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Multilayer rail track model with two nonlinearities

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Abstract

Seeking of new models, especially highly valued analytical ones which are useful for parametrical analysis of rail track dynamics, is still of importance. The simplest model is represented by a beam resting on elastic or viscoelastic foundation. This model is good enough for the analysis of rails but all parameters of rail foundation must be averaged and therefore it is impossible to study the behaviour of specific components of rail track structure. In the case of two layer model, in which the first layer represents rails and the second one sleepers, the solution becomes very complicated. Both mentioned models have been already solved and validated under assumption of nonlinear foundation stiffness. Nonlinear stiffness of rail fastening systems is a factor arising from experimental measurements. It can have significant influence on the rail track behaviour and this phenomenon is not checked so far. Therefore inclusion of additional nonlinearity in the two layer model based on double beam system is an interesting direction of further investigations being natural extension of previous works. In this paper, a model consisting of two infinitely long beams representing rails and sleepers, and connected by a viscoelastic nonlinear layer, rests on nonlinear foundation. The existence of two nonlinearities makes the system computationally difficult and the detailed analysis of its stability and solution convergence is needed before application to the real rail track investigation. The two layer model of rail track built on the basis of a double beam system has been solved under assumption of two nonlinearities describing foundation and fastening system stiffness. The proposed computational approach using a hybrid semi-analytical method, involving the wavelet analytical approximation combined with the Adomian's decomposition, allows obtaining the solution in the physical domain for vertical vibrations of rails and sleepers. The model solution together with computational examples are presented along with methodological guidelines allowing its engineering application. The form of solution makes the parametrical analysis efficient, leading to reliable discussion of the investigated factors influence on the rail track system dynamic response to moving train. The obtained solution for the two layer model with two nonlinear factors is an important novelty giving possibility of the particular rail track components effects investigation..

Keywords: two layer model, rail track dynamics, nonlinear response, semi-analytical method.

1 Introduction

Existing analytical models for vertical rail track vibrations are mainly linear. Although the nonlinear stiffness of foundation does not influence significantly the structure response, in terms of engineering, nonlinear features of other components, such as e.g. fastening systems, can affect the rail track behaviour more noticeably. This phenomenon is not investigated deeply enough so far, mainly due to a lack of appropriate models and methods of their solution. Thus seeking of new models, especially highly valued analytical ones which are useful for parametrical analysis, is still of importance. The simplest model is represented by a beam resting on viscoelastic foundation [1, 2]. This model is good enough for the analysis of rails but all parameters of rail foundation must be averaged and therefore it is impossible to study the behaviour of specific components of rail track structure. Some additional assumptions, like e.g. nonlinear foundation stiffness, can be introduced but even relatively simple modifications lead to complex models unable to be solved with classical methods [1, 2]. In the case of two layer model, in which the first layer represents rails and the second one sleepers, the solution becomes very complicated. Both mentioned models have been already solved and validated under assumption of nonlinear foundation stiffness [2-4]. Their solutions have been obtained by using hybrid method based on semi-analytical approach [2, 5]. Nonlinear stiffness of rail fastening systems is a factor arising from experimental measurements. Its influence on the rail track behaviour is not checked so far. Therefore inclusion of additional nonlinearity in the two layer model based on double beam system [6] is an interesting direction of further investigations. In this paper, a model consisting of two infinitely long beams representing rails and sleepers, and connected by a viscoelastic nonlinear layer, rests on nonlinear foundation. The system is subjected to a load moving along the upper beam. Configuration of this load comes from a real axles configuration of rail vehicle. The existence of two nonlinearities makes the system computationally difficult and the detailed analysis of its stability and solution convergence is needed before application to the real rail track investigation. The model solution together with computational examples are presented. This solution, under assumption that the first layer is a beam and the second layer is a rigid body, taking into account two described nonlinearities, is the main novelty of this paper and forms important contribution to the field.

