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Time Integration with Subcycling in Lattice Discrete Particle Model

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Abstract

This paper concerns with multi-time step methods, also called subcycling, in lattice discrete particle models which can be used in numerical simulation of concrete, polymers and many other materials. Domain solved is split into two parts, where different time steps are used in order to save computational time.

Keywords: multi-time step methods, lattice discrete particle model.

1 Introduction

Failure behaviour of concrete is a highly complex phenomenon, and different material models have been developed. There are many constitutive models based on concepts of plasticity, fracture mechanics, damage mechanics, or combinations of them. In the paper, we limit our attention to Lattice discrete particle models (LDPM), an alternative to standard finite element models.

LDP models of concrete are based on the mesoscale level, where the size distribution of aggregates is utilized. In reference [1] and [2], an LDPM is formulated in the framework of discrete models for which the unknown displacement field is not continuous but only defined at a finite number of points representing the centre of aggregate particles.

LDP models are tightly connected with nonlinear dynamic problems, and an explicit time integration scheme is typically used. It is well known [3] that time increments in

explicit methods have to satisfy stability conditions that are affected by element size and material parameters on particular finite elements.

Failure zone in concrete can be localized into a relatively small area in comparison with the whole domain and very different time increments can be used in different parts of the domain solved. For decades, the idea of at least two different time steps in a problem has been known [3,4,5] and is denoted as a subcycling or multi-time step method. It can significantly reduce computational time.

In LDP models, evaluation of the internal forces is very demanding in comparison with the majority of continuum models, and the stiffness matrix is not explicitly available. Therefore, many algorithms in literature have to be at least rewritten because many of them are based on an explicitly assembled stiffness matrix.

2 Methods

There are the following main steps in LDPM: (a) generation of particle sizes and positions, interparticle connection and definition of potential material failure positions (facets); (b) discrete compatibility conditions and equilibrium conditions have to be assembled; (c) constitutive law has to be defined, and the vector of internal forces has to be evaluated whenever required.

Because the evaluation of the vector of internal forces is relatively computationally demanding, any reduction of the number of time steps decreases the computational time. Time integration with subcycling is very often described for linear problems where several simplifications can be used. LDP models of concrete failure are nonlinear in the whole range of loading and unloading.

The subcycling is based on the standard finite difference method. Nodes are split into two groups. Group S contains nodes, where the short time step Δt is used and group L contains the nodes, where the long time step mΔt is prescribed, where m is the ratio between long and short time steps. In this contribution, m is assumed to be an integer. All vectors and matrices can be decomposed into blocks with respect to the nodal partitioning. The superscript S denotes the blocks connected to DOFs of the S group while L denotes the DOFs connected to the L group. In the group L, quantities are expressed at time $km\Delta t$ and one subscript k is used. In the group S, quantities are expressed at time $km\Delta t + j\,\Delta t$ and therefore, two subscripts k and j are used.

$$a_k = \frac{1}{\Delta t}(d_{k-1} - 2d_k + d_{k+1})$$

where d denotes the vector of nodal displacement. New displacements in the whole domain at time $(k+1)m\Delta t$ is evaluated in the form

$$d_{k+1} = \left(\frac{1}{m^2\Delta t^2}M\right)^{-1}\left(f_k^E - f_k^I + \frac{1}{m^2\Delta t^2}M(2d_k - d_{k-1})\right)$$

where M is the mass matrix, f_k^E is the vector of external (prescribed) forces and f_k^I is the vector of internal forces which depend on the displacement vector, d_k. The vector

d_{k+1}^{L} is extracted from the d_{k+1}. In the group S, the time increment $m\Delta t$ is split into m substeps and the displacement vectors at particular times have the form

$$d_{k+1,j}^{S} = \left(\frac{1}{\Delta t^2} M^S\right)^{-1} \left(f_{k,j}^{E,S} - f_{k,j}^{I,S} + \frac{1}{\Delta t^2} M^S \left(2d_{k,j}^{S} - d_{k,j-1}^{S}\right) - \left(1 - \frac{j}{m}\right) f_{k}^{I,B} - \frac{j}{m} f_{k+1}^{I,B}\right)$$

where E denotes external, I denotes internal, S denotes the group S and B denotes interface between the group S and L. The vectors $f_{k}^{I,B}$ and $f_{k+1}^{I,B}$ are vectors of internal forces assembled from elements which contain the interface between the group S and L. It means, some element displacements are from the group L and some of them are from S.

3 Results

The subcycling algorithm was tested on a simple example based on bar elements, where initial displacements are prescribed. The benchmark was solved by the standard finite difference method and by the finite difference method with subcycling. Their comparison is depicted in Figure 1.

