Proceedings of the Sixteenth International Conference on
Computational Structures Technology
Edited by: P. Iványi, J. Kruis and B.H.V. Topping
Civil-Comp Conferences, Volume 14, Paper 7.2
Civil-Comp Press, Edinburgh, United Kingdom, 2026
ISSN: 2753-3239, doi: 10.4203/ccc.14.7.2

Evolutionary Form-Finding of Lattice Domes via a Constrained Force Density Method in Rhino-Grasshopper

B. Tóth[1,2], M. Bruggi[2] and J. Lógó[1]

[1] Department of Structural Mechanics, Budapest University of Technology and Economics , Hungary
[2] Department of Civil and Environmental Engineering, Politecnico di Milano, Italy

Abstract

This contribution presents a form-finding approach for the optimal design of reticulated domes implemented within a commercial parametric environment. The proposed methodology leverages Galapagos, the evolutionary solver embedded in the Rhino-Grasshopper platform, to explore optimal funicular configurations of lattice shells under self-weight. The equilibrium of the spatial network is enforced through a compact Python script that implements the Force Density Method (FDM), interfaced with the parametric model: by expressing the ratio of axial force to length for each branch as the design variable, the FDM reduces the equilibrium equations of the unrestrained nodes to a linear system, from which the shape of the network is directly recovered. The evolutionary solver optimizes the force densities of the branches, subject to geometric constraints on member lengths and node positions, effectively driving the shape of the dome toward the optimal funicular configuration. The Maxwell number, defined as the sum of the force-times-length products over all branches of the network, is adopted as the objective function to be minimized. The approach is tested on a Schwedler dome topology, and the results demonstrate that the evolutionary algorithm converges to solutions in close agreement with those obtained via gradient based optimization techniques. The key advantage of the proposed workflow lies in its accessibility: by exploiting widely adopted commercial tools and requiring only minimal scripting, the method lowers the barrier for practitioners and researchers to perform structurally informed shape optimization of lattice shells without the need for custom numerical solvers.

Keywords: form-finding, structural optimization, force density method, evolutionary algorithms, Galapagos, Grasshopper, lattice domes, lightweight structures, Maxwell number

1 Introduction

Lattice shells are doubly curved structures whose load-carrying efficiency relies on the predominantly axial response of their branches [1, 2]. The search for efficient shapes for this class of structures is naturally framed within equilibrium-based design strategies, such as funicular analysis, see e.g. [3, 4, 5]. From a modelling standpoint, a reticulated shell can be regarded as a statically indeterminate network of vertices and edges with a prescribed topology: boundary conditions are introduced at the restrained nodes, while the remaining ones are kept in equilibrium with the applied point loads. By introducing the force density of each branch, defined as the ratio between its axial force and its length [6], the equilibrium equations of the unrestrained nodes turn into a linear system that decouples along the three coordinate directions, from which the shape of the network is directly recovered. In the close past, the combination of the Force Density Method (FDM) with optimization routines has proven to be a powerful strategy for the form-finding of gridshells and reticulated domes, see e.g. [7]. Most of the existing formulations rely on gradient-based solvers, such as sequential convex programming [8], which were originally developed for large-scale size optimization of elastic structures [9] and require the sensitivities of the objective function and of the constraints with respect to the design variables.

While very effective, such solvers typically call for dedicated numerical implementations and a non-negligible amount of analytical derivation, which may discourage their adoption outside specialized research groups. The present contribution explores an alternative route, aimed at bringing FDM-based shape optimization of lattice shells within the reach of the parametric design community. The present work builds on the constrained FDM formulation proposed in [10], which is here recast within a derivative-free evolutionary framework. The multi-constrained minimization problem is solved by means of an evolutionary algorithm, namely the Galapagos solver natively available in the Rhino-Grasshopper environment [11], which is an easily accessible, general tool for parametric modelling in architectural and structural design [12, 13].

The FDM kernel is implemented as a compact Python script interfaced with the parametric model: the force densities of the branches are taken as design variables, and the evolutionary solver drives them toward the configuration that minimizes the objective function, while the geometric requirements on member lengths and nodal positions are enforced through penalty terms. Because the strategy is derivative-free, no sensitivity analysis has to be coded, and the whole workflow can be deployed with only a minimal amount of scripting, lowering the entry barrier for practitioners and researchers who wish to perform structurally informed shape optimization without resorting to custom numerical solvers. As in [10], the Maxwell number is adopted as the quantity to be minimized, defined as the sum of the products of axial force and length over all the edges of the spatial network. By virtue of Maxwell's theorem [14], this measure equals the sum of the products between the applied loads and their distances from an arbitrary reference point, see also the discussions and examples reported in [15, 16]. The constraints retained in the formulation are purely geometric: bounds are prescribed on the minimum and maximum length of the bars and on the admissible range of the vertical coordinates of the nodes.

