



Proceedings of the Seventh International Conference on
Railway Technology: Research, Development and Maintenance
Edited by: J. Pombo

Civil-Comp Conferences, Volume 15, Paper 3.9
Civil-Comp Press, Edinburgh, United Kingdom, 2026

ISSN: 2753-3239, doi: 10.4203/ccc.15.3.9

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Some Tribological Aspects of Interaction of Wheels and Rails

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Abstract

Tribological aspects of interaction of wheels and rails have a major impact on traffic safety, energy consumed on traction, dynamic characteristics of a train, maintenance, repair costs, and more. In this work, the tribological aspects of interaction of wheels and rails are considered taking into account properties and degree of destruction of the third body. The causes of negative, neutral and positive friction, as well as mild, severe and catastrophic wear, and methods of friction management are identified. In addition, the paper evaluates impact of the wheel wear on the friction path, sliding velocity and the frequency of zigzag motion.

Keywords: wheel, rail, third body, destruction, loading, wear.

1 Introduction

The performance properties of rolling stocks are largely determined by the interaction conditions between the wheels and rails. The movement of the rolling stock is based on the rolling of the wheel on a rail, but this is accompanied by slippage in both the longitudinal and transverse directions. The increase of wheel slippage on the rail leads to the increase of various undesired phenomena, among which are: thermal loading of the contact zone, destruction rate of the third body and direct contact between the juvenile surfaces, which create conditions for their seizure. This leads to the instability of the friction coefficient; increased energy consumption for traction, wear intensity of the rubbing surfaces, and environmental pollution from vibration, noise and other undesirable phenomena. Studying the tribological properties of rails and wheels is a

highly complex task, encompassing numerous aspects and factors, some of which are stochastic or are not identified. By testing alone, it is extremely difficult to identify the various factors influencing these properties and to determine the contribution of each factor to the overall process. To date, no modern testing facility can fully reproduce the tribological characteristics of wheels and rails and their impact on rolling stock performance. Such indicators of rail transport efficiency as the probability of derailment of a wheel, energy consumed for traction, environmental pollution, service life, and maintenance costs of wheelsets, rails, and friction brakes largely depend on the tribological properties of the contact zones of the corresponding interacting elements [1-3]. Thus, predicting the tribological properties of wheels and rails, as well as controlling friction are highly relevant for improving the performance of rolling stocks. However, despite numerous efforts by scientists and engineers worldwide, the processes occurring in the wheel-rail contact zone remain insufficiently studied, and prediction and control of tribological properties remain challenging.

Intensive work is underway in many countries to address these issues. New friction modifiers for rolling and steering surfaces of wheels and rails, as well as methods and devices for their application are being developed; new wheel profiles, wheelsets, bogies, and brake designs are being developed. These measures increase speeds, axle loads, and traffic safety. However, the expected and desired results are not always achieved: wheels and brake pads remain the most vulnerable components of rolling stocks, and the energy consumed for traction is considered excessive, as are vibration, noise, and other undesirable phenomena. These issues are interrelated, and their solution requires a unified, well-founded approach and confirmation of the obtained results in laboratory and field conditions.

Currently, the operating conditions of rolling stocks and tracks, and the requirements for them have changed dramatically – speeds and axle loads have been increased, as have the requirements for traffic safety, environmental friendliness and efficiency.

The friction surfaces of wheels and rails are sensitive to mechanical and thermal loads, which can be significantly affected by slippage. In such conditions, stress can reach up to 3 GPa, and speeds of 100 m/s. The slip occurs when: the traction increases; the friction coefficient on the rolling surfaces decreases; during sudden braking; or when passing curves due to differences in the travel distances of the inner and outer wheels of a wheelset; it also occurs when friction forces are unstable; during zigzag movement on straight sections; due to design features of the track and running gear of the rolling stock; due to rigid installation of the wheels on the axles of the wheelset, due to the changing geometric parameters of the friction surfaces during operation due to wear, etc.

Friction brake pads exert significant forces on the wheel tread surface. Under prolonged braking, they become very hot, which leads to an increase in the wear rate, changes in the structure of the surface layers, and the appearance of melting marks on the tread surface. Axial forces acting on the wheelset in curves and lateral movements

of the wheelset and vehicle can cause extremely undesirable lateral forces. They can exceed 90 kN and the total lateral force of the wheelset in curves can lead to rail overturning. In addition, an increase in axial forces dramatically increases friction forces and the energy consumed for traction, the likelihood of wheel derailment, the intensity of wheel and rail wear, environmental pollution from vibrations, noise and wear products, as well as other factors [4-6], which lead to an increase in operating costs. Thus, predicting the tribological properties of wheels and rails, and the ability to control friction are of great importance for improving the performance. However, the increased speed and axle loads of modern rail vehicles have increased both the power and thermal loads in the wheel-rail contact zone. Under these conditions, traditional methods for predicting and solving such problems may not yield the expected results and will require new approaches and solutions. Until railway transport overcomes key obstacles, it will be impossible to fully realize its potential and capabilities.

