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Inverse estimation of catenary geometry from pantograph-catenary contact force

M. Tur, N. Aldaz, J. Gil, S. Gregori, A. M. Pedrosa and F. J. Fuenmayor

Institute of Mechanical Engineering and Biomechanics, Universitat Politecnica de Valencia, Spain

Abstract

This work addresses the inverse reconstruction of a CA-220 twin contact wire catenary geometry from the measured contact force data. It analyses the impact of installation errors and proposes a Bayesian optimization framework to estimate geometric parameters. Five algorithms are proposed and the convergence is assessed using synthetically generated data with a known solution. Results show that dropper lengths can be accurately identified, while other variables have limited influence.

Keywords: pantograph, inverse problem, catenary, twin contact wire, contact force, bayesian optimization

1 Introduction

Currently, advanced numerical simulation codes are available to model the dynamic interaction between the pantograph and the catenary, enabling the estimation of performance indicators for the design of new railway electrification systems, such as steady arm uplift and contact force [1]. The accuracy and predictive capability of these simulations are primarily governed by three factors: the time integration algorithm employed in the numerical scheme, the fidelity of the pantograph model, and the level of detail of the catenary representation. In general terms, these codes use the finite element method to model the different components, and there are several valid alternatives for the integration scheme, including the HHT method. Current computational capabilities allow real catenaries to be simulated with sufficient accuracy and within reasonable computational times. For the pantograph, lumped-mass models are typically used, adjusted from the Frequency Response Function obtained in a test bench.

This work focuses on analysing the influence of the simplifications introduced in the catenary model. It analyses the effect of installation errors on the simulation results. It is well known that the final geometric configuration of the catenary differs from the nominal design due to various sources of error, such as inaccuracies in the positioning of masts and cantilevers, which affect the anchoring points of the messenger wire and steady arms, as well as errors in the placement and effective lengths of the droppers, among others. Additional factors influencing the simulation, such as the efficiency of the tensioning system [2] or the material properties, are not considered in this study.

The aim of this work is to demonstrate that the actual static configuration of the catenary can be identified from measurements of the pantograph–catenary interaction force, and to propose a methodology for estimating this real static configuration. To validate the proposed approach, a catenary model incorporating installation errors will be generated, including variations in dropper lengths and inaccuracies in the anchoring points of both the steady arm and the messenger wire at the cantilevers. A dynamic simulation will then be performed, and the resulting contact force will be used, starting from the nominal design geometry, to reconstruct the actual catenary configuration.

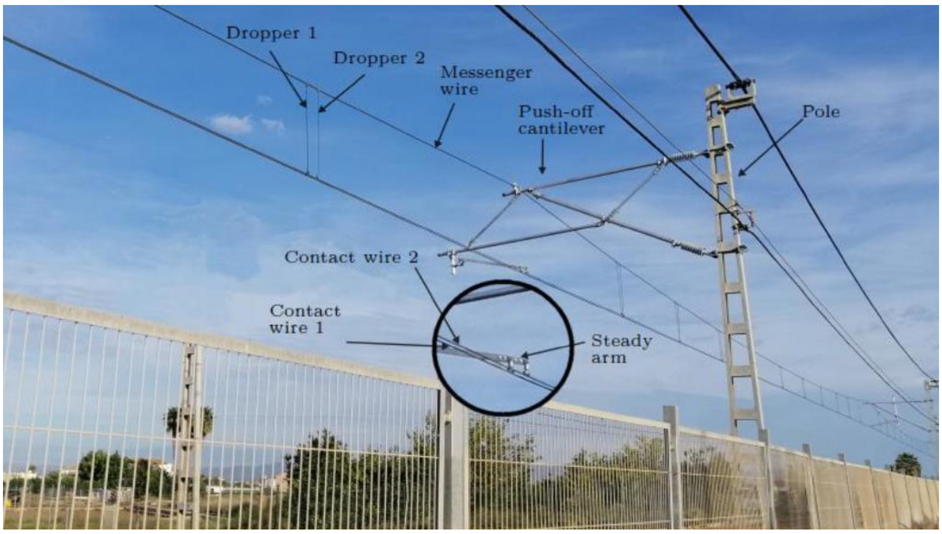


Figure 1: Components of the catenary CA-220.

