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Optimal topologies considering fatigue with reliability constraint

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Abstract

The process of design optimization is generally accompanied by a decrease of the level of reliability. It can be even more problematic when one can elaborate topology optimization tasks, where the optimization process is based on material removal in least stressed areas. In order to control the safety level of the structure, it is necessary to include estimation of the probability of failure in the optimization procedure by adding constraint(s) for the reliability index. This approach is called Reliability Based Topology Optimization (RBDO). The lighter/thinner design is more likely to fail due to random factors. Among them, wind or accidental loads are most often mentioned as uncertain effects for structures. In the case of mass production of some parts, there may also be a material parameters or geometrical dimensions, which are characterized by a certain scatter caused by a production accuracy.

The aim of this paper is to formulate an iterative algorithm for topology optimization taking into account fatigue based reliability constraints. As the performance function number of cycles has been taken for low cycle fatigue problem. Combination of recently proposed topology optimization method and reliability analysis of the Authors of this presentation may lead to very complex and time consuming procedure. To tackle with this problem a First Order Reliability Method (FORM) is applied. There are several kinds of procedures within the first order approach such as: Reliability Index Approach (RIA) or the family of Performance Measure Approach (PMA) algorithms.

Keywords:, topology design, reliability, fatigue, optimisation, plasticity, probability.

1 Introduction

The process of design optimization is generally accompanied by a decrease of the level of reliability. It can be even more problematic when one can elaborate topology optimization tasks, where the optimization process is based on material removal in least stressed areas. In order to control the safety of the structure, it is necessary to include estimation of the probability of failure in the optimization procedure by adding constraint(s) for the reliability index. This approach is called Reliability Based Topology Optimization (RBDO) [6]. The lighter/thinner design is more likely to fail due to random factors. Among them, wind or accidental loads are most often mentioned. In the case of mass production of some parts, there may also be a material parameters or geometrical dimensions, which are characterized by a certain scatter caused by a production accuracy. The authors of this paper have also several papers in this topic for more than a decade (Lógó et al. [1, 2], Blachowski et al. [3], Tauzowski et al. [4])). This presentation is a result of their novel research.

The aim of this paper is to formulate an iterative algorithm of topology optimization taking into account fatigue based reliability constraints. As the performance function number of cycles has been taken for low cycle fatigue problem. Combination of recently proposed topology optimization method [3] and reliability analysis [4] of the Authors of this presentation may lead to very complex and time consuming procedure. To tackle with this problem a First Order Reliability Method (FORM) is applied [5]. There are several kinds of procedures within the first order approach such as: Reliability Index Approach (RIA) or the family of Performance Measure Approach (PMA) algorithms.

Other aspect of the topology optimization is the fact that the recent papers on that topic present mostly elastic solution procedures. Plasticity in topology optimization is almost forgotten direction. This paper presents a recently developed topology optimization procedure extended with reliability constraint. The presented numerical examples show dependence of the volume fraction on probability of failure. All aspect of numerical analysis, finite element formulation, topology optimization as well as reliability analysis library are performed by our own software implemented in MATLAB and C++.

2 Methods

The proposed methodology consists of two parts: a return mapping-based algorithm for topology optimization and a first order reliability method (FORM) for reliability assessment. Both methods operate in nested loop where at each iteration step update in the topology of the structure is performed and then reliability constraint is checked if required level of safety is assured.

The above nested optimization problem can be mathematically written as follows:

$$
\begin{aligned}
&\min_{\boldsymbol{\rho}} V(\boldsymbol{\rho}) \\
&\text{subject to } \beta \leq \beta_t. \\
&\text{where} \quad \beta = \min_{\mathbf{u}} \|\mathbf{u}\| \\
&\qquad\qquad \text{subject to } G(\mathbf{u}) = 0
\end{aligned} \tag{1}
$$

Within the first order approach, there are different methods to determining the reliability limit. In Reliability Index Approach (RIA) safety factor is determined as follow:

$$
\begin{aligned}
&\text{where} \quad \beta = \min_{\mathbf{u}} \|\mathbf{u}\| \\
&\qquad\qquad \text{subject to } G(\mathbf{u}) = 0.
\end{aligned} \tag{2}
$$

Performance Measure Approach (PMA) computes safety constrains in different way:

$$
\begin{aligned}
&\text{where} \quad \min_{\mathbf{u}} G(\mathbf{u}) \\
&\qquad\qquad \text{subject to } \|\mathbf{u}\| = \beta.
\end{aligned} \tag{3}
$$

Where $\boldsymbol{\rho}$ denotes design variable vector, $V(\boldsymbol{\rho})$ is structural volume, β and β_t are actual and target reliability index, respectively, $\mathbf{u}$ is standardized random variable vector and finally $G(\mathbf{u})$ is a performance function related to the reliability of the structure.