2 The wheel-rail interaction features

Complex physical, mechanical, and tribo-chemical phenomena occurring in the wheel-rail contact zone under direct environmental influences complicate the solution to problems of efficient operation. Much work has been devoted to the analysis of these phenomena, but they are not always suitable for solving various engineering problems. This applies to problems of predicting and controlling friction forces, types and rates of wear, environmental factors, the predisposition and prevention of wheel derailments, reducing energy consumption and the costs of preventive maintenance and repair.

One of the most vulnerable parts of wheel and rail during interaction are wheel flange root and rail corner. Figure 1 shows the tread surfaces designed for rolling, traction, and braking; the steering surfaces (the rail head and wheel flange side surfaces); the wheel flange root and rail corner surfaces designed for rolling, traction, and braking.

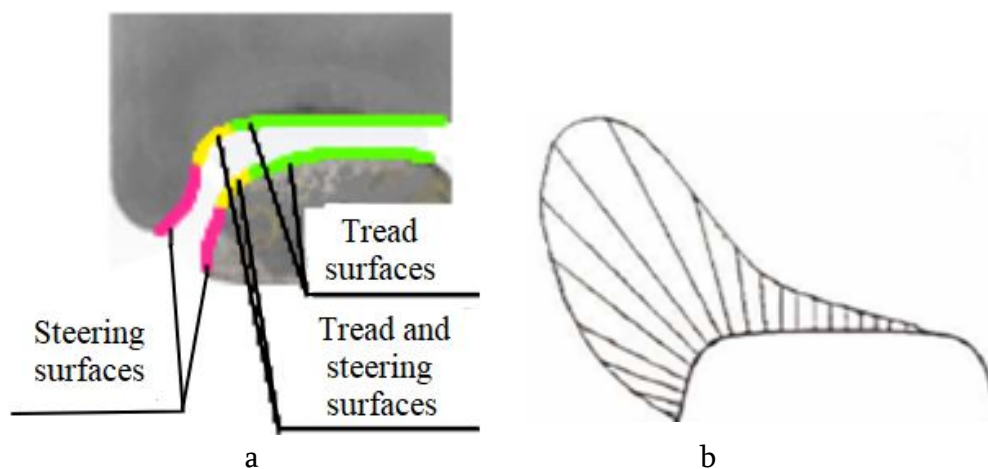


Figure 1: Components of the wheel and rail interacting surfaces (a) and friction work value (b) [7].

But traction, braking and steering require mutually exclusive properties, and the “ideal” value of the friction coefficient ($\mu < 0,1$) in the contact zone of the flange root and the rail corner, while desired for steering, is not acceptable for both other cases.

As can be seen from Figure 2, the maximum frictional work is observed in the area of the base of the flange and rail corner, which simultaneously act as rolling and steering surfaces.

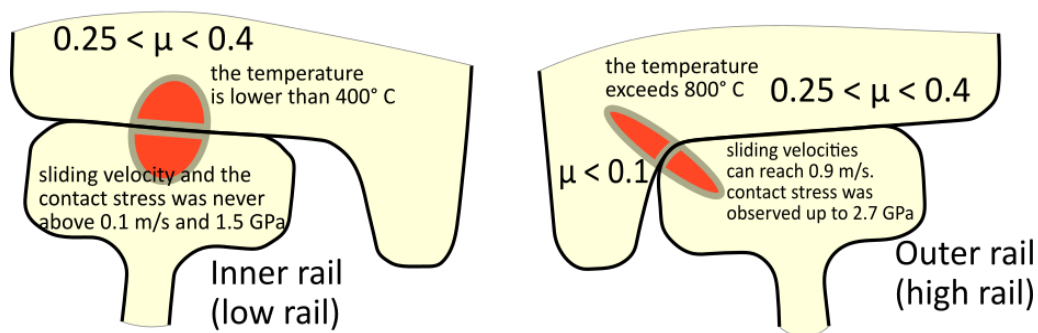


Figure 2: The ideal values of the friction coefficients and stress distribution in the contact zone of wheel and rail according to [8] and the thermal loads.

