Proceedings of the Seventh International Conference on
Railway Technology: Research, Development and Maintenance
Edited by: J. Pombo
Civil-Comp Conferences, Volume 15, Paper 3.4
Civil-Comp Press, Edinburgh, United Kingdom, 2026
ISSN: 2753-3239, doi: 10.4203/ccc.15.3.4

Statistical Approach to Anomaly Detection of Derailment Coefficient Considering Inner-Rail L/V Correlation

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Abstract

Monitoring the running condition of railway vehicles in curves and detecting abnormalities in bogies are essential for ensuring safe and stable railway operations. The derailment coefficient is widely used as a fundamental indicator for evaluating vehicle running safety. It is conveniently obtained as the ratio of lateral force to wheel load (L/V) acting on the outer rail. However, the derailment coefficient is influenced not only by vehicle performance but also by various factors such as track conditions (e.g., curve radius) and running conditions. Therefore, it is difficult to detect bogie abnormalities based solely on the value of the derailment coefficient.

On the other hand, the derailment coefficient on the outer rail (outer L/V) exhibits a strong correlation with that on the inner rail (inner L/V). Based on this correlation, it is hypothesized that normal conditions correspond to maintaining this relationship, whereas deviations indicate abnormal conditions. This paper proposes a statistical approach for anomaly detection of derailment coefficients considering the inner rail L/V.

By constructing a scatter plot of inner L/V versus outer L/V using a dataset comprising 4,518 measurements measured in revenue service, and defining ±3σ and ±5σ boundaries based on deviations from a regression line, anomalous conditions can be mechanically identified. The results demonstrate that the proposed method can detect bogie faults, such as wheel load imbalance, that cannot be identified from the derailment coefficient alone.

Keywords: anomaly detection, derailment coefficient, statistical approach, Nadal's formula, lubrication condition, inner-rail L/V, bogie faults, L/V monitoring system, condition monitoring, correlation.

1 Introduction

The bogie of a railway vehicle is a critical component governing running safety. Unlike electrical systems, it does not possess redundancy in the event of failure. Therefore, railway operators ensure the integrity of bogies through designs tailored to operating conditions, strict quality control during manufacturing, the establishment of maintenance intervals appropriate to service environments, and the reliable execution of bogie maintenance.

However, in actual operation, unexpected events or external factors may cause bogie failures. Therefore, to ensure safe and stable railway operations, it is considered desirable to monitor the running conditions of vehicles in revenue service and to detect anomalies in bogies (Figure 1).

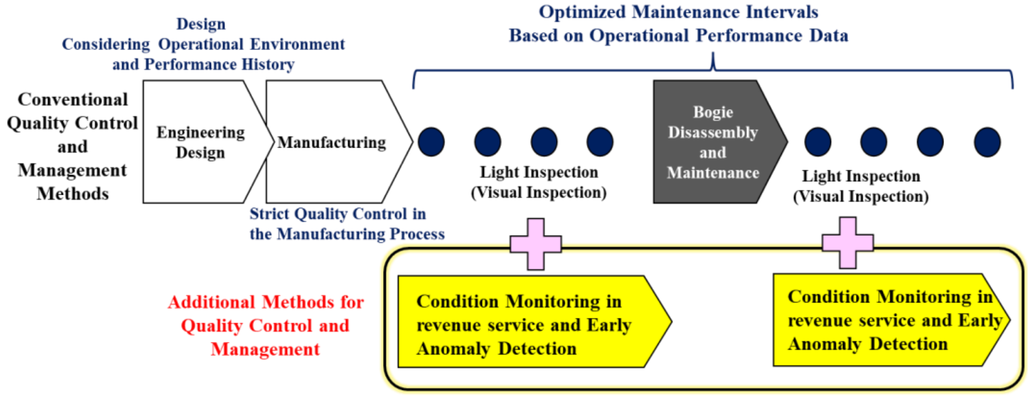

Figure 1: Conceptual diagram of quality maintenance and management for railway bogies.