2 Methods

The mathematical coupled model consists of two dynamical differential equations [1, 4, 8]:

$$EI_r \frac{\partial^4 u}{\partial x^4} + m_r \frac{\partial^2 u}{\partial t^2} + c_r \left(\frac{\partial u}{\partial t} - \frac{\partial w}{\partial t}\right) + k_r(u-w) + k_{Nr}u^3 - k_{Ns}w^3 = P(x,t) \quad (1a)$$

$$m_s \frac{\partial^2 w}{\partial t^2} + c_s \frac{\partial w}{\partial t} + k_s w - c_r \left(\frac{\partial u}{\partial t} - \frac{\partial w}{\partial t}\right) - k_r(u-w) + k_{Ns}w^3 - k_{Nr}u^3 = 0 \quad (1b)$$

where $u(x,t)$ [m] and $w(x,t)$ [m] are vertical vibrations of rails and sleepers, respectively; EI_r [Nm^2], m_r [kg/m] – bending stiffness and unit mass of rails; EI_s [Nm^2], m_s [kg/m] – bending stiffness and unit mass of sleepers; k_{Nr} [N/m^4], k_r [N/m^2], c_r [Ns/m^2] – nonlinear part of stiffness, linear stiffness and viscous damping of the layer between rails and sleepers (including fastening system); k_{Ns} [N/m^4], k_s [N/m^2], c_s [Ns/m^2] – nonlinear part of stiffness, linear stiffness and viscous damping of the rail track foundation; $P(x,t)$ [N/m] – a set of loads $Q_k(x,t)$ generated by axles of train moving uniformly along rails with constant speed V [m/s].

The load can be composed of several terms, e.g. a quasi-static stationary constant in time part generated by the weight of vehicle, a part varying in time generated mainly by geometrical irregularities of rail-wheel contact surface or track stiffness changes. In this paper, each separated load includes only the quasi-static part and the part varying in time associated with regular corrugated wear of rail rolling surface:

$$Q_k(x,t) = (P_0 + \Delta P \cdot \exp(i(\Omega t + \varphi_l))) \frac{1}{2a} \cos^2\left(\frac{\pi(x - Vt - s_k)}{2a}\right) H(a^2 - (x - Vt - s_k)^2) \quad (2)$$

$H(.)$ is the Heaviside function, $2a$ [m] – an interval of the load distribution under a single wheel, s_k [m] – a distance from the first axle, $\Omega = 2\pi \cdot f_\Omega$ – the load frequency related to rail imperfections, φ_k – phase shift linked to positions of particular wheels.

The system has been solved by using a hybrid semi-analytical method [5, 6]. The nonlinear parts are represented by the Adomian series [8] and the solution in the physical domain can be found as an analytical wavelet based approximation [2, 4, 5]. This set of approximated factors makes the solution difficult for convergence analysis, mainly due to a need of adjustments of the algorithm computational efficiency to particular sets of parameters. Appropriate conditions are already defined and checked for similar multi beam systems [1, 8] and further difficulties can be diminished by a detailed analysis of the wavelet approximating procedure resulting in a good determination of coefficients appearing in computational formulas. This paper shows examples of solutions for the EMU250 train and different values of nonlinear factors related to the stiffness of rail track layers.

3 Results

The following system of parameters is used in computational examples [1, 4]:

$P = 78.33 \cdot 10^4$ N/m, $EI_r = 6.4 \cdot 10^6 \mathrm{Nm}^2$, $m_r = 60$ kg/m, $k_r = 8.8 \cdot 10^7 \mathrm{N/m}^2$, $c_r = 0.06 \cdot \sqrt{k_r \cdot m_r}$, $m_s = 266.67$ kg/m, $k_s = 8.5 \cdot 10^7 \mathrm{N/m}^2$, $c_s = 0.06 \cdot \sqrt{k_s \cdot m_s}$, $V = 200$ km/h, $\Omega_k = 2\pi \cdot f_{\Omega_k}$, a = 0.0075 m, configuration of EMU250 bogie, i.e. 2700 mm between axles, the length of the rail surface irregularity 0.8 m.