The friction coefficient for wheel-rail interaction can vary in the range 0.05 - 0.8. The values of the friction coefficient for the tread and steering surfaces must be correspondingly in the ranges of 0.25-0.4. The optimal value of the friction factor for tread surfaces is 0.35 [9] and for steering surfaces - as low as possible. The scuffing on the wheel and rail steering surfaces causes rise of the friction coefficient, energy consumed on rolling, vibrations, noise, wear intensity and probability of derailment.

As Figure 2 shows, the power and thermal loads on the rolling surfaces are relatively low. However, when wheels operate in traction and braking modes, creep and relative sliding velocity increase. This leads to the third body destruction, increased tangential stresses, and various types of destruction of the interacting surfaces. In the contact zone of the flange and rail corner, increased slippage is characteristic, which, when the third body is destroyed, leads to an increase in tangential stresses and temperatures. The shear deformation generated on the surface sharply decreases towards the depth and multiple repetition of such processes results in superficial plastic deformations, lamination and fatigue damage (Figure 3) [9, 10].

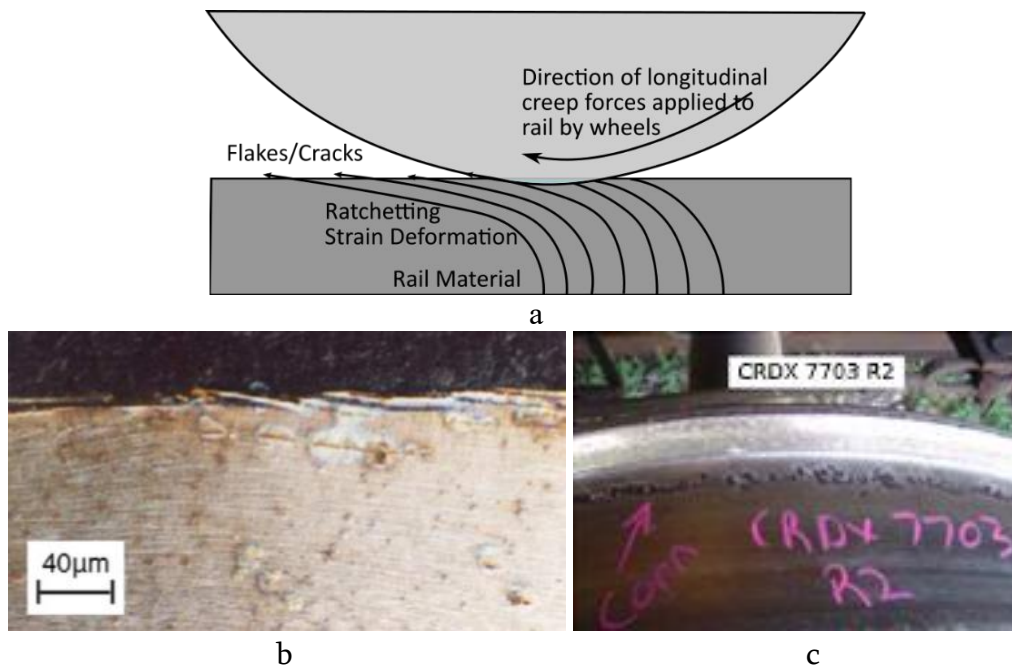


Figure 3: The scheme of the surface plastic deformation (a); appearance of cracks and lamination (b); appearance of fatigue pits (c).

Thus, for providing the interacting surfaces with due tribological properties, their separation from each other by the continuous third body with corresponding properties is necessary. It should be noted that various types of surface damage take place simultaneously, proceed with various intensity and a dominant type of damage is ascertained visually. It should also be noted that the absence of information about signs of the surface damage, the types of which are determined visually, makes it difficult to obtain an objective picture. This applies to scuffing, scoring, fatigue and other types of damage.

The experimental researches have shown that surface damage intensity and types are especially sensitive to the relative sliding velocity and shear stresses. Thereby, at low relative sliding velocities of the surfaces, when power of the thermal action, velocity and resistance of the shear deformation in the contact zone are comparatively small, stability of the third body and its resistance to scuffing are high and a main type of damage is fatigue wear [11]. With the increase of the total and relative sliding velocities of surfaces, thermal load of the actual contact zone and destruction rate of the third body increase. However, time of action of this load, thickness of the heated-up layer and sizes of micro-asperities generated because of the scuffing and subsequent rupture of the seized places, decrease. At the increase of the relative sliding velocity, share of the adhesive wear and scuffing increases and it often becomes a dominant type of damage.