Catenary model

The analyses are carried out using a model of the Spanish CA-220 twin contact wire catenary system. Its typical operating speed ranges from 160 to 200 km/h, although its design allows operation at speeds of up to 220 km/h while maintaining satisfactory performance in the presence of track irregularities and height variations associated with structures such as overpasses, underpasses, and other infrastructure singularities [3]. Figure 1 shows the main components of the overhead contact line considered in the model, including the messenger wire, the two contact wires, the droppers, and the steady arm. To ensure electrical balance between the two contact wires, equipotential droppers arranged in pairs are used; their function is to limit potential differences and promote a uniform current distribution, thereby preventing imbalances that could affect current collection quality.

A real catenary section consisting of 22 spans, with varying lengths as well as different numbers and spacing of droppers, has been modelled, as detailed in Table 1. The main mechanical parameters are summarised in Table 2. The mass of the clamps connecting the droppers and the steady arm to the wires is 0.208 kg. Figure 2 shows the static configuration of the model as designed, in which the contact wire height is maintained at 5.3 m at the steady arms, while a prescribed presag is present at mid-span.

Span	1	2	3	4	5	6	7	8	9	10	11
Length (m)	53.5	38	38	40	60	53	60	55	60	60	60
N droppers	0	2x5	2x5	2x6	2x8	2x7	2x8	2x8	2x8	2x8	2x8
Span	12	13	14	15	16	17	18	19	20	21	22
Length (m)	60	58	60	60	60	60	60	50	38	38	49
N droppers	2x8	2x8	2x8	2x8	2x8	2x8	2x8	2x7	2x5	2x5	0

Table 1: Span length and number of droppers.

	EA (N)	EI (Nm^2)	Density (kg / m)	Tension (kN)
Contact wire	19.0 10^6	279	1.64	24
Messenger wire	18.8 10^6	272	1.33	18
Droppers	3.72 10^6	--	0.224	--
Steady arm	45.7 10^6	--	1.76	--

Table 2: Mechanical properties of the catenary.

The model includes the messenger and contact wires, the droppers, and the steady arms. Cantilevers are replaced by an equivalent support with stiffness and damping. Static and dynamic analyses are carried out using the PACDIN code [4]. For the dynamic analysis, the Newmark integration scheme is employed, and Rayleigh damping is assumed with mass and stiffness coefficients of 0.0125 s^{-1} and 10^{-4} s, respectively. The dynamic problem is solved in real time using a modal basis comprising with 12 000 modes together with a procedure based on the offline precomputation of certain matrices (see [5] for details).

A constant pantograph speed of $V = 192$ km/h is considered. The pantograph comes into contact with the catenary from mast 3. The dynamic analysis sector is shadowed in grey in Figure 2, comprising 10 spans, from mast 8 to mast 19. A nonlinear behaviour of the droppers is assumed, in which the compressive stiffness is 0.3% of the tensile stiffness value.

