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## **Energy harvesting and sensitivity analysis of vibrating piezoelectric composite beam**

**W. Larbi<sup>1</sup>, R. Aloui<sup>2</sup> and M. Chouchane<sup>2</sup>**

<sup>1</sup>**Conservatoire National des Arts et Métiers (CNAM), France**

<sup>2</sup>**Ecole Nationale d'Ingénieurs de Monastir, Tunisia**

### **Abstract**

Vibration energy harvesting using the direct effect of piezoelectricity has attracted increasing attention during the last two decades. Different modeling techniques have been applied to describe the electromechanical coupling effect of a piezoelectric harvester and to predict its electrical output. This study aims to identify the most important properties of both harvester substrate material and piezoelectric material that cause uncertainty in the predicted performances of the harvester. Global sensitivity analysis, applied in this paper, is a promising method used to identify systems parameters which have significant impact on the system output. In this paper, the Elementary Effects method (EEs), a particular implementation of the global sensitivity method, is used to identify the impact of substrate and piezoelectric material properties on the voltage frequency response function of a typical bimorph piezoelectric energy harvester with fixed geometry. With a small number of model evaluations at selected ranges of material properties, it has been found that the elastic modulus and density of the piezoelectric layer are the parameters which lead to the largest output variability. Furthermore, it has been found that the order of importance of the parameters can change from short-circuit to open-circuit conditions.

**Keywords:** global sensitivity analysis, elementary effects, piezoelectric energy harvesting, finite element method

## 1 Introduction

Sensitivity analysis of dynamic structures and mechatronic systems is very useful tool in many engineering problems, such as: parametric identification, structural optimization, model updating problems and others [1]. In previous research studies, Lasecka-Plura et al [2] studied the sensitivities of steady-state vibration response and the FRF of planar frames including viscoelastic dampers. A comparison was made between the first-order, the second-order and the exact solution sensitivity measures. R. Aloui et al. [3] introduced the first order sensitivity to evaluate the frequency response functions of piezoelectric energy harvesters to the variation of the load resistance from short circuit to open circuit conditions. Ruiz et al [4] analyzed the global sensitivity of the frequency response function of the output voltage of piezoelectric harvesters then the uncertainty propagation referring to the physical parameters of the harvester. Their approach is based on the identification of Sobol' indices to assess the robustness of the stochastic prediction. Peralta et al [5] presented a procedure to update the electromechanical properties of the piezoelectric energy harvesters based on Bayesian updating approaches and global sensitivity studies. This technique constitutes a powerful method for the robust design and prediction of the performances of vibration energy harvesters using piezoelectric material.

In this research paper, we propose a global sensitivity analysis of a bimorph piezoelectric energy harvester. First, it is necessary to establish a deterministic model to identify the output responses of the harvester. The bimorph piezoelectric energy harvester is modeled using the finite element method. For the sensitivity analysis, the Morris method is applied using the modulus of the voltage frequency response function of the harvester as an output and a set of uncertain physical parameters. The considered numerical example shows the robustness of the EEs method compared to others global sensitivity analysis methods.

## 2 Methods

The harvester consists of a piezoelectric bimorph cantilever beam based on Euler Bernoulli beam theory, which is composed of two piezoelectric layers bonded to an elastic substrate as shown in Figure 1. The harvester excited under harmonic base motion at the clamped end.

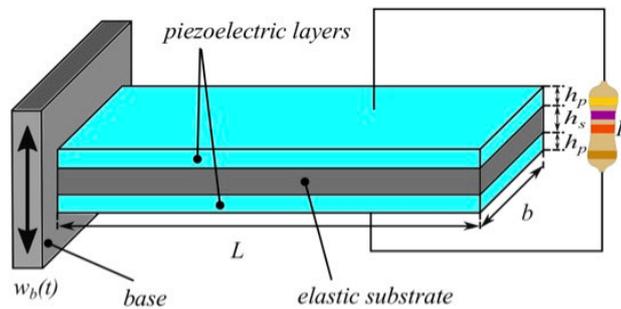


Figure 1: Bimorph piezoelectric energy harvester

The voltage frequency response function (FRF) of the system is derived from a finite element formulation applied to a piezoelectric laminated beam and formulated using Euler-Bernoulli theory. The Elementary Effects Method is then applied to study the effect of the variability of the material properties of the substrate and piezoelectric layers on the voltage FRF output of the harvester. This method constitutes a simple way for screening the effect on the output responses of a few important input factors among the many factors that can be contained in the studied model. The fundamental idea behind this method is given by Morris [6, 7], who introduced the concept of elementary effects, proposing the construction of two sensitivity measures: the mean and the standard deviation of the EEs for each input parameter. In practice, this method is conducted by: (a) defining the model and its input parameters, (b) assigning Probability Density Function (PDF) to each input parameter, (c) generating an input matrix using an appropriate random sampling method, (d) calculating the corresponding output response vector, and (e) computing the EEs and then the sensitivity measures of each input/output relationship [8]. The average and standard deviation are then computed for a set of elementary effects for each factor.

### 3 Results

This section presents a numerical example of the harvester introduced before, which is composed of two identical piezoceramic layers of PZT-5A bonded on the top and the bottom surfaces of an aluminum substrate. For the finite element discretization, the harvester is modeled in 1D using 30 linear elements with three degrees of freedom per node. The model is validated for the frequency range from 0 to 5000 Hz by comparing the first three resonance frequencies computed by the finite element model to those given by the analytical solution computed by Erturk and Inman [9] for the same harvester for the short-circuit and open circuit conditions. The selected uncertain parameters are the Young modulus and the material densities for both the substrate and the piezoelectric layers, also the permittivity and piezoelectric constants for the piezoceramic layers. A tolerance of  $\pm 20\%$  is reported for the piezoceramic PZT-5A electromechanical properties and  $\pm 10\%$  for the Young's modulus and density of the aluminum substrate material. An uniform distribution is assumed for all materials properties of the harvester.