Currently, a large number of experimental and theoretical studies are being conducted to improve the efficiency of wheels and rails, and there have been some successes in this area. However, due to insufficient study of the processes occurring in the contact zone, achieving the desired results is not always possible. The various

aspects of the complex physical, tribo-chemical and mechanical processes taking place in the contact zone are not properly studied yet, which is accordingly reflected on the operation quality and service life. As an example, can be cited the interaction of the wheel and rail that occurs on: the tread surfaces during rolling, traction and braking; steering surfaces mainly in curves; flange root and rail corner at rolling, traction, braking and steering. For interacting surfaces of wheels and rails, unpredictable change of tribological properties (sharp increase of the friction coefficient and wear rate and so-called catastrophic wear) is typical. As the main cause of the latter is considered to be the heaviest form of the adhesive wear – scuffing, that is not properly studied yet [12] and whose signs are appearance of pits and scratches on the surfaces and transfer of the material from one surface on the other.

At the presence of a continuous film of the third body in the contact zone, there is a stable friction force and low intensity of damage to the surfaces. But the destruction of the third body in the contact zone of heavily loaded surfaces leads to seizure, which leads to an increase in tangential stress and deformations in the seized areas. During friction, the seized areas can be separated without removing material from the surfaces or with the removal of this material in the form of wear products, as well as with the removal of material from one surface and its adhesion to another surface (Figure 4).

At heavy working conditions, when destruction of the third body is irreversible and seizure is spread over the factual contact area, the relative displacement of the surfaces causes sharp increase of: the shear stresses, corresponding deformations, values and instability of the friction forces, and destruction of the seized places. Strength of the seized places may exceed the strength of the interacting bodies because of which the cracks may appear on the seized places of surfaces, material pulled out from one surface can form a wear product or can be transferred on the other surface and attach to it, which is followed by development of the scuffing process (Figure 4).

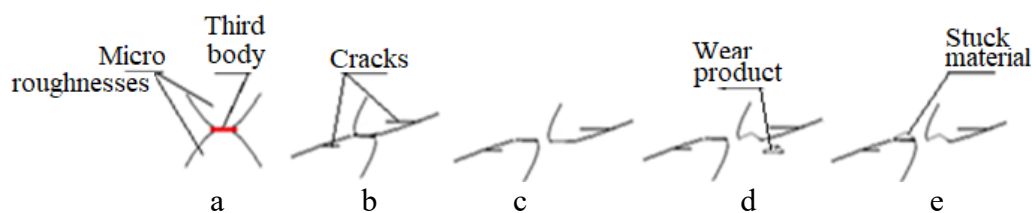


Figure 4: Scheme of interaction, seizure and destruction of micro-roughnesses: a - interaction of stationary micro roughnesses in the presence of the third body; b - their interaction during friction, destruction and absence of the third body, seizure and appearance of cracks on micro roughnesses; c - separation of plastically deformed microroughnesses without removing material from them; d - separation of micro roughnesses with removal of material in the form of wear product; e - separation of micro roughnesses with removal of material from one surface and its adhesion to another surface.

As can be seen from the figure 4, the interaction of micro-roughnesses on heavily loaded friction surfaces can lead to the destruction of the third body, seizure, and increased shear stresses and adhesive wear. Seizure can involve a group of adjacent

micro-roughnesses and lead to cracks in individual micro-roughnesses or the surface of the group of adjacent micro-roughnesses, leading to fatigue failure of the surfaces. Increased tangential deformations can also lead to its critical form—seizure. Thus, fatigue and adhesive processes in the contact zone occur simultaneously, and the dominant one depends on the operating conditions.

The actual contact zone is 5-10% of the nominal contact zone [13, 14] and with direct contact of surfaces on the actual contact zones, which are stochastically distributed, significant tangential stresses and corresponding deformations and heat release arise. This causes a difficult to predict, destructive effect of surface layers and many studies are devoted to their prevention [15, 16].

The heavy power loading of wheels and rails, and rolling-sliding of interacting surfaces in the contact zone create high thermal loads and conditions for intensification of micro- and macro-scale physical and chemical processes, which stipulates the formation and destruction of the third body. Their study requires an interdisciplinary approach [1]. However, the essence of the processes (especially micro-scale ones) occurring in the contact zone has not been studied sufficiently, which complicates their mathematical modeling, hinders prediction and control of tribological characteristics of wheels and rails, and attempts at improving their performance and can lead to various unfounded decisions. The examples of this are the lubrication of the rail corner and the bottom of the wheel flange, which act simultaneously as steering and rolling surfaces. Also, the reduction in the distance between rails in the Russian standard gauge from 1524 mm to 1520 mm while maintaining the same distance between wheels, which significantly increases the rate of the wheel and rail wear [17]. With a track width of 1520 mm, reaching 1524 mm due to wear, increases the "over wear" of rails and flanges [18]. This is explained not by the advantage of the 1520 mm track width in increasing wear resistance, but by changes in the wheel and rail profiles due to wear [19] and emphasizes the importance of proper limitation of the degree of wear of the wheels and rails for their efficient use. Besides, the role of physical and chemical processes occurring in the contact zone are not taken into account, similarly with the properties of lubricants and third bodies in the formation of boundary and oil layers. While micro- and macro-scale processes occurring in the real contact zone - the formation, properties and stability of the oil and boundary layers - are largely determined by the properties of modern lubricants [5].