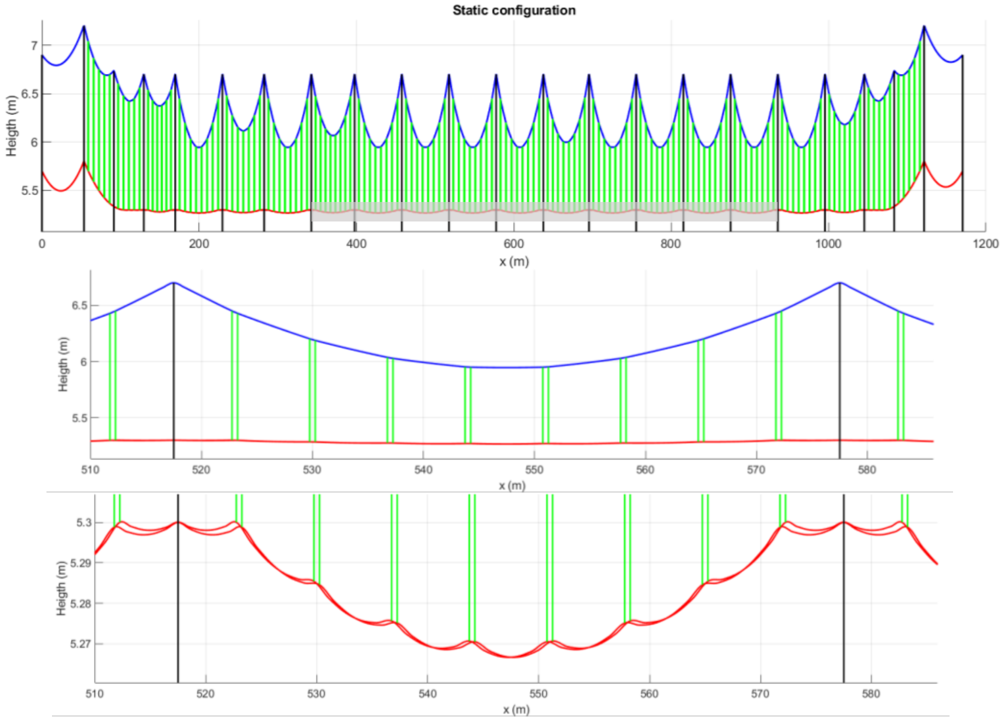
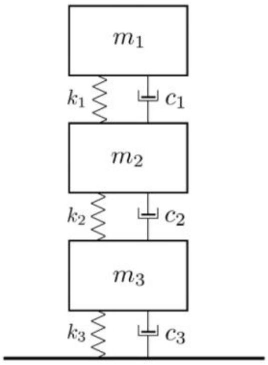


Figure 2: Static configuration of the catenary section, and detail of a central span.

Pantograph Model

The pantograph is a lumped mass model with 3 degrees of freedom as shown in Figure 3.



	m (kg)	c (Ns/m)	k (N/m)
1	9.9	152	10 150
2	21.1	0.15	18 160
3	12.5	92	650

k_h = 50.000 N/m

Figure 3: Lumped mass pantograph model.

2 Methods

First, a reference catenary is generated, incorporating deviations from the design geometry due to installation errors (as-built geometry). A numerical simulation is then carried out to obtain the contact force. This force is taken as a reference, as if it were the force measured in service. The objective is to define an algorithm to determine the reference (as-built) geometry from the contact force.

The following parameters are selected to define the catenary geometry (see Figure 4), each within a specified admissible range of variation:

- L_d. Length of droppers from span 8 to 18. Given that there are 8 pair of droppers per span (one for each contact wire), the total number of variables is 176. An admissible variation of ±25 mm is allowed
- H_{mw}. Height of the messenger wire attachment point to the cantilever at masts 8 to 18: 11 variables. An admissible variation of ±50 mm is allowed.
- H_{sa}, Y_{sa}. Height and lateral position of the attachment point of the steady arm to the cantilever at masts 8 to 18. The total number of variables is 22. Variations of ±50 mm are allowed.

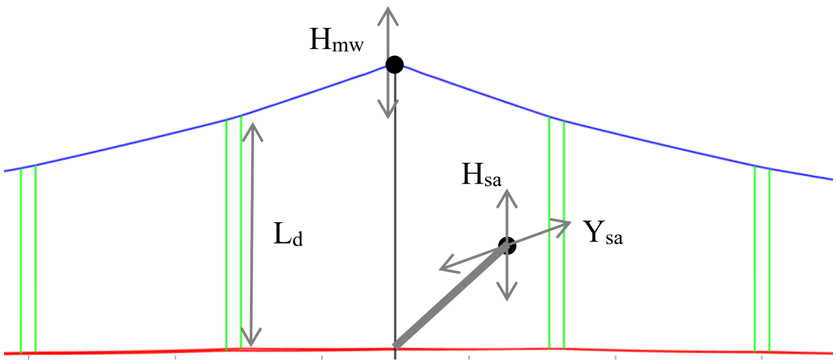


Figure 4: Definición de los parámetros de diseño.

