schuëller, and H. A. Jensen, "Computational Methods in Optimization Considering Uncertainties: An Overview", Comput Methods Appl Mech Eng, 198(1), 2–13, 2008. doi: 10.1016/j.cma.2008.05.004

[7] A. Csébfalvi, J. Lógó, "A critical analysis of expected-compliance model in volume-constrained robust topology optimization with normally distributed loading directions, using a minimax-compliance approach alternatively", ADV ENG SOFTW, 120, 107-115, 2018. doi: 10.1016/j.advengsoft.2018.02.003

[8] A. Csébfalvi, "A new compliance-function-shape-oriented robust approach for volume-constrained continuous topology optimization with uncertain loading directions", PERIOD POLYTECH-CIV, 62(1), 219-225, 2018. doi: 10.3311/PPci.11398

[9] Andreassen E, Clausen A, Schevenels M, Lazarov BS, Sigmund O (2011) Efficient toplogy optimization in MATLAB using 88 lines of code, Struct Multidisc Optim 43 (1) 1-16

[10] G.I.N. Rozvany, "On Symmetry and Non-uniqueness in Exact Topology Optimization", STRUCT OPTIM, 43, 297–317, 2011. doi: 10.1007/s00158-010-0564-0