It should also be noted that existing designs of the undercarriage of rolling stocks and tracks, train operating modes, lubrication technologies and modification of wheels and rails cannot always ensure the proper operational properties of wheels and rails due to the complexity of controlling tribological properties under constantly changing relative sliding speeds, normal and shear stresses and other parameters.

Modern modification technologies have significantly improved the tribological properties of wheels and rails, but they still remain vulnerable and require further improvement [4].

The research has shown that the determining parameter for performance of a heavily loaded friction contact is a degree of the third body destruction, which depends on the number of kinematic, geometric, power, thermal and other parameters [20]. Among the many factors, the power and thermal loading of the contact zone and the stability of the third body (especially the boundary layers) are particularly noteworthy.

It is known that when the wheelset moves in a straight segment, due to the conical shape of the rolling surface of the wheel, it performs periodic lateral movements (zigzag movement), which affect the friction path, sliding velocity and wear rate [21]. As an example, let's consider new standard wheelset with the diameter of the rolling circumference $D=957$ mm (radius $R=D/2=478.5$ mm) and a slope of the profile $n=\tan\alpha=1/20=0.05$, and the same wheelset worn-out by 7 mm with the profile slope of $n=0.34$. In such conditions the amplitude and frequency of lateral movement and, accordingly, the friction paths of a new and worn wheels are different. The calculated values of the friction path per 1 km, zigzag movement frequency, and sliding speed for a freely rotating wheelset (without bogie) at a speed of 100 km/h moving along a straight section of the track show that the sliding speed, zigzag movement frequency, and friction path for worn wheels exceed those of unworn wheels by 8.0, 2.6 and 7.96 times, respectively [19].

3 The experimental researches into variation of tribological properties of the interacting surfaces

For more detailed study of the properties and state of the third body in the contact zone, we performed the experimental researches on the high speed, serial twin disk machine SMC-2-E and TRIBOLOG with the use of existing lubricants (used for Stadler train in Georgia) and ecologically friendly friction modifiers, developed by us. Figure 7 shows the results of experimental studies on determining the influence of the friction path on the friction coefficient with a single application of the lubricant to the rubbing surfaces of the rollers rolling with 5% sliding on the SMC-2-E machine.