Reference geometry (as-built):

Figure 5 and Table 3 present the values of the variables used to define the reference geometry, while Figure 6 shows the resulting profile of the contact wires in the region of interest. The simulation output is the contact force shown in Figure 7. The standard deviation of this force is 23.7 N and the mean value is 142 N.

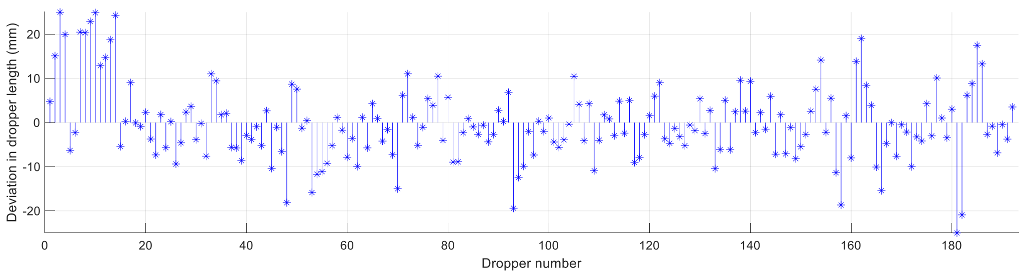


Figure 5: Deviation in dropper length.

Mast	7	8	9	10	11	12	13	14	15	16	17	19
H_{mw}	-13	14	14	11	0	2	9	-2	-1	19	14	0
H_{sa}	-50	34	-50	50	50	21	50	-8	8	12	49	9
Y_{sa}	50	2	-49	-50	43	-3	39	-16	-44	-42	6	28

Table 3: Deviation in bracket position in mm.

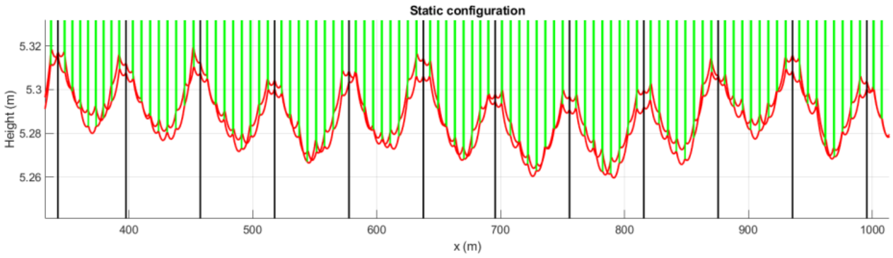


Figure 6: Static configuration of the reference catenary in the region of interest.

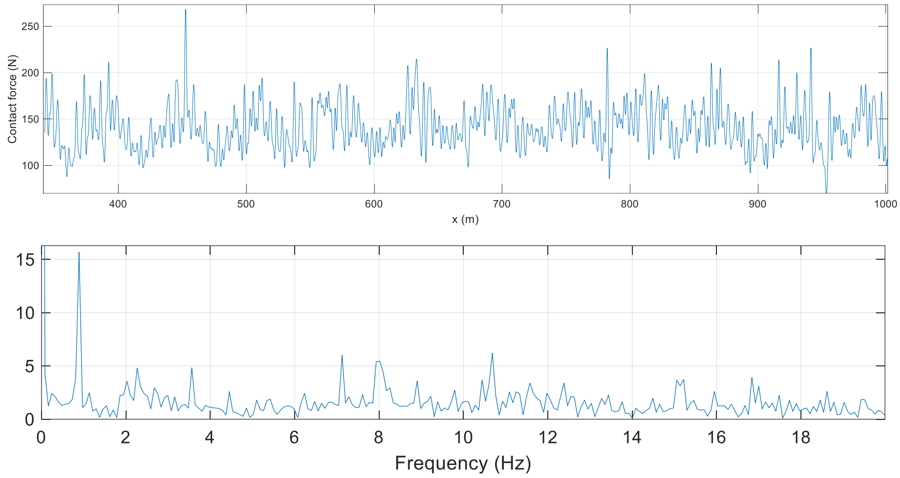


Figure 7: Contact force of the reference catenary in the time and frequency domain.