The proposed workflow is assessed on a benchmark Schwedler dome topology under self-weight. The results show that the evolutionary solver converges to configurations in close agreement with those obtained by gradient-based techniques [10], thereby supporting the viability of the approach as an accessible alternative for form-finding of reticulated domes.

The remainder of the paper is organized as follows. The Force Density Method is briefly recalled, and the multi-constrained problem together with its penalized counterpart is then introduced. A numerical example based on the Schwedler dome topology is finally presented to illustrate the workflow and to discuss the quality of the solutions retrieved by the evolutionary solver.

2 Force density method

The equilibrium of spatial networks is handled through the Force Density Method (FDM) [6]. The network is composed of $n_s = n + n_f$ nodes and m branches acting as bars, embedded in a Cartesian reference frame with axes x, y, z. The full set of nodal coordinates is gathered in vectors x_s, y_s, z_s, where subscripts distinguish between the n free nodes, those loaded by external forces, and the n_f supported nodes where reactions develop. The connectivity of the grid is encoded in the matrix C_s, with C and C_f denoting its subsets associated with free and restrained nodes, respectively. The coordinate differences between the connected nodes along each axis are expressed as follows:

$$\mathbf{u} = \mathbf{C}_s\mathbf{x}_s, \quad \mathbf{v} = \mathbf{C}_s\mathbf{y}_s, \quad \mathbf{w} = \mathbf{C}_s\mathbf{z}_s. \tag{1}$$

For each branch of the network, the ratio of axial force to length defines the force density, and these are stored in the vector $\mathbf{q} = \mathbf{L}^{-1}\mathbf{s}$, where $\mathbf{s}$ is the vector collecting the forces in the m branches, $\mathbf{L} = \mathrm{diag}(\mathrm{l})$ and $l_i = \sqrt{u_i^2 + v_i^2 + w_i^2}$. Only gravitational loading is considered, with vertical point forces at the free nodes assembled in $\mathbf{p}_z$. A key property of the FDM is that substituting $\mathbf{q}$ into the nodal equilibrium conditions yields a system that is linear and decoupled across the three spatial directions:

$$
\begin{aligned}
&\mathbf{C}^T\mathbf{Q}\mathbf{C}\mathbf{x} + \mathbf{C}^T\mathbf{Q}\mathbf{C}_f\mathbf{x}_f = \mathbf{0},\\
&\mathbf{C}^T\mathbf{Q}\mathbf{C}\mathbf{y} + \mathbf{C}^T\mathbf{Q}\mathbf{C}_f\mathbf{y}_f = \mathbf{0},\\
&\mathbf{C}^T\mathbf{Q}\mathbf{C}\mathbf{z} + \mathbf{C}^T\mathbf{Q}\mathbf{C}_f\mathbf{z}_f = \mathbf{p}_z,
\end{aligned}
\tag{2}
$$

with $\mathbf{Q} = \mathrm{diag}(\mathbf{q})$.

It is important to mention that self weight is dependent on the shape while the shape is determined by the solution of (2). To address this interdependency, the FDM procedure is applied iteratively: at each step, the nodal loads are updated based on the member lengths obtained in the previous solution, and the process is repeated until the total applied load shows only minor variation. Preliminary tests on the initial grid indicated that the change in total self-weight becomes negligible after a few iterations; therefore, a small fixed number of inner iterations (four in this work) was adopted as a reasonable balance between accuracy and computational effort. This iterative update is performed within each evaluation of the penalized objective function carried out by the evolutionary solver.

