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Plane Performance Calculated with a Less Complex Wing Theory

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Abstract

In this paper the author uses a less complex wing theory in ideal fluids that connects lift force and drag force of finite 3- dimensional wings with mass flow into the generated vortex cores.
Suggesting an unusual induced drag in ideal fluids for 2- dimensional wing sections, leads to a modified version of the equation Prandtl has derived for the relation between lift force F_y and drag force F_x of a 3- dimensional finite wings. This new equation goes over into Prandtl's equation, if we have low aspect ratios of the wings.
To test the theory, the author calculated the measured and published performance of two high performance gliders and the human powered Gossamer Condor. The input parameters are the gross weight, the span, the velocity, the air density and the aspect ratio. The results shows good agreement with the published results, for example the glide ratio or the expected power consumption of the man driven Gossamer Condor.

Keywords: aerodynamic wing performance, vortex core, wing theory, lift coefficient, drag coefficient, ideal fluids.

1 Introduction

Wake vortices behind planes are a typical topic in fluid dynamics. Lanchester [1] and Prandtl [2] described the 3-dimesional flow behind wings with finite span in ideal fluids, resulting in an induced drag out of the ability to generate a circulation, in a relation between lift force F_y and expected drag force F_x:

$$F_x = F_y \cdot \frac{w_\infty}{2u_\infty} \tag{1}$$

where u_∞ is the velocity of the plane and w_∞ is the induced downward velocity of the fluid flow.
As described in [3] the relation between C_l and C_d is stated as an induced drag in 3-dimensional flow in ideal fluids and now additionally an induced drag in 2-dimensional flow:

$$C_d = \left(\frac{C_l}{2\pi}\right)^2 \cdot \frac{\pi \cdot \ln(1+\sqrt{2})}{2\sqrt{8}} + \frac{C_l^{\,2}}{\pi\Lambda} \tag{2}$$

This is leading to the equation:

$$F_x = F_y \cdot \frac{w_\infty}{2u_\infty} \cdot \left[\frac{\Lambda \cdot \ln(1+\sqrt{2})}{8 \cdot \sqrt{8}} + 1\right] \tag{3}$$

where Λ is the aspect ratio of the wing. This is an extension of formular (1). Formular (2) is gradually going over into formular (1) when Λ is close or less 1. The theory is explained in [8] and here the author is calculating the expected the aerodynamical performance of two high performance sailplane wings, the SB 10 and the ETA and the wing of the human powered Gossamer Condor.

2 Level flight of a wing of finite span in ideal fluids

As shown in [8] we calculate C_l out of the lift force F_y and the speed u_∞ of the plane together with the density ρ and the area A of the wing:

$$C_l = \frac{2 \cdot F_y}{\rho \cdot u_\infty^{\,2} \cdot A} \tag{4}$$

C_d the drag coefficient of the wing is then calculated according to equation (2).

Alternatively the author suggests to formulate the drag coefficient with help of the mass flow $\dot{m}_{core}$ into the vortex cores generated from the wing and the descent velocity w_∞.

$$C_d = \frac{2 \cdot \dot{m}_{core} \cdot w_\infty}{\rho \cdot u_\infty^{\,2} \cdot A} \tag{5}$$

The mass flow $\dot{m}_{core}$ is defined as

$$\dot{m}_{core} = C_d \cdot \frac{\rho}{2} \cdot A \cdot \frac{u_\infty^{\,2}}{w_\infty} \tag{6}$$

and w_{oo} is defined as

$$w_\infty = \frac{\Gamma_0}{b_1} \tag{7}$$

where b_1 is the wingspan, and Γ_0 the circulation

$$\Gamma_0 = 4 \cdot T_1 \cdot u_{HK} \tag{8}$$

where T_1 is the average wing width, the wing area is $A_1 = b_1 \cdot T_1$
with

$$u_{HK} = \frac{C_l \cdot u_\infty}{2 \cdot \pi} \tag{9}$$

We can then write the drag force

$$F_x = \dot{m}_{core} \cdot w_\infty \tag{10}$$

and the lift force

$$F_y = \dot{m}_{core} \cdot \frac{C_l}{C_d} \cdot w_\infty \tag{11}$$

The vertical mass flow generating the lift force is according to Prandtl the flow through a circle area with the wing span as the diameter. Fluid particles are entering this area with u_∞, perpendicular to this area. This mass flow is moving down with w_∞, parallel to the area

$$F_y = \dot{m}_y \cdot w_\infty \tag{12}$$

where $\dot{m}_y$ is defined (as explained above)