Geometry finding algoritm:

The problem addressed consists of determining the values of the 209 design parameters that define the geometry such that the simulated contact force matches the reference value as closely as possible. The starting point is the as-designed catenary, i.e., all parameter deviations are initially set to zero. To achieve this, a sequence of optimization problems will be defined using the Bayesian Optimization algorithm, which requires the simulation of dynamic problems for different values of the design parameters. For the pantograph–catenary interaction problem, the optimal number of parameters is at most five for every optimization problem, since a larger number would require too many iterations of the algorithm.

The proposed strategy is based on the idea [6] that the geometric parameters at a given kilometer point (kp) along the track affect the contact force from the instant at which the pantograph passes that location, whereas their influence on the contact force at preceding kp can be neglected.

The optimization algorithm is based on generating individuals, each defined by a set of design parameters. Let $F_c^{(i)}(t_j)$ denote the contact force of individual (i) at time instant t_j, and $F_c^{(Ref)}(t_j)$ the contact force of the reference catenary. The following error indicator is defined for individual (i):

$$ErrorFc^{(i)} = \sqrt{\frac{1}{N}\sum\forall t_j \left(F_c^{(i)}(t_j) - F_c^{(Ref)}(t_j)\right)^2} \tag{1}$$

N is the number of points used to compute the contact force, ie the number of time steps in the analysis sector (ten central spans, Figure (2)). The contact force is filtered with a 20 Hz low-pass filter.

3 Results

Five algorithms are compared. The first three use as variables the 228 parameters described in the previous section. Algorithm A1 has two levels of iterations. The outer level runs until the ErrorFc is lower than tolerance. At the inner level, a sequence of optimization problems is solved, in which H_{mw}, H_{as}, and Y_{as} associated with a mast are defined as optimization variables. The iteration proceeds from mast 8 to mast 18. Next, the dropper lengths are determined by defining optimization problems with two variables (pair of droppers), starting from the smallest pk of the sector of analysis. Once the iteration is completed, the process is repeated until the error becomes zero or no longer decreases. Algorithm A2 performs the inner iterations by mast and droppers associated with the span located immediately after the mast. Algorithm A3 performs the iterations by spans, defining as variables those associated with the two mast and the droppers of each span. Table 4 schematically show these algorithms.

A1
While Error < Tolerance
Loop for mast m=8 to 18
Optimization mast m:
-3 variables:
-H_{mw}, H_{sa},Y_{sa}
Loop for spans 8 to 18
Loop for dropper pair p=1…8
Optimization dropper pair p:
-2 variables:
-L_d two consecutive Droppers

A2
While Error < Tolerance
Loop for spans s=8 to 18
Optimization mast s:
-3 variables mast m:
-H_{mw}, H_{sa},Y_{sa}
Loop for dropper pair p=1…8
Optimization for dropper:
-2 variables:
-L_d two consecutive Droppers

A3
Loop for spans s=8… 18
While Error < Tolerance
Optimization mast s:
-3 variables:
-H_{mw}, H_{sa},Y_{sa}
Loop or dropper pair p=1…8
Optimization for dropper:
-2 variables:
-L_d two consecutive Droppers
Optimization mast s+1:
-3 variables:
-H_{mw}, H_{sa},Y_{sa}

Table 4: Algorithms A1, A2 and A3.

Figure 8 shows the evolution of ErrorFc as a function of the number of iterations. It should be noted that each iteration corresponds to an optimization problem, which in turn requires an average of 50 solutions of the dynamic problem.