2 Derailment coefficient and monitoring system from track side

2.1 Derailment coefficient and Nadal's formula

The derailment coefficient is widely used to evaluate running safety. It is defined as the ratio of the lateral force (L) to the vertical wheel load (V), i.e., L/V, obtained by measuring both quantities (Figure 2).

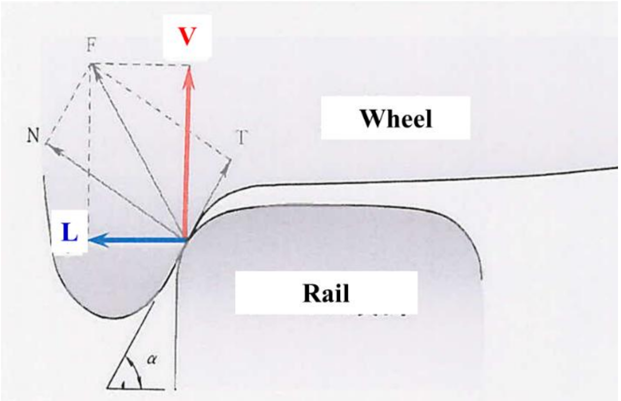

Figure 2: Derailment Coefficient (L/V).

The running safety assessment based on the derailment coefficient is commonly evaluated using Nadal's formula in Equation (1) [1][2][3]. Here, μ denotes the coefficient of friction between the wheel and rail, θ represents the wheel flange angle, and α is the safety factor. Nadal's formula describes the condition for wheel climb onto the rail, formulated based on static equilibrium.

$$\frac{Q}{P} < \alpha \cdot \frac{tan\theta - \mu}{1 + \mu \cdot tan\theta} \tag{1}$$

The derailment coefficient at the onset of wheel climb is referred to as the critical derailment coefficient. In running safety assessment, a reference value of the derailment coefficient is adopted as the evaluation criterion by applying a safety margin of 15% to this critical value. In calculating this reference value, the coefficient of friction between the wheel and rail is generally assumed to be μ = 0.3. In this study, since the flange angle of the vehicle considered is 70°, the reference derailment coefficient is calculated to be 1.14.

If the measured derailment coefficient is below 1.14, no countermeasure is required; otherwise, corrective actions such as vehicle modification, track improvement, or installation of guard rails are necessary.

2.2 L/V monitoring system from track side

Tokyo Metro, which operates nine subway lines in Tokyo, the capital of Japan, has introduced L/V monitoring systems across all nine lines [4] [5].

The system is installed on circular curves with radius below 300 m. Figure 3 shows that strain gauge sensors attached to the rails measure lateral force and wheel load on both the inner and outer rails. In addition, a transmission system for acquiring the identification number of the measured train set is installed alongside the system.

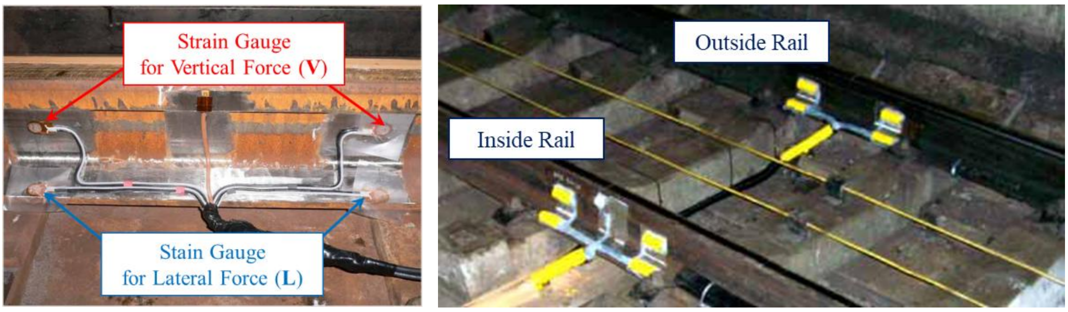

Figure 3: Strain gauge measurement.

Figure 4 shows overview of the L/V monitoring system. Each time a train passes, lateral force and wheel load are measured, and the derailment coefficient is automatically calculated. The data can be remotely monitored at depots. In addition, the data measured at each location are stored on a server, and approximately 14 years of data have been accumulated. This enables statistical analysis of the derailment coefficient data.