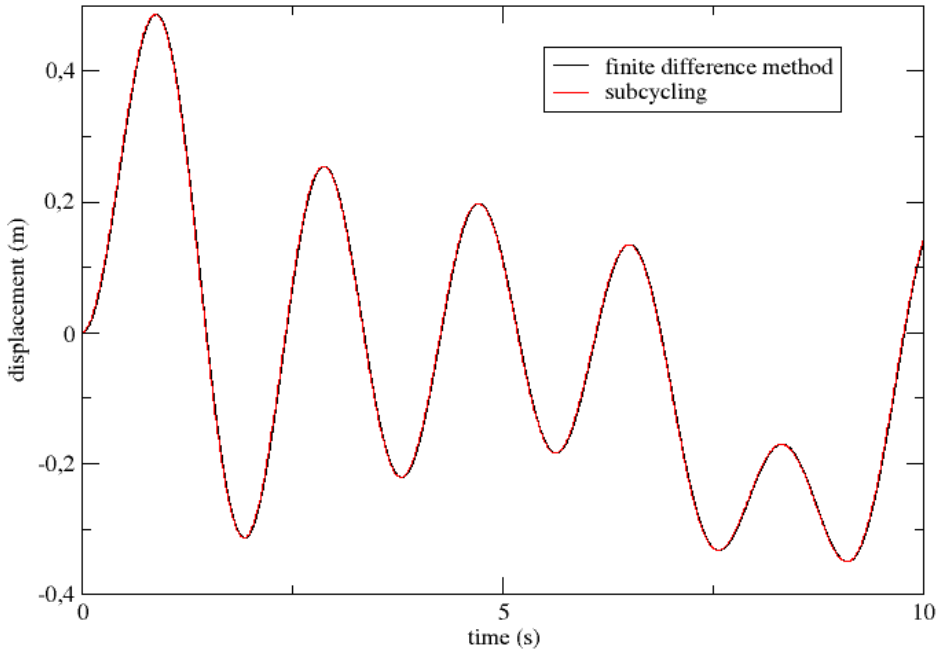

Figure 1: Benchmark on bar elements.

The subcycling algorithm was also used in the analysis of a cube with the edge length 2 mm modelled by LDPM with 47 facets. It is a very coarse model utilized only to test the implementation of the LDPM and the time integrator. More specifically, it involves only one central LDPM cell, and the remaining cells are on the boundary of the cube sample, where displacements are prescribed.

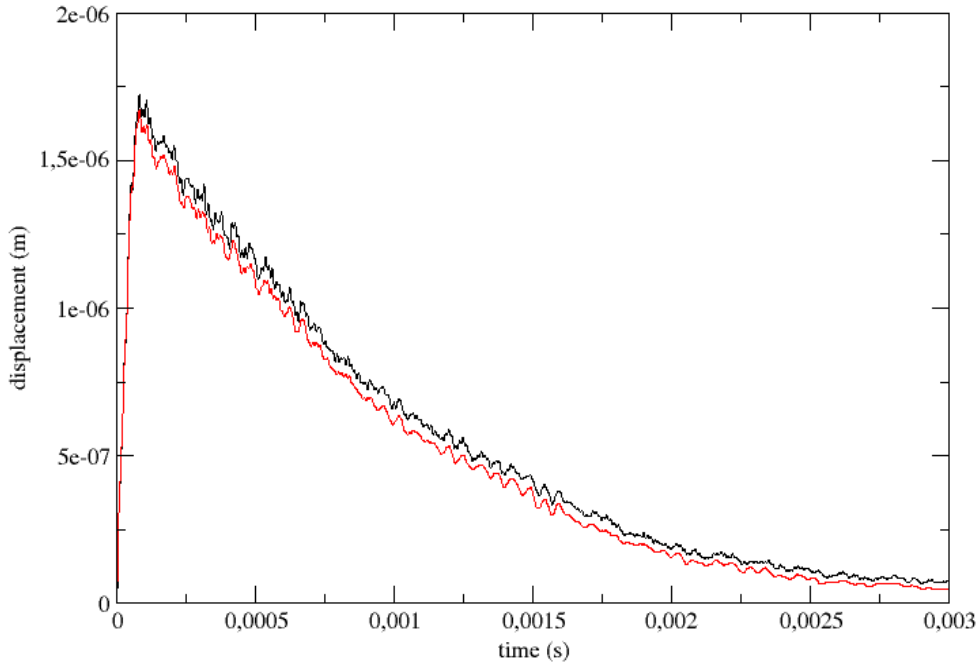

Figure 2: Comparison of vertical displacement obtained from the standard finite difference method and by subcycling algorithm.

4 Conclusions and Contributions

In this contribution, the finite difference method with the subcycling was used for the time integration of lattice discrete particle models intended to describe the failure of concrete. Evaluation of the internal forces in the LDPM is very computationally demanding, and any reduction of the number of time increments is crucial. The finite difference method with subcycling was implemented into computer code SIFEL and some preliminary numerical tests were performed. Comparison of response obtained by the standard finite difference method and by the finite difference method with subcycling is satisfactory. In the future, optimization of the implementation is needed. Additional improvement of the time integration can be achieved by parallelization of the subcycling.

Acknowledgements

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