23–25 August, 2022 | Montpellier, France

Proceedings of the Fourteenth International Conference on Computational Structures Technology Edited by B.H.V. Topping and J. Kruis Civil-Comp Conferences, Volume 3, Paper 20.1 Civil-Comp Press, Edinburgh, United Kingdom, 2022, doi: 10.4203/ccc.3.20.1 ©Civil-Comp Ltd, Edinburgh, UK, 2022

# Data-driven characterization of viscoelastic materials using hydroacoustic measurements L. del Río-Martín<sup>1,2</sup> and A. Prieto<sup>2</sup>

# <sup>1</sup>Laboratory of Applied Mathematics, DICAM, University of Trento, Italy <sup>2</sup>CITMAga, Department of Mathematics, University of A Coruña, Spain

# Abstract

Any numerical procedure in computational acoustics requires choosing an appropriate model for the constitutive law of the vibroacoustic material under consideration. Regarding the linear wave propagation in a viscoelastic material, the most common model assumptions are the classical Maxwell and Kelvin-Voigt models or the most recent fractional derivative models. Usually, once the frequency-dependent constitutive law is fixed, the intrinsic parameters of the mathematical model are estimated to fit the available experimental data with the mechanical response of that model. This modelling methodology potentially suffers from the epistemic uncertainty of a priori inadequate model selection. However, in this work, the mathematical modelling of linear viscoelastic materials and, consequently, the choice of their frequency-dependent constitutive laws is performed based only on the available experimental measurements without imposing any functional dependence on the parameters. This data-driven approach requires the numerical solution of an inverse problem for each frequency of interest. The acoustic response of a viscoelastic material due to the time-harmonic excitations generated by a transducer has been calculated numerically. In these numerical simulations, the non-planar directivity pattern of the transducer has been taken into account. In addition, the acoustic pressure field has been approximated using a plane wave discretisation to avoid numerical pollution errors at a high-frequency regime and reduce the computational cost of the calculations solving each inverse problem. To illustrate the proposed methodology for selecting the visco-elastic model, experimental measurements of insertion loss and fractional power dissipation in underwater acoustics have been used.

**Keywords:** data-driven techniques, material characterisation, hydroacoustic data, high-frequency regime

#### **1** Introduction

Elastomeric materials appear in many applications in the automotive, aerospace, or naval industries because they can be used in passive structural vibration control or noise radiation techniques [1]. These materials are polymers with a viscoelastic mechanical behaviour at ultrasound frequencies [2]. The continuous rise of new materials in industrial problems (see Figure 1), many of them with unknown properties, makes it necessary to give a complete description of their acoustic behaviour. In the present work, a polymeric material with a planar surface has been numerically characterised by using the frequency response of the insertion loss and the fractional power dissipation at ultrasonic frequencies.



Figure 1: Detail of the absorbing tile of Apltile SF5048 material (available at https://www.acoustics.co.uk/product/apltile-sf5048/).

To perform the material characterisation, a suitable choice of the viscoelastic model is fundamental: the more appropriate the model is, the more accurate its mechanical response will be compared to the experimental data. Well-known viscoelastic material models such as Maxwell, Zener, and Kelvin-Voigt models [3] or the more recent fractional derivative viscoelasticity models [4] are common choices for modelling linear wave propagation in viscoelastic materials. Usually, to estimate the unknown parameters, the constitutive laws are first fixed, and then the available experimental data are fitted with the response of the mathematical model. However, in the present work, a data-driven approach is considered [5]. This methodology avoids the need to choose a constitutive law for fitting. Instead of this, the fitting problem consists of minimising the distance between a set of experimental data and the computed values. Therefore, the choice of the viscoelastic model is based only on the experimental ultrasound measurements and not on imposing any functional dependence of the parameters in terms of frequency.

#### 2 Methods

In this work, a viscoelastic material has been characterised by using a data-driven approach instead of a classical parametric model. This material is part of a coupled problem formed by the material surrounded by water. An analysis of the mathematical modelling of the problem has been performed. First of all, the mathematical models of the problem under consideration are described, including the classical parametric models, emphasising the differences between the parametric and the non-parametric approaches. Then, the coupled problem under consideration is described, and the acoustic quantities of interest, such as the reflection and the transmission coefficients, the insertion loss, and the fractional power dissipation, are defined. The direct problem of wave propagation in the multilayer medium is solved, and the pressure fields (incident, reflected, and transmitted) are wholly described using an integral representation. Moreover, the reflection and the transmission coefficients are computed in a plane wave framework. Finally, the inverse problems for parametric and non-parametric approaches are described, taking into account different constitutive laws for the primal unknowns of the fitting problem, and emphasising the advantages and disadvantages of each used law. When the real and the imaginary parts of the Young modulus are used as unknowns, the frequency response of the levels under consideration, this is, echo reduction (ER), insertion loss (IL) and fraction power dissipation (FPD) presents spurious oscillations. Therefore, a change in the primal unknowns is necessary, and the fitting problem in terms of these new unknowns is described and solved considering a trust-region reflective.