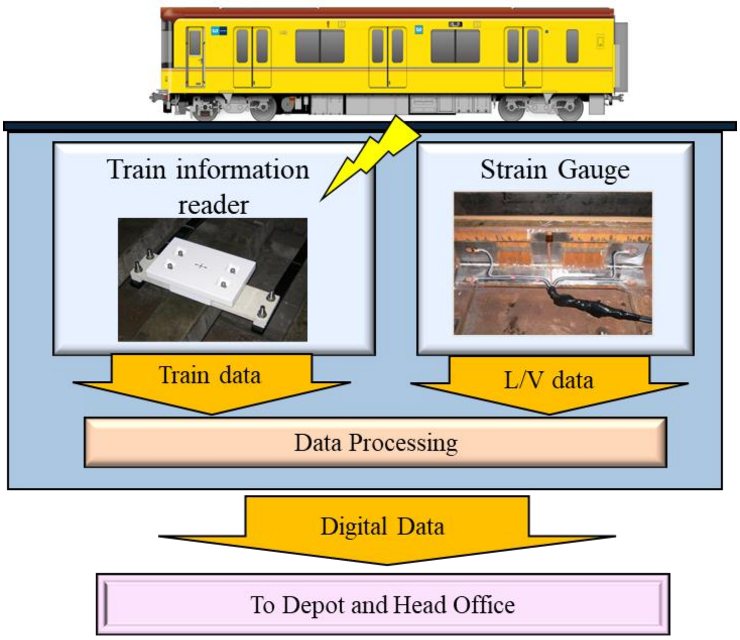

Figure 4: Overview of the L/V Monitoring System.

3 Challenges in anomaly detection using derailment coefficient

3.1 Variation of derailment coefficient

To examine the behavior of the derailment coefficient in revenue service, the vehicle running condition data shown in Table 1 were analyzed. In this study, data spanning four years were obtained for a specific bogie of a particular train set at the same measurement location. The total number of data points is N= 4,518.

Item	Details
Measurement Location	a selected location on a Tokyo Metro line (curve radius: 160 m)
Target Vehicle	a selected trainset (20m length vehicle with bolster bogie)
Target Bogie	Car No. 1, leading bogie, leading axle
Measurement Period	January 1, 2020 – December 31, 2023 (4 years)
Number of Data Points	N =4,518
Filtering Condition	Data with speeds below 30 km/h were excluded due to the influence of speed fluctuations

Table 1: Overview of measurement data.

Figure 5 shows the variation of the derailment coefficient over time. Periods with missing data are attributed to the train not being in revenue service due to scheduled inspections or vehicle maintenance work.

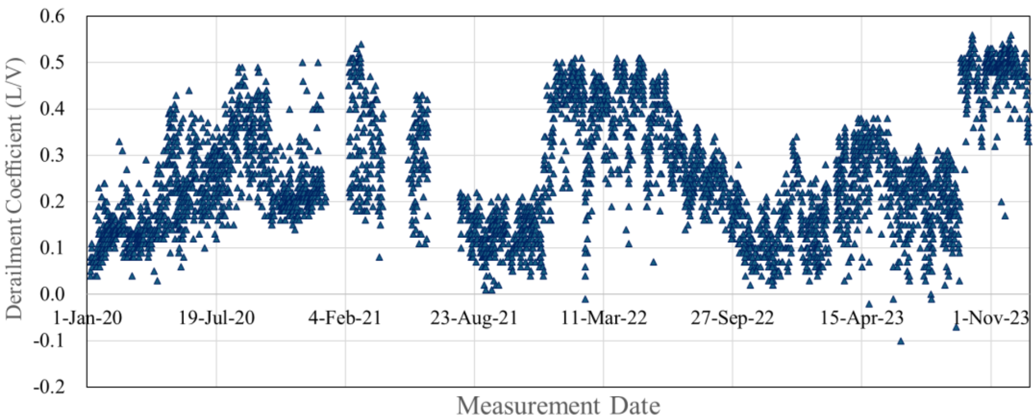

Figure 5: Variation of the derailment coefficient over time.