Low cycle fatigue problem is based on continuum damage approach where scalar (in isotropic case) field D is micro pores volume fraction caused by plastic material deterioration. Assuming the value of plastic damage ratio representing fatigue failure D_{C} the number of cycles N_r to failure can be determined:

$$
N_r = \frac{D_{\mathrm{C}} - D}{\frac{\delta D}{\delta N}}, \tag{4}
$$

where $\frac{\delta D}{\delta N}$ is increment of damage ratio per one load cycle. Performance function based on number of cycles has the form: $g(x) = N_{\mathrm{r}} - N_{\mathrm{TARGET}}$, where N_{TARGET} is the threshold value of number of cycles.

The reliability procedure above can be interchanged any appropriate formulation of the reliability assessment (second order reliability method or different simulation techniques).

3 Results

To demonstrate the correctness of the optimization procedure presented above a cantilever beam and an L-shape structure are investigated (Figure 1). The following material constants are assumed: E = 1 and ν = 0:3. Vertical load is deterministic f_v = 1, while horizontal one is probabilistic with standard normal distribution f_h = N(0; 0:4).

Comparison of deterministic and reliability-based optimal topologies for two different structures obtained using the proposed methodology above has been shown in Figure 1. One can see the optimal topologies are almost the same ones, however the stress distributions have significant differences.

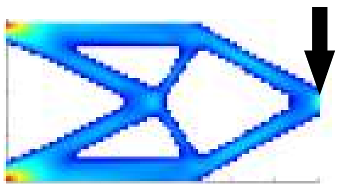
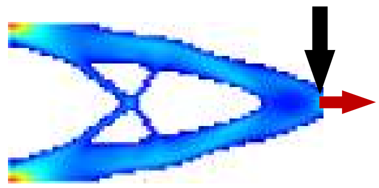
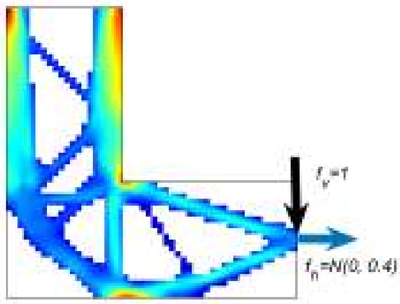
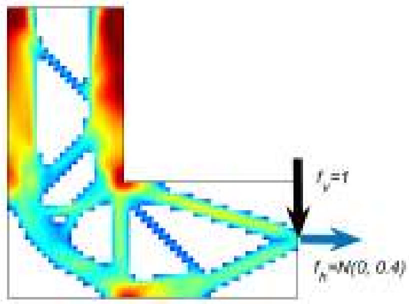
	
Deterministic optimal topology of cantilever structure	Reliability-based optimal topology of cantilever structure
	
Deterministic optimal topology of L-shaped structure	Reliability-based optimal topology of L-shaped structure

Figure 1. Comparison between deterministic and reliability-based optimal topologies. Force in black colour represents deterministic load, while the horizontal force corresponds to probabilistic one.

4 Conclusions and Contributions

This work presents a new iterative, nested (double loop) algorithm for topological optimization with regard to fatigue based reliability constraint. The external topological optimization loop implements a heuristic algorithm of material removal in the least stressed areas. The internal loop controls whether the failure probability is not exceeded. The applied first-order approach (FORM) to the safety control in the topological optimization process is not a high time overhead. Both Reliability Index Approach (RIA) and Performance Measure Approach (PMA) algorithms estimate the reliability index in a relatively short time especially when the algorithm is implemented using parallel computing. The effectiveness of the presented method has been confirmed by a numerical example cantilever and known from the literature as L-shape benchmark. Topologies taking into account reliability conditions have not only higher volume fraction but also their different shape is more resistant to random load conditions. The application of reliability analysis in conjunction with the low-cycle fatigue problem allows to ensure that the designed structure has sufficiently high number of cycles.

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