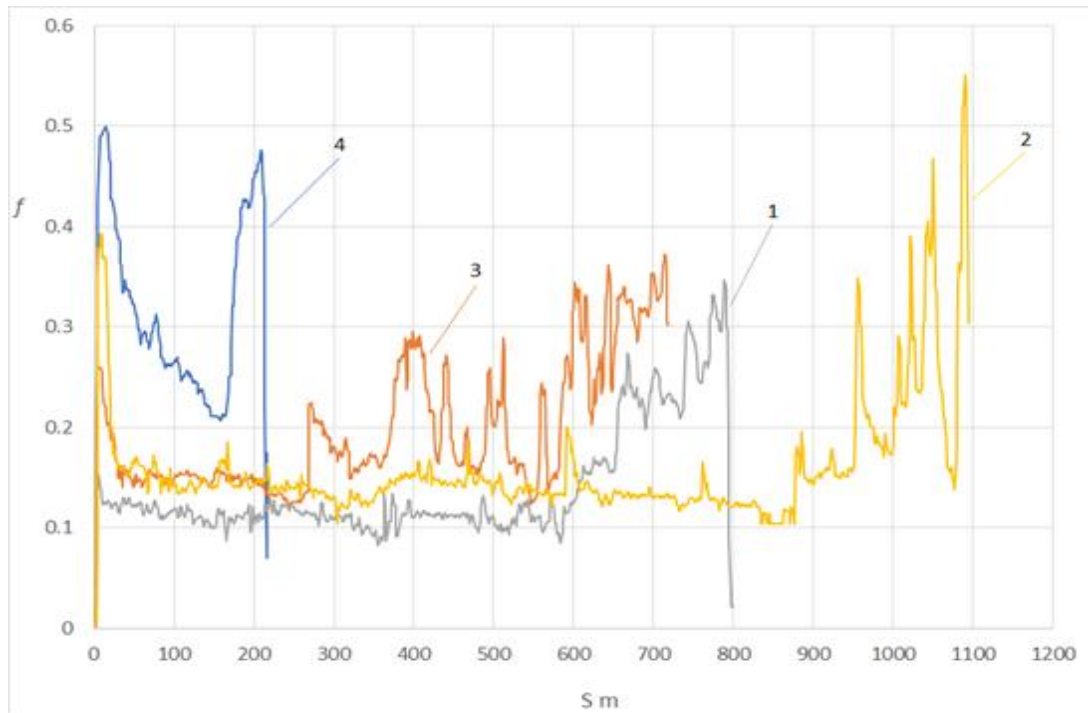


Figure 7: Dependence of the friction coefficient (f) on the friction path (S) under conditions of sliding for various friction modifiers applied once to the friction surface under the load of 271 MPa: 1, 2, 3 – lubricants developed by us; 4 – Lubricant used for Stadler train in Georgia.

The contact zone must support axial load, frictional load, and thermal load arising from the interaction of the wheels and rails. As can be seen from the graphs, in terms of the coefficient of friction and stability of the third body, the friction modifiers developed by us are characterized by higher tribological properties.

Of particular importance are the stability and tribological properties of the third body at high speeds, when the power, thermal loads and damage to the contact area of the wheel and rail increase.

The lubricant that we created was tested on the machine “TRIBOLOG” under conditions of pure sliding of the sphere with a diameter of 5 mm. Figure 8 shows the results of experimental studies of the lubricant made by us, carried out under initial point contact and pure sliding.

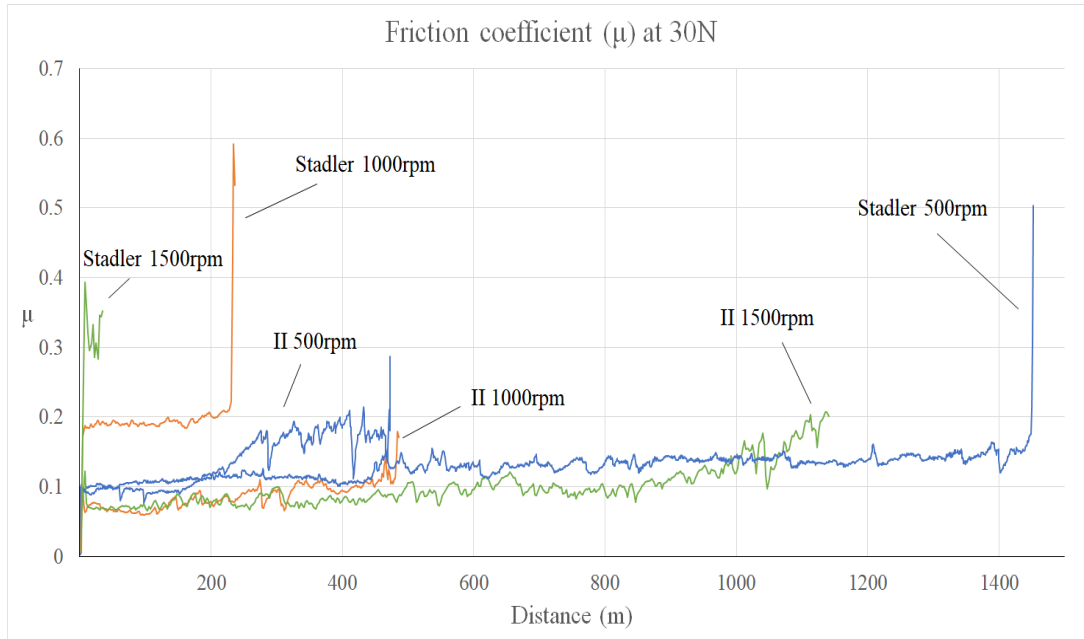


Figure 8: Dependence of the friction coefficient on the sliding distance under normal load and different sliding speeds for the lubricant developed by us (II) and for the lubricant used for Stadler train.

With increasing force and thermal effects, the rate of atomic and molecular processes occurring in the contact zone and facilitating the formation and destruction of the third body increases [22, 23]. As can be seen from the graphs, for the Stadler lubricant, with the increase of sliding speed, the friction path decreases till the first signs of seizure; while for the lubricant developed by us, with the increase of sliding speed, the friction path increases till the first signs of seizure.

In stress-augmented thermal activation, the activation energy barrier that controls the rate of atomic and molecular processes is reduced by the application of stress, with the result that the rate of these processes rises exponentially with the rise of applied stress. This can be explained by the use of special additives to lubricants and the results of several recent studies, which have shown that the formation of several types of boundary films on rubbing surfaces is due to thermal activation enhanced by the stress. [23].