First, although the curve radius of 160 m represents a severe track condition, the maximum derailment coefficient over the four-year period is approximately 0.6, which is significantly lower than the reference value of 1.14. In this case, if the reference value of the derailment coefficient is used as an anomaly detection criterion, it becomes a lenient threshold relative to the measured values. Consequently, it is expected that the detection accuracy of bogie anomalies would be reduced, or that anomalies may not be detected.

Another notable feature observed in Figure 5 is that the derailment coefficient exhibits variability within a certain range. Although the data correspond to a specific bogie of a particular train set at a specific location, the derailment coefficient does not remain constant. Previous studies using PQ monitoring bogies also reported significant variations in derailment coefficients in revenue service [6] [7].

3.2 Factors affecting derailment coefficient

Previous studies have demonstrated that the derailment coefficient is significantly influenced by the lubrication condition at the top surface of the inner rail on curves [8] [9]. This is because lubrication of the inner rail top surface reduces the tangential force at the trailing axle of the bogie, which in turn decreases the anti-steering moment within the bogie. As a result of this reduction, the lateral force on the outer rail at the leading axle is reduced, as shown in Figure 6.

Thus, since the derailment coefficient varies significantly depending on the lubrication condition at the wheel–rail interface on the inner rail, it is evident that variations in the derailment coefficient alone do not necessarily indicate bogie anomalies.

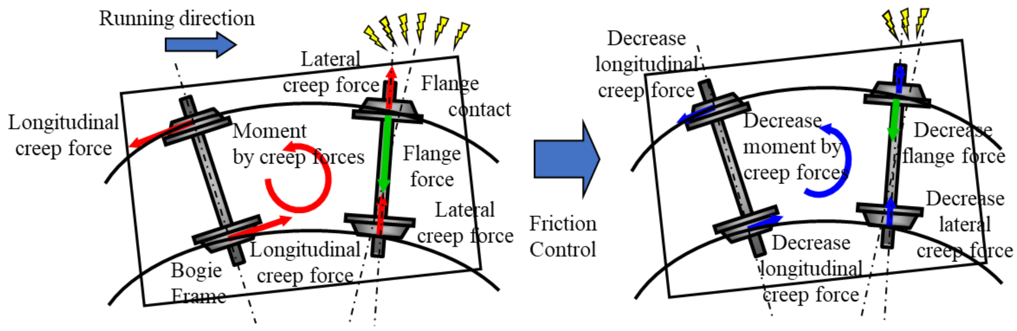

Figure 6: Mechanism of outer rail lateral force reduction by wheel–rail lubrication.

3.3 Limitations of conventional approach

In applying derailment coefficient–based anomaly detection to bogies, a key challenge is the significant variability of the derailment coefficient. Nadal's formula considers the wheel flange angle and the coefficient of friction between the outer wheel flange and the outer rail; however, it does not account for the lubrication condition between the inner rail top surface and the inner wheel tread. In addition, the curve radius at the measurement location is not considered. Previous studies have evaluated the relationship between curve radius and derailment coefficient, indicating a tendency for higher derailment coefficients to be observed as the curve radius decreases [6] [7].

Therefore, when applying the derailment coefficient for bogie anomaly detection, it is desirable to take into account the influence of the lubrication condition at the wheel–rail interface, as well as the effect of track geometry at the measurement location. Accordingly, the necessity and effectiveness of incorporating these factors are investigated in this study.

4 Proposed method considering lubrication conditions

4.1 Influence of inner rail L/V

Section 3.2 described the influence of the lubrication condition of the inner rail on the derailment coefficient. This lubrication condition can be approximately represented by the derailment coefficient on the inner rail (inner L/V); therefore, the influence of inner L/V on outer L/V is examined. For the data shown in Table.1, a scatter plot with inner L/V on the x-axis and outer L/V on the y-axis is presented in Figure 7. Figure 7 indicates a strong correlation between inner L/V and outer L/V. The correlation coefficient is 0.88, which can be statistically evaluated as a strong correlation.