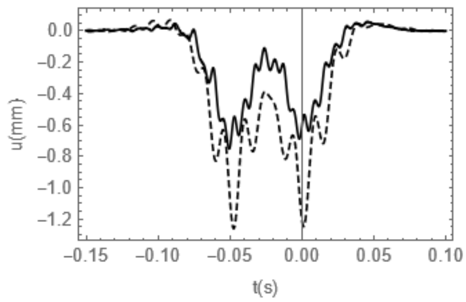

Figure 1. Vertical vibrations of rails: linear – dashed, nonlinear – solid ($k_{Nr} = 5 \cdot 10^{13} \mathrm{N/m}^4$, $k_{Ns} = 10^{13} \mathrm{N/m}^4$).

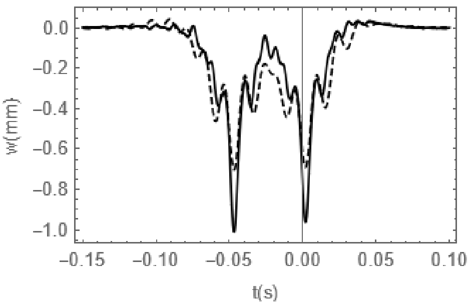

Figure 2. Vertical vibrations of sleepers: linear – dashed, nonlinear – solid ($k_{Nr} = 5 \cdot 10^{13} \mathrm{N/m}^4$, $k_{Ns} = 10^{13} \mathrm{N/m}^4$).

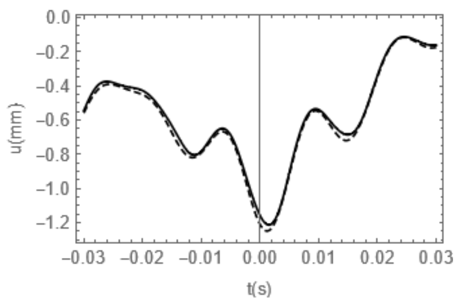

Figure 3. "Zoomed" vertical vibrations of rails: linear – dashed, nonlinear – solid ($k_{Nr} = 10^{13} \mathrm{N/m}^4$, $k_{Ns} = 5 \cdot 10^{13} \mathrm{N/m}^4$).

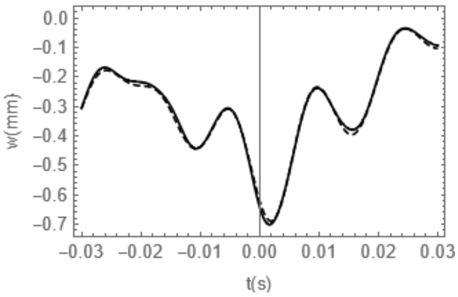

Figure 4. "Zoomed" vertical vibrations of sleepers: linear – dashed, nonlinear – solid ($k_{Nr} = 10^{13}\mathrm{N/m^4}$, $k_{Ns} = 5 \cdot 10^{13}\mathrm{N/m^4}$).

Figures 1-4 shows examples of rails and sleepers vertical vibrations. One can see that significant differences between linear and nonlinear response appear in the case of stronger nonlinearity under rails, compared to rail track foundation. In addition, sleepers vibrate stronger while rails vibrations have lower amplitude. In the opposite case, i.e. with stronger nonlinearity in the track bed, differences are hardly noticeable and can be neglected from engineering point of view. However, these observations must be confirmed by the parametrical analysis for a wide range of physical parameters, especially for different values of nonlinear factors k_{Nr} and k_{Ns}. Further investigations should also include different descriptions of loads, especially consideration of its inertial properties (e.g. an assumption of moving masses instead of forces only) [9].

4 Conclusions and Contributions

The two layer model of rail track built on the basis of a double beam system has been solved under assumption of two nonlinearities describing foundation and fastening system stiffness. The proposed computational approach using a hybrid semi-analytical method, involving the wavelet analytical approximation combined with the Adomian's decomposition, allows obtaining the solution in the physical domain for vertical vibrations of rails and sleepers. The form of solution makes the parametrical analysis efficient, leading to reliable discussion of the investigated factors influence on the rail track system dynamic response to moving train.

The obtained solution for the two layer model with two nonlinear factors is an important novelty giving possibility of the particular rail track components effects investigation. The detailed parametrical analysis for various vehicles and sets of physical parameters describing the rail track is left for future work.

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