$$\dot{m}_y = \dot{V}_1 \cdot \rho \tag{13}$$

with

$$\dot{V}_1 = A_{01} \cdot u_\infty \tag{14}$$

Here A_{01} is a circle with the wing span as the diameter

$$A_{01} = \frac{\pi \cdot {b_1}^2}{4} \qquad (15)$$

The relation between $\dot{m}_y$ and $\dot{m}_{core}$ is

$$\frac{\dot{m}_y}{\dot{m}_{core}} = \frac{C_l}{C_d} \qquad (16)$$

$$r_{core} = \sqrt{\frac{A}{4} \cdot \frac{C_l}{2\pi}} \qquad (17)$$

with A the wing area and C_l the lift coefficient

3 Power calculation

In [8] we have written down the power that is consumed to keep the wing flying on level consisting of the drag force multiplicated with the velocity of the wing

$$P^*{}_{tot} = F_x \cdot u_\infty \qquad (18)$$

Alternatively we can see on the descent velocity v_y of the unpowered wing in relation to the horizontal velocity u_{oo} of the wing with the known relation between C_d and C_l. We use the relation

$$\frac{C_d}{C_l} = \frac{v_y}{u_\infty} \qquad (19)$$

That leads to the descent velocity

$$v_y = \frac{C_d \cdot u_\infty}{C_l} \qquad (20)$$

The lifting force multiplicated with the descent velocity is the power to keep the plane on level according to lift and drag.

$$P^*{}_{tot} = F_y \cdot v_y = \dot{m}_y \cdot w_\infty \cdot v_y \quad (21)$$

The kinetical energy of the vortex cores (seen as cylinders with radius r_{core}) is calculated

$$E_{kin} = \frac{1}{2} \cdot \Theta \cdot \omega^2 \tag{22}$$

Where Θ is the momentum of inertia of a cylinder (the vortex core) which is growing in length in time leading to $\dot{\Theta}$
The rotational power of the vortex cores (kinetical energy per time) is calculated

$$P^*{}_{core} = \dot{E}_{kin} = \frac{1}{2} \cdot \dot{\Theta} \cdot \omega^2 \tag{23}$$

where ω is the angular velocity

$$\omega = \frac{u_{HK}}{r_{core}} \tag{24}$$

Inertial mass Θ of a cylinder (representing the vortex cores)

$$\Theta = \frac{1}{2} m_{core} \cdot {r_{core}}^2 \tag{25}$$

Ongoing flow into the vortex core (the vortex core is a cylinder with constant radius r_{core} increasing the mass with growing length)

$$\dot{\Theta} = \frac{1}{2} \dot{m}_{core} \cdot {r_{core}}^2 \tag{26}$$

Leading to the rotational power of the vortex cores

$$P^*{}_{core} = \frac{1}{4} \cdot \dot{m}_{core} \cdot {u_{HK}}^2 \tag{27}$$

3 Results

Plane parameter from [6] calculated values in Table 1

Description	Parameter	Value	
Lift force	F_y	7651.8 N	
Air density	ϱ	1.2 kg/m^3	
Plane velocity	u_∞	25 m/s	
Wing span	b_l	29 m	
Wing area	A_l	22.978 m^2	
Circulation	Γ	11.198 $\frac{m^2}{s}$	
Aspect ratio	Λ	36.6	
	Calculated values		Used equation
Drag force	F_x	143.342 N	(3)
Mass descent velocity	w_∞	0.386 m/s	(7)
Area of influenced mass	A_{01}	660.519 m^2	(15)
Downward accelerated volume	$\dot{V}_1$	16512.5 $\frac{m^3}{s}$	(14)
Lift coefficient	C_l	0.888	(4)
Average wing section	T_l	0.79 m	
Trailing edge velocity	u_{HK}	3.533 m/s	(9)
Drag coefficient	C_d	0.0166	(2)
Mass flux into the cores	$\dot{m}_{core}$	371.20 $\frac{kg}{s}$	(6)
Mass flow vertical	$\dot{m}_y$	19815.59 $\frac{kg}{s}$	(13)
Core radius	r_{core}	0.901 m	(17)
Glide ratio	E	53.38	Measured 53

Table 1: Input parameter sailplane SB 10 [6] and from the author calculated values show good agreement with the published measured value E = 53