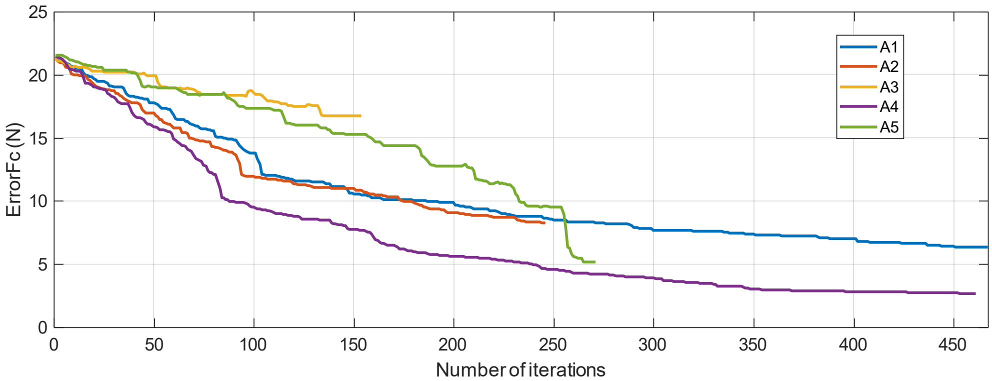


Figure 8: Global error in the contact force as a function of the number of iterations for all algorithms.

The variables H_{mw}, H_{ra}, and Y_{ra} have little influence on the contact force, and it is not possible to infer the exact solution of the problem using the proposed algorithms alone. Some geometric measurement should be used instead to estimate them. An alternative problem is therefore formulated, in which the geometry is determined considering only the dropper lengths as design variables. Two additional algorithms are defined, both based on span-by-span iterations. In algorithm A4, the error used as the objective function in the optimization problem is computed over all spans, whereas in algorithm A5 the error is evaluated only over the span in which the optimization problem is being solved. These algorithms are summarized in Table 5. Figure 8 shows the convergence results. The convergence achieved with algorithm A4 is better than that of the other algorithms. This algorithm is able to recover the exact geometry. Figure 9 compares the contact force with that obtained for the reference catenary.

A4	**A5**
Loop for spans s=8 to 17 While **GlobalError** < Tolerance Loop or dropper pair p=1…8 Optimization for dropper: -2 variables: -L_d two consecutive droppers	Loop for spans s=8 to 17 While **SpanError** < Tolerance Loop or dropper pair p=1…8 Optimization for dropper: -2 variables: -L_d two consecutive droppers

Table 5: Algorithms A4 and A5.

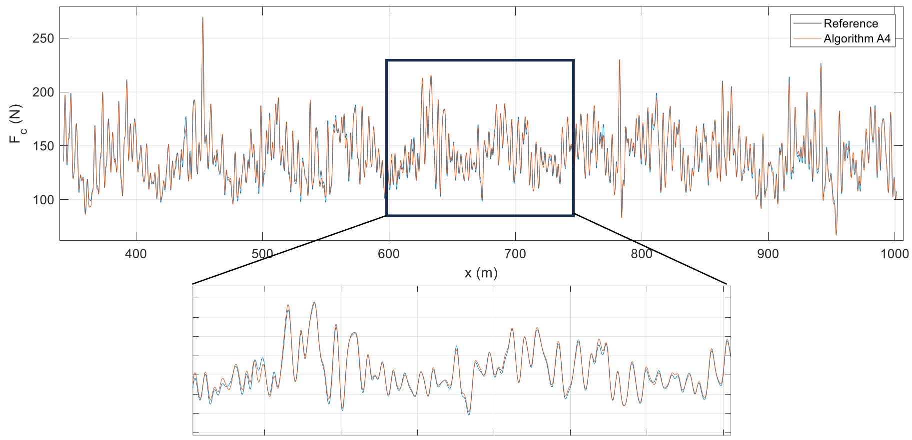


Figure 9: Comparison of contact force of the reference catenary and algorithm A4.

4 Conclusions and Contributions

This work addresses the inverse problem of reconstructing the installed geometry of a CA-220 twin-contact-wire catenary from measured contact force data. In this study, the measurement data generated synthetically, i.e., by means of a simulation; however, future work will consider contact forces obtained from on-track measurements. The main contributions and conclusions are as follows:

(i) An inverse identification framework has been proposed to link contact force measurements with the geometric parameters of the overhead contact line.
(ii) Five algorithms using Bayesian Optimization strategy have been defined to iteratively update the representative geometric variables.
(iii) Some geometric variables (H_{ra}, Y_{ra} and H_{mw}) have a negligible influence on the contact force and, therefore, cannot be reliably identified from the measured force.
(iv) Dropper lengths are the most significant parameters and can be identified with sufficient accuracy.

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