One possible factor contributing to the variation in inner L/V is the influence of a trackside lubricator installed at the entrance of the curve. The lubricator operates under predefined conditions, such as being activated for x seconds after the passage of a total of y train sets. Accordingly, the lubrication condition can vary significantly depending on whether the measured train (bogie) passes immediately after lubrication or just before lubrication. In addition, since the primary purpose of the lubricator is to prevent rail wear—such as corrugation on the rail head and side wear at the gauge corner—the operating conditions are adjusted as appropriate in response to the rail condition.

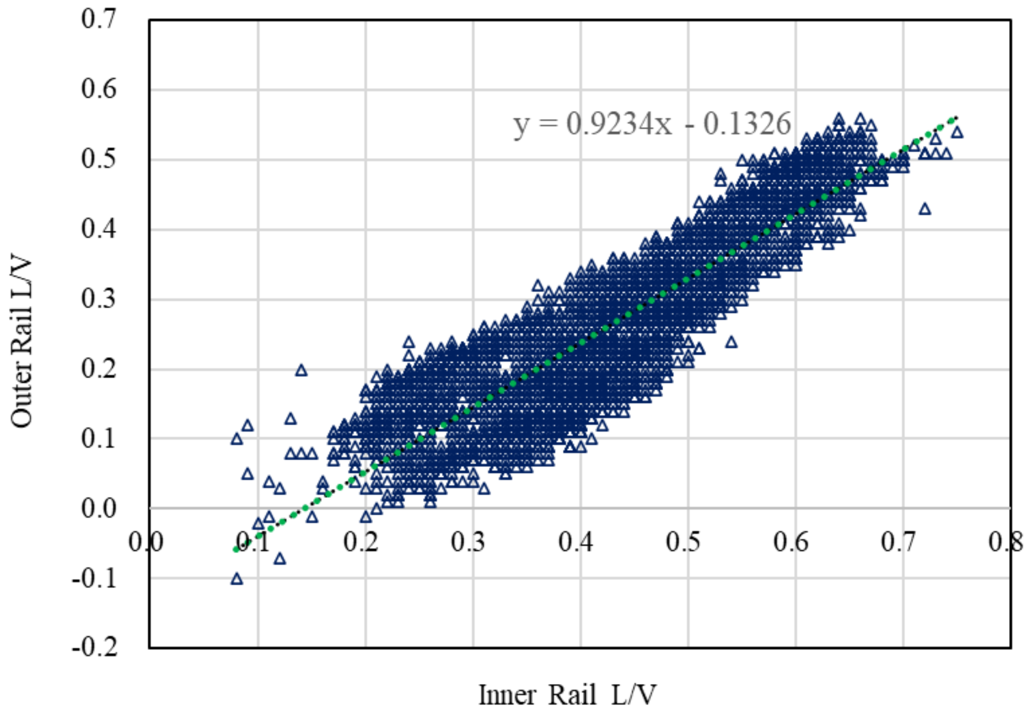

Figure 7: Correlation between inner L/V and outer L/V.

4.2 Evaluation of outer L/V considering variations in inner L/V

As described in the previous section, a strong correlation exists between inner L/V and outer L/V. By utilizing this correlation, the following hypothesis can be formulated: if the bogie is in a normal condition, this correlation is maintained, whereas deviations from this relationship may indicate abnormal conditions [10]. For example, although an outer L/V value of 0.4 is within the feasible range, a measured pair of inner L/V = 0.2 and outer L/V = 0.4 deviates significantly from the data distribution shown in Figure 7. Provided that the track conditions remain unchanged and no variation is observed in data from other train sets, it can be inferred that some form of anomaly has occurred in the measured vehicle.

By visualizing the data, it becomes possible to qualitatively assess whether the measured derailment coefficient deviates from the data distribution. However, a key challenge is the lack of a quantitative criterion for determining the degree of deviation that should be regarded as abnormal. Therefore, in this study, the establishment of a decision criterion based on statistical methods is investigated.