Parameters from [7], calculated values in Table 2

Description	Parameter	Value	
Lift force	F_y	8338.5 N	
Air density	ϱ	1.2 kg/m^3	
Plane velocity	u_∞	31 m/s	
Wing span	b_1	30.9 m	
Wing area	A_1	18.6 m^2	
Circulation	Γ	9.236 $\frac{m^2}{s}$	
Aspect ratio	Λ	51.33	
	Calculated values		Used equation
Drag force	F_x	120.577 N	(3)
Mass descent velocity	w_∞	0.299 m/s	(7)
Area of influenced mass	A_{01}	749.9 m^2	(15)
Downward accelerated volume	$\dot{V}_1$	23247.086 $\frac{m^3}{s}$	(14)
Lift coefficient	C_l	0.777	(4)
Average wing section	T_1	0.602 m	
Trailing edge velocity	u_{HK}	3.835 m/s	(9)
Drag coefficient	C_d	0.0112	(2)
Mass flux into the cores	$\dot{m}_{core}$	403.3930 $\frac{kg}{s}$	(6)
Mass flow vertical	$\dot{m}_y$	27896.5 $\frac{kg}{s}$	(13)
Core radius	r_{core}	0.758 m	(17)
Glide ratio	E	69.15	published 70

Table 2: Input parameter sailplane ETA [7] and from the author calculated values show good agreement with the published value E = 70

Parameter from [10] , calculated values in table 3

Description	Parameter	Value	
Lift force	F_y	997,677 N	
Air density	ϱ	1.2 kg/m^3	
Plane velocity	u_∞	5.6 m/s	
Wing span	b_l	29.3 m	
Wing area	A_l	66.54 m^2	
Circulation	Γ	6.45 $\frac{m^2}{s}$	
Aspect ratio	Λ	12.9	
	Calculated values		Used equation
Drag force	F_x	29.47 N	(3)
Mass descent velocity	w_∞	0.22 m/s	(7)
Area of influenced mass	A_{0l}	674.26 m^2	(15)
Downward accelerated volume	$\dot{V}_1$	3775.83 $\frac{m^3}{s}$	(14)
Lift coefficient	C_l	0.796	(4)
Average wing section	T_l	2.27 m	
Trailing edge velocity	u_{HK}	0.71 m/s	(9)
Drag coefficient	C_d	0.0235	(2)
Mass flux into the cores	$\dot{m}_{core}$	133.84 $\frac{kg}{s}$	(6)
Mass flow vertical	$\dot{m}_y$	4531 $\frac{kg}{s}$	(13)
Core radius	r_{core}	1.452 m	(17)
Power to fly horizontal	P*tot	165 Nm/sec	
Glide ratio	E	33.8	(21)

Table 3: Input parameter for the human powered Gossamer Condor [10] and from the author calculated values show good agreement with the published value of the expected power to keep the plane in a horizontal flight.

4 Conclusions

A wing section has no drag in ideal fluids according to the potential theory. We can calculate the circulation which is leading to the lift force. The start vortex is moved to vicinity and has lost the influence in the theoretical model. The author is suggesting the image of having a vortex pair in 2- dimensions which is continuously fed with attracted fluid particles. In the random walk source model [9],[3] it is shown, that one can identify an induced drag from wing sections in ideal fluids in 2- dimensions which is resulting in an equation that connects lift coefficient and drag coefficient. This is unexpected, but shown in [3] with calculating the drag coefficient of a rotating cylinder out of the known lift coefficient, in ideal, non-viscous fluids. W. G. Bickley, had made that before, placing a vortex behind a rotating cylinder in an empirical defined position. We could reproduce his equation but without using empirical values. Do we have induced drag in 2- dimensions, so will this, until now, forgotten part of the drag, be a part of the drag of a wing in 3 dimensions. To formulate this it is used an equation (3) which is a modification of Prandl's relation (1) between lift force and drag force of wings in ideal fluids. In the presented equations, the relation between lift coefficient and drag coefficient, is assumed to be the same relation as between the mass that is moved downward from the plane (to generate the lift) compared to the mass that is flowing into the vortex cores (16). How can the author prove, that the assumptions lead to a useful understanding of the underlying aerodynamical effects? The wing theory shows the theoretical achievable aerodynamical values of a wing generating the lift of a finite wing. The parts of the plane that do not contribute to the lift will lead to an efficiency less than 100% compared to the ideal values calculated here. High performance sailplanes should come close to the calculated ideal values, which is shown in table 1 and table 2, by calculating the expected glide ratio of a SB 10 and a ETA with the here used wing theory, leading to good agreement with published values. The expected performance of the human powered Gossamer Condor is calculated because the concept is chosen to realize a plane with extreme low power consumption. Here the calculated power to fly horizontal calculated with the chosen theory, is close to the published values.

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