# 3 Results

In the present work, a viscoelastic material has been numerically characterised using the frequency response of the insertion loss and the fractional power dissipation. A full non-parametric (data-driven based) methodology is used to characterise the viscoelastic material, that is, to determine the frequency-dependent parameters of the material under consideration. By using this approach, the choice of a parametric model for fitting is avoided, and the fitting is performed minimising the distance between the experimental data and the computed values.

Taking into account the setup, which is used to measure the experimental data, a multilayer medium formed by the viscoelastic material, surrounded by a fluid, is studied. For this purpose, the mathematical models of the compressible fluid and the viscoelastic solid are reviewed at the beginning of this work. Then, the coupled problem and the acoustic quantities of interest for this problem have been described. The direct problem of wave propagation in the multilayer medium has been studied. Since the available experimental data are measured in a setup where the acoustic source is a parametric array with a non-planar directivity pattern, the incident, reflected, and transmitted fields have been described through an integral representation. These integrals involve a plane wave spectrum and the reflection and transmission coefficients in a plane wave propagation problem. This approach has been validated with manufactured models where the exact mechanical behaviour

is known analytically and in real-world scenarios where experimental measurements have been used as input data of this non-parametric methodology. In both cases, the numerical results confirm the material characterisation's robustness and accuracy in terms of the complex-valued Young's modulus (storage modulus and loss factor) as it is shown in Figure 2.



Figure 2: Frequency response of the insertion loss (left) and the frequency response of the real and imaginary part (E' and E'', respectively) of the Young's modulus (right) of a viscoelastic material with manufactured mechanical properties.

#### **4** Conclusions and Contributions

Since some inverse problems are solved to characterise the material, the choice of the primal unknowns for the fitting problem is highly relevant. In this work, various constitutive laws over the unknowns have been studied, such as considering that the Young modulus is a linear function of the frequency or that is governed by an arbitrary frequency-dependent function. With these assumptions, when the fitting results present good agreement with the experimental data, the unknowns present an oscillatory behaviour. Then, a new pair of unknowns, depending on the wavenumber and the thickness of the material, has been considered. A trust-region reflective algorithm has been used to solve this fitting problem. To reduce the simulation execution-time, the adjoint method is used to calculate the derivatives of the functional cost, which are required by the used algorithm. Finally, numerical simulations have been performed. The code has been validated by using manufactured data. Then, a real-world viscoelastic material has been characterised, considering a parametric model and a non-parametric approach. The results with both methodologies have been compared, showing that the non-parametric approach allows us to improve the frequency response of the unknowns, achieving relative errors similar to the parametric methods. Concluding, the viscoelastic material has been characterising, with errors less than 10% and with a smooth frequency response of the real and imaginary parts of the Young modulus similar to those appearing in the literature. Despite the problem being ill-posed from a mathematical point of view, the proposed methodology has contributed to characterised engineered sophisticated layers as a viscoelastic material.

### Acknowledgements

The second author has been partially supported by the Xunta de Galicia project "Numerical simulation of high-frequency hydro-acoustic problems in coastal environments - SIMNUMAR" (EM2013/052) co-funded with European Regional Development Funds (ERDF). The first author acknowledges funding from the Spanish Ministry of Universities and the European Union-Next GenerationEU under the project RSU.UDC.MS15. Moreover, this work has been supported by MICINN projects MTM2014-52876-R, MTM2017-82724-R, PID2019-108584RB-I00, and also by ED431C 2018/33 - M2NICA (Xunta de Galicia & ERDF) and ED431G 2019/01 - CITIC (Xunta de Galicia & ERDF).

# References

- M. D. Rao. "Recent applications of viscoelastic damping for noise control in automobiles and commercial airplanes". Journal of Sound and Vibration, 262(3), 457-474, 2003. 2001 India-USA. Symposium on Emerging Trends in Vibration and Noise Engineering. doi: 10.1016/S0022-460X(03)00106-8
- [2] J. D. Ferry. "Viscoelastic Properties of Polymers". Wiley, 1980.
- [3] R. M. Christensen. "Theory of Viscoelasticity". Dover Civil and Mechanical Engineering. Dover Publications, 2nd edition, 2013
- [4] S. Y. Kim, D. H. Lee. Identification of fractional-derivative-model parameters of viscoelastic materials from measured FRFs. Journal of Sound and Vibration, 324:570–586, 2009. doi: 10.1016/j.jsv.2009.02.040
- [5] T. Kirchdoerfer, M. Ortiz. Data-driven computational mechanics. Computer Methods in Applied Mechanics and Engineering, 304:81–101, 2016. doi: 10.1016/j.cma.2016.02.001