The anomaly detection procedure is as follows:

(1) Remove outliers from the measured data (e.g., data obtained under abnormally low speeds).
(2) Construct an empirical distribution of inner L/V versus outer L/V.
(3) Fit a regression line to this distribution.
(4) Calculate the deviation of each data point from the regression line for each 0.01 increment in inner L/V.
(5) Compute the standard deviation (σ) from the deviations obtained in Step 4.
(6) Derive and plot the ±3σ and ±5σ thresholds based on the calculated σ.
(7) Define data exceeding ±3σ as “warning” and those exceeding ±5σ as “anomalous.”

Regarding the statistical interpretation of the ±3σ and ±5σ thresholds derived from the standard deviation σ, assuming that the data follow a normal distribution, the probability of occurrence beyond ±3σ is 0.27% (approximately once in 370 observations), while that beyond ±5σ is 0.000057% (approximately once in 1.74 million observations).

Figure 8 shows the application of the proposed detection method to the data presented in Figure 7. By statistically drawing the ±3σ and ±5σ boundaries on the scatter plot representing the relationship between inner L/V and outer L/V, it becomes possible to map the occurrence probability of the measured derailment coefficients on the graph. This enables a mechanical determination of “anomalous (non-normal)” conditions when the measured data deviate from the normal region.

Note that anomaly detection boundaries are not defined for regions where inner L/V is less than 0.20 or greater than 0.67, as the number of available data points in these ranges is insufficient for reliable statistical evaluation, and these regions are therefore excluded from the analysis.

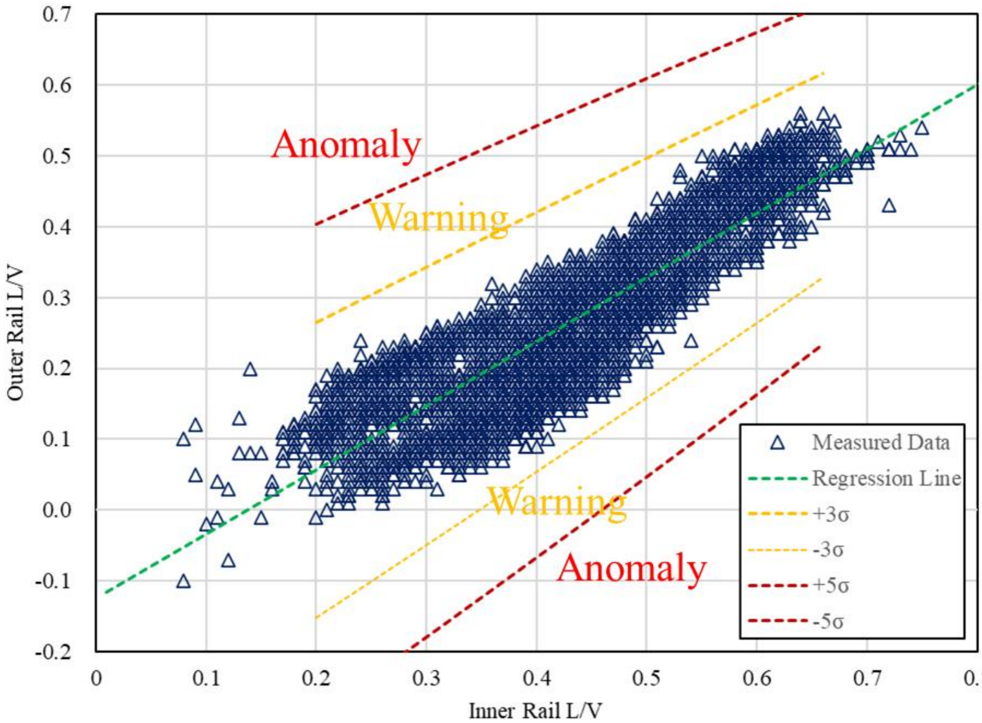

Figure 8: Threshold-based relationship between inner L/V and outer L/V.