4 Analysis of results of the experimental researches

Worsening of the working conditions caused by the partial, non-progressive damage of the third body in the separate unit places corresponds to the separate small impulses of the friction moment. The continuous third body, or the one that is at the initial stage of destruction and self-restorable, corresponds to the negative friction. Destruction of the third body in the multiple places leads to formation of multiple adhesive junctions of micro-asperities, disruption of these junctions, as well as comparatively increased impulses of the friction moment and to “neutral friction”. The progressing of the third body destruction leads to extended discontinuous third body, adhesive junctions of

micro-asperities, disruptions of these junctions and increase of the friction forces corresponding to “positive friction” [20] (Figure 9).

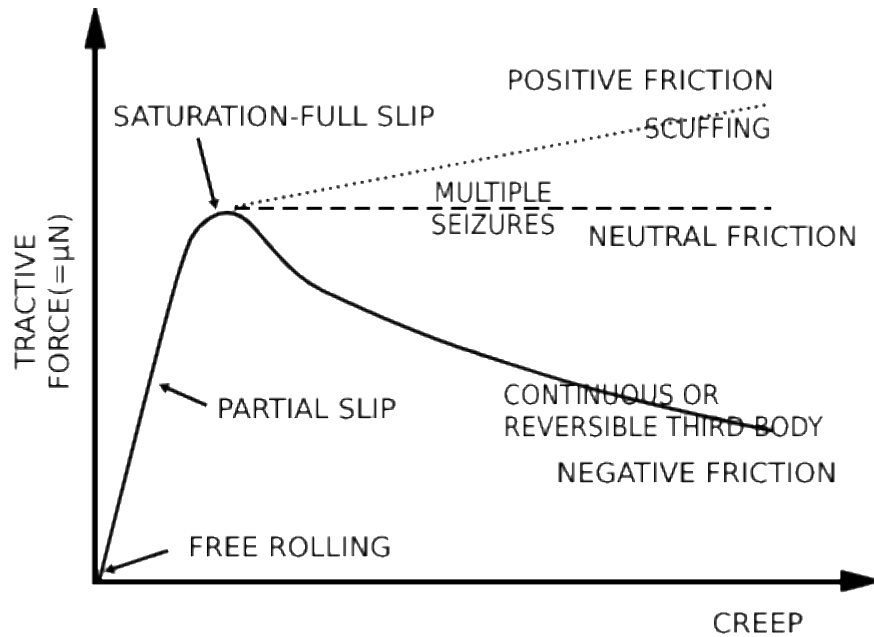


Figure 9: Friction/creep relationship.

The destruction of third body in the separate unit places corresponded to the separate small impulses of the friction moment. And adhesive wear of the mean intensity and the balance of destruction and restoration of the third body was observed, which corresponds to the “mild” wear type (Figure 10 a) [8]. The destruction of the third body in multiple places corresponded to the multiple, comparatively increased impulses of the friction moment and a heavy form of the adhesive wear – scuffing; separate seizures of the micro-asperities (or unit seizures located in one or several narrow strips) that corresponds to the “severe” wear type (Figure 10 a). But then, without additional feeding or insufficient renovation of the third body, continuation of operation leads to extension of destruction of the third body and appearance of multiple seizures. Then the friction moment and its variable component are rapidly growing and the unit and multiple seizures (causing scuffing) are becoming uninterrupted, after which, they propagate on the whole width of the interacting surfaces and the corresponding wear becomes “catastrophic”.