The aim of the study is to identify the effect of the excitation frequency on the importance order of the parameters particularly in the vicinity of the first mode. Furthermore, the system is be studied for both a short-circuit condition simulated using a load resistance  $R = 100 \Omega$  and an open-circuit condition simulated using for a load resistance  $R = 107 \Omega$ .

Figure 2 shows the voltage and tip displacement FRFs responses of the harvester versus the excitation frequencies in the vicinity of the first mode for 70 samples. One can be observe the variability in the first natural frequency and in amplitude peaks. The voltage FRF peak varies between 0.02032 and 0.01152 V/g whereas the tip displacement FRF peak varies between 442.46 and 756.55  $\mu\text{m/g}$ .

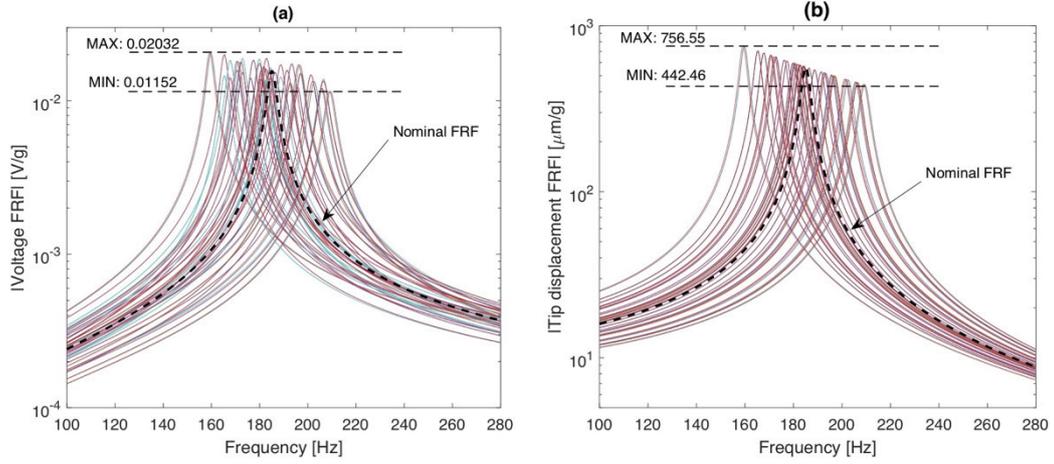


Figure 2: Modulus of FRFs versus excitation frequencies of the first mode for the 70 simulations of: (a) voltage and (b) Tip displacement for the short circuit condition  $R = 100\Omega$ .

Figure 3 shows the average  $\mu$  and the standard deviation  $\sigma$  of the EEs to the electromechanical characteristics of the bimorph piezoelectric energy harvester for the output voltage FRF at the selected excitation frequencies in the vicinity of the first natural frequency. One can notice that the hierarchical influence of the input parameters is unchangeable and independent of the excitation frequency.

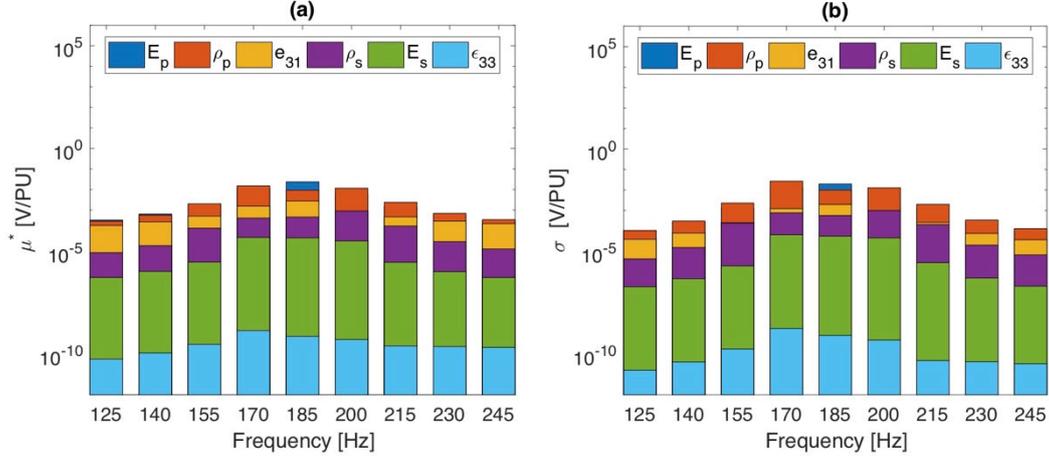


Figure3: EEs measures for the voltage FRF.

## 4 Conclusions and Contributions

In this work, a finite element formulation of the electromechanical coupling problem for a laminated piezoelectric cantilever beam introduced. The finite element formulation is applied in the case of a symmetric bimorph piezoelectric energy harvester. The FRFs of the harvester are used as the performance predictors. The Morris global sensitivity method using the elementary effects is used to study the

effect of the variability of the material properties of the substrate and piezoelectric layers on the voltage FRF out- put of the harvester. The EEs method can estimate the effect of parameters and their interactions by considering both the mean and the variance of the elementary effects. Using only 70 simulations, the sensitivities measures show that the piezoelectric elastic modulus and its density are the most influential parameters on the voltage FRF. These results are in agreement with the published research of Ruiz et al [10] which used the Sobol' indices as a variance based sensitivity analysis method. The method of Morris correctly screens the most and least sensitive parameters among few selected parameters for a spatially distributed bimorph piezoelectric energy harvester FE-model with fewer model evaluations than the Sobol's method.

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