4.3 Bogie faults detectable by the proposed anomaly detection method

This section describes the types of phenomena that can be detected by the anomaly detection method shown in Figure 8. One of the causes of derailment on sharp curves is the reduction in outer wheel load due to deterioration of the left–right wheel load balance of the vehicle. Specifically, this may be caused by failures in the secondary suspension system, such as a punctured air spring or malfunction of an automatic leveling valve that controls the air supply and exhaust of the air spring.

In Japan, following the derailment accident on the Hibiya Line in 2000, railway operators have implemented management practices for left–right wheel load imbalance. Although the management criteria for wheel load imbalance are defined by each operator, there are cases where a target value of within 10% and a management limit of 15% under static conditions are adopted.

To examine the condition of left–right wheel load imbalance in the data analyzed in this study, the ratio of inner wheel load to outer wheel load (inner V / outer V) was calculated, and its statistical characteristics were evaluated. Figure 10 presents the histogram of this ratio. The distribution exhibits an approximately normal shape, with a minimum value of 0.79 and a maximum value of 1.22.

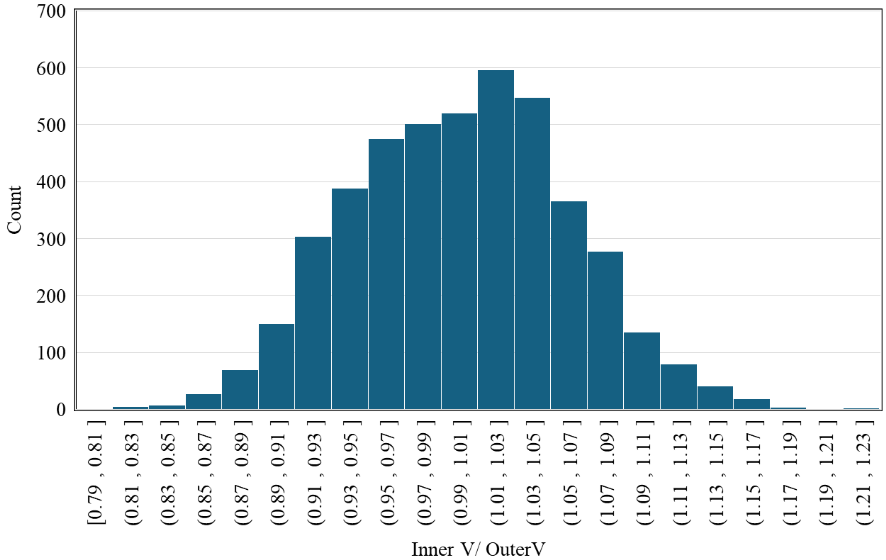

Figure 10: Histogram of inner V / outer V.

Since a reduction in outer wheel load leads to an increase in the inner V / outer V ratio, particular attention is paid to the maximum value. The value of inner V / outer V = 1.22 corresponds to a left–right wheel load imbalance of 9.91% (under curved track conditions).

As an illustrative example, the level of wheel load variation that can be detected as an anomaly is presented. Table 2 shows actual data with inner L/V = 0.50 together with hypothetical data in which outer L/V reaches the +3σ threshold. Although the +3σ value of outer L/V is 0.50, the corresponding inner V / outer V ratio becomes 1.79, which corresponds to a wheel load imbalance of 28.3% (for simplicity, it is assumed that only the left–right wheel loads vary).

While a derailment coefficient of 0.50 is relatively small compared with the reference value of 1.14 based on Nadal's formula, the use of the proposed anomaly detection criterion, as shown in Figure 10, enables the detection of bogie faults—such as deterioration of wheel load balance—that cannot be identified from the derailment coefficient alone.

	Actual data	Hypothetical data
Inner V (kN)	48.2	58.8
Outer V (kN)	43.3	32.8
Inner V / Quter V.	1.11	1.79
Wheel load imbalance (%)	5.2	28.3
L/V	0.38	0.50

Table 2: Validation of anomaly detection (variation in wheel load data).