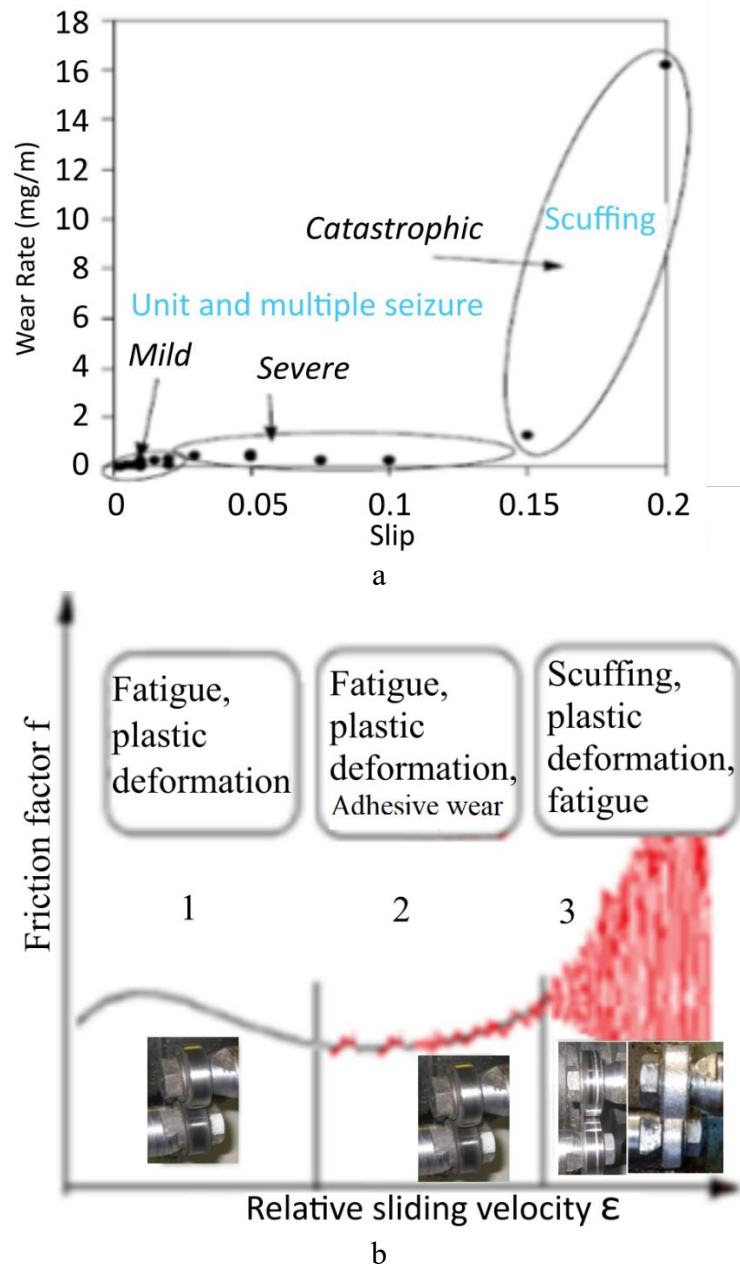


Figure 10: Dependences of the wear rate (types) on slip (a) and the friction factor on relative sliding velocity and various damage types (b).

At full separation of the interacting surfaces by the third body, the tribological properties of the contact zone mostly depend on the properties of the third body and they provide high wear resistance of the interacting surfaces and relatively stable friction coefficient. The destruction of the third body in the separate unit places corresponds to the separate small impulses of the friction moment and adhesive wear of the mean intensity. and the balance of destruction and restoration of the third body corresponds to the “mild” wear type [24]. The progressive destruction of the third body corresponds to the multiple, comparatively increased impulses of the friction

moment and a severe form of the adhesive wear – scuffing. The latter leading to the “catastrophic wear” (Figure 10b).

4 Conclusions

Tribological properties of the interacting surfaces mainly depend on tribological properties of the third body, degree of its destruction, disposition of the surfaces to seizure etc. Improving the tribological properties of heavily loaded friction surfaces can be achieved primarily by providing the contact zone with a continuous or minimally destructed third body that has appropriate tribological properties.

The continuous or discontinuous but restorable third body at the initial stage of destruction, and progressively destructing third body have quite different properties. In the first case, the tribological properties are relatively stable and mainly depend on the properties of the third body. In the second case, these properties are unstable and worsened which are characterized by increasing friction coefficient, catastrophic wear and typical noise.

The prediction of destruction of the third body is possible in the laboratory conditions by estimation of the friction torque, vibrations, noise variation and with the use of the criterion of destruction of the third body.

Fatigue and adhesion processes in the contact zone can occur simultaneously, and the dominant one depends on the operating conditions.

For the Stadler modifier, with the increase of sliding speed, the friction path till the first signs of seizure decreases. While for the modifiers developed by us, the friction path till the first signs of seizure and the stability of the third body increase because of use of special additives that form boundary films on rubbing surfaces due to thermal activation enhanced by stresses.

The friction coefficient (negative, neutral and positive), wear rate of the interacting surfaces (mild, severe and catastrophic), damage types (scuffing, fatigue, plastic deformation, adhesive wear), vibrations and noise generated in the contact zone depend on tribological properties and degree of destruction of the third body, and seized places inside the area of the factual contact zone.

For the improvement of tribological properties of the interacting surfaces, it is necessary to provide their contact zone with continuous or restorable third body having due tribological properties at the initial stage of destruction.

Acknowledgements

This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) [AR-25-3639, Improvement of tribological and ecological properties of the railway wheels and rails].

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