# Parametric Design Tool for Frameworks with Minimal Rigidity Condition 

A. Ackermann and O. Gáspár<br>Department of Mechanics, Materials and Structures, Budapest University of Technology and Economics, Hungary


#### Abstract

This paper shows an algorithm designed in the Rhino-Grasshopper environment, which can create multiple minimally rigid bracing schemes for a given cubic grid structure defined by the user a priori. The algorithm is based on an additive construction method, it creates minimally rigid structures in each step. The algorithm can help the designer in the preliminary design stages to find bracing schemes suitable for the design objectives which all meet the necessary and the sufficient conditions of minimal rigidity by definition. It is an easy-to-use, semi-intuitive integrated design tool, the vast number of variations can be explored in a simple CAD environment with low computational cost. The parametric environment and the generative construction process allows the users to investigate a predefined design space, while it also allows to them to control the constraints rather easily.


Keywords: minimal rigidity, grid bracing, generative design, parametric design, topology optimization, scaffolding, integrated structural design, grammar based rules

## 1 Introduction

Scaffolding architecture is an emerging field of design. There is a pressing need for temporary structures to be built with reusable elements. Reusable elements reduce both the cost and the environmental impact of such structures. Scaffolding systems offer a ready-made toolkit that becomes increasingly popular among architects looking for a sustainable solution for their temporary designs. Scaffolding frameworks are lightweight and adaptable to various needs and layouts due to their modularity.

The complexity of the resulting net of nodes and bars is architecturally appealing. An organized chaos is exciting from an aesthetical point of view but creates a demanding task for the engineer. The emerging question is how to brace a cubic grid to best serve architectural purposes while using minimum material?

The underlying theoretical problem is the minimal bracing of cubic frameworks, or simply put, their rigidity. This paper describes a method for the generation of minimally rigid cubic grid structures. A cubic grid is minimally rigid, if removing any of its bars leads to the loss of the structure's rigidity. Both the necessary and the sufficient condition of minimal rigidity is long known [1,2] but it can only be checked during the structural analysis, or post-processing phase of design. The theoretical results offer alternatives on how to check if a given arrangement of bracing element is adequate or not, but it offers little guidance to the designer on how to place them in the first place. An interactive method to generate minimally rigid bracing schemes suiting the designers' needs is missing. The standardized bracing solutions of scaffolding systems are constrained for specific arrangements of the framework and result repetitive patterns. There is a wide variety of applicable bracing schemes for a given three-dimensional cubic framework. We propose here an algorithm that is capable of exploring this design space.

## 2 Methods

In the following we are considering 3-dimensional cubic frameworks, which are made of unit cubes. The grid consists of rigid rods connected by hinged joints. The structure is examined as a rigid body, supports are not taken into consideration. The grid structure is braced by stiff diagonals only, placed in the plane of the sides of the cubic elements, not spanning across more than one cubic element [3]. The number of the minimally required diagonals can be easily calculated based on Maxwell's formula [1], which describes the necessary condition for the rigidity of the framework. However, the grid will only be rigid, if the diagonals are arranged adequately. For the planar problem, Bolker and Crapo [4] gave an easily comprehensible sufficient condition, despite promising initial results [5] such an elegant and simple description of the sufficient condition for the three-dimensional problem has not been presented.

We introduce an algorithm written in the Rhino-Grasshopper environment, which can generate multiple minimally rigid bracing schemes for a given cubic grid geometry defined by the user a priori. The algorithm is based on an additive construction method, which creates minimally rigid structures in each step. In the present implementation the starting unit is a rigid cube with 6 diagonals-but various initial, minimally rigid units can be readily impleneted. The algorithm generates the structure by attaching and bracing unit cube elements one by one based on a predefined set of rules. These rules control the number and adequate arrangement of the diagonals in relation to how it is attached to the rigid structure (with only one side, two sides etc.). By carefully studying the possible variations, we were able to limit the set of the possible building steps to five types (as in Table 1). This allows an effective implementation of the method from an algorithmic point of view, while offering a great variety of solutions, hence freedom of design.

## 3 Results

There are multiple minimally rigid bracing formations for a defined geometry. The algorithm can create various bracing schemes by choosing the construction sequence of the cube units and the placement of the diagonals quasi randomly. The sequence of steps can also be influenced by the user, hence allowing more control over the arrangement of the diagonals The algorithm first creates a simplified 3D model of the input geometry as a cubic grid.. The input cubic geometry, which is defined by the user, can be both convex or non-convex, describing 'blocks', 'towers' or 'frames' with overhanging parts and/or voids within the hull. In the current implementation the unit elements are cubes, but the method is readily applicable to unit elements made of multiple cubes as well. Due to the additive logic of construction, the solution space is constrained. The choice of the initial unit and the building blocks also limit the number of bracing schemes that can be achived by the method described. However, a vast number of variations can be explored in a simple CAD environment with low computational cost.


Figure 1. Bracing variations for a $3 \times 3 \times 3$ unit cubic grid structure

Thanks to the parametric design elements, the geometry can be easily changed, and it is easy to navigate between the generated bracing variations. All generated variations are stored within the algorithm, therefore it is quick and easy for the designer to navigate between them and to choose the one that best fits the user's design objectives. In the design practice this tool could be used for the preliminary design of temporary buildings such as festival stages, lookout towers and various pavilions.

|  | line...existing rod dashed line...new rod | Number of new joints | Number of new rods | Change of the necessary bracing rods in the structure globally |
| :---: | :---: | :---: | :---: | :---: |
| 1 |  | 8 | 12 | +6 |
| 2 |  | 4 | 8 | +4 |
| 3 |  | 2 | 5 | +1 |
| 4 |  | 1 | 3 | 0 |
| 5 |  | 3 | 7 | +2 |

Table 1. Building steps used by the algorithm

## 4 Conclusions and Contributions

The underlying problem of the present investigation, the minimal bracing of cubic grids has mainly been of theoretical interest to researchers of the field. However, as architects started to investigate the creative potential of scaffolding applied to various temporary architectural projects a more practical aspect of minimally (or as close as possible) rigid grids, and the wide variety of the arrangement of diagonals become apparent. A grid with 'as few diagonals as possible', arranged in accordance with the users need is more transparent and easier to 'furnish' with various functional elements.

The presented algorithm is an easy-to-use, semi-intuitive integrated design tool which offers insight into the wide variety of possible minimally rigid bracing schemes in the initial design process of cubic grid stuctures. It allows the architectural and structural aspects to be considered simultaneously. Admittedly, it only targets minimal rigidity and neglects further relevant problems (i.e. internal force distribution) of structural design. However, its main novelty is that it can generate multiple bracing schemes for a given geometry, which by definition all meet both the necessary and the sufficient conditions of minimal rigidity. The parametric environment and the generative construction process allows the users to investigate a predefined design space, while it also allows to them to control the constraints rather easily.

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