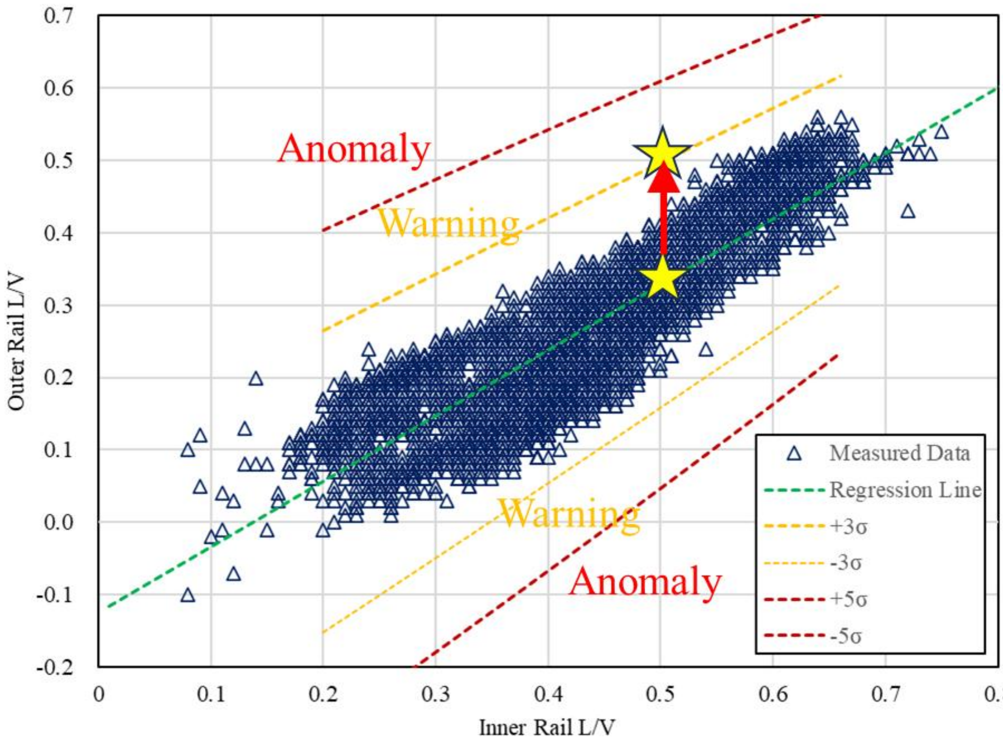

Figure 10: Conceptual illustration of anomaly detection.

5 Conclusions and contributions

The reference value of the derailment coefficient derived from Nadal's formula is widely used as an index for evaluating the running safety of railway vehicles. However, the derailment coefficient is influenced not only by the condition of the vehicle but also significantly by the lubrication condition at the wheel–rail interface on the inner rail, resulting in substantial variability. In addition, it is affected by track geometry at the measurement location, such as curve radius. Therefore, it is difficult to detect bogie anomalies based solely on this reference value.

Accordingly, when applying the derailment coefficient to bogie anomaly detection, it is desirable to consider both the influence of the lubrication condition at the wheel–rail interface and the effect of track geometry at the measurement location.

Since the derailment coefficient exhibits a strong correlation with the lubrication condition at the wheel–rail interface on the inner rail, the absence of this correlation can be regarded as an indication of abnormal conditions. Furthermore, continuous monitoring of the derailment coefficient at specific locations in revenue service enables statistical evaluation that accounts for both curve radius and wheel–rail lubrication conditions.

By constructing a scatter plot representing the relationship between inner L/V and outer L/V from the measured data, and statistically defining ±3σ and ±5σ boundaries based on deviations from the regression line, the occurrence frequency of the derailment coefficient can be mapped on the graph. This enables a mechanical determination of anomalous conditions when the measured data deviate from the normal region.

By applying the proposed anomaly detection method, it becomes possible to detect bogie faults—such as deterioration of wheel load balance—that cannot be identified from the derailment coefficient alone. Early detection of such faults enables the prevention of major failures and accidents, thereby contributing to the maintenance and improvement of safe and stable railway operations.

Acknowledgements

The data used in this study were provided by the Rolling Stock Department of Tokyo Metro Co., Ltd. The authors also acknowledge ABeam Consulting Ltd. and the CBM Team at the Nakano Depot for their technical and analytical support.

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