3 Optimization problem

A multi-constrained minimization is stated in terms of any set of force densities $\mathbf{q}$:

$$
\begin{cases}
\min\limits_{q_j \leq 0} \quad \mathcal{M} & \text{(3a)}\\
\text{s.t.} \quad \mathbf{C}_x^T\mathbf{Q}\mathbf{C}_x\mathbf{x} + \mathbf{C}_x^T\mathbf{Q}\mathbf{C}_{fx}\mathbf{x}_f = \mathbf{0}, & \\
\qquad \mathbf{C}_y^T\mathbf{Q}\mathbf{C}_y\mathbf{y} + \mathbf{C}_y^T\mathbf{Q}\mathbf{C}_{fy}\mathbf{y}_f = \mathbf{0}, & \text{(3b)}\\
\qquad \mathbf{C}_z^T\mathbf{Q}\mathbf{C}_z\mathbf{z} + \mathbf{C}_z^T\mathbf{Q}\mathbf{C}_{fz}\mathbf{z}_f = \mathbf{p}_z, & \\
\qquad \dfrac{l_j}{l_{min}} \geq 1 \quad \text{for} \;\; j = 1...m, & \text{(3c)}\\
\qquad \dfrac{l_j}{l_{max}} \leq 1 \quad \text{for} \;\; j = 1...m, & \text{(3d)}\\
\qquad z_i(\mathbf{q}) \geq z_i^{min} \quad \text{for} \;\; i = 1...n, & \text{(3e)}\\
\qquad z_i(\mathbf{q}) \leq z_i^{max} \quad \text{for} \;\; i = 1...n, & \text{(3f)}
\end{cases}
$$

Since an evolutionary solver is employed, the inequality constraints (3c)–(3f) are enforced via penalty terms on the objective function. The penalized objective reads:

$$
\mathcal{M}_p = \mathcal{M} + \alpha \sum_{i=1}^{m} \left\langle 1 - \frac{l_i}{l_{min}} \right\rangle + \beta \sum_{i=1}^{m} \left\langle \frac{l_i}{l_{max}} - 1 \right\rangle + \gamma \left\langle 1 - \frac{z_i^{min}}{z_{lim}} \right\rangle + \delta \left\langle \frac{z_i^{max}}{z_{lim}} - 1 \right\rangle \tag{4}
$$

where $\langle \cdot \rangle = \max(0, \cdot)$ and α, β, γ, δ are penalty weights.

The penalized counterpart of (3) is obtained by replacing $\mathcal{M}$ with $\mathcal{M}_p$ and dropping the inequality constraints.

In the formulation above, the objective function is given by the sum, over all branches of the network, of the product between axial force and length [14]. Because the focus is on anti-funicular networks, as enforced by the side constraints in (3), this quantity reads:

$$\mathcal{M} = -\mathbf{s}^T \mathbf{l} = -\mathbf{q}^T \mathbf{L} \mathbf{l}. \tag{5}$$

(3b) expresses nodal equilibrium of the free nodes along the three coordinate axes and is used to recover $\mathbf{z}$ once $\mathbf{q}$ is known. (3c) and (3d) impose, respectively, a lower bound l_{min} and an upper bound l_{max} on the length of every branch; these geometric requirements are conveniently formulated through the branch coordinate differences already introduced in Eqn. (1). Finally, Eqs. (3e–3f) collect two sets of inequalities that constrain the vertical nodal coordinates $\mathbf{z}$: within the prescribed design domain, each of the n coordinates z_j is required to lie between a lower limit z_j^{min} and an upper limit z_j^{max}.

The penalized minimization problem is tackled by means of an evolutionary algorithm, namely the Galapagos solver natively available within the Rhino-Grasshopper environment, see the discussion in Section 1. Unlike gradient-based schemes such as the Method of Moving Asymptotes [8], which would require the sensitivities of the objective function and of the constraints with respect to the design variables collected in $\mathbf{q}$ (see e.g. [7]), the adopted population-based strategy is derivative-free: at each generation, candidate sets of force densities are evaluated only through the penalized objective $\mathcal{M}_p$, where the equilibrium equations (2) are solved by the FDM kernel and the violations of the geometric requirements (3c)–(3f) are accounted for via the penalty terms in Eqn. (4). This makes the implementation particularly straightforward within the parametric workflow, as no analytical or numerical sensitivity analysis has to be coded.

4 Numerical example

The workflow outlined in the previous sections is now applied to the optimal shape design of lattice domes, see e.g. [17]. As a benchmark topology, a Schwedler dome [18] is selected, whose grid combines meridional ribs, parallel rings, and diagonal bracings, as shown in Figure 1(a).

The Schwedler grid possesses a cyclic symmetry. This property is exploited to reduce the number of independent design variables: the force densities are grouped into equivalence classes according to the cyclic symmetry of the grid, so that all branches belonging to the same orbit share a common value of q. The FDM kernel is still assembled and solved on the full network, but the evolutionary solver only operates on the reduced set of independent force densities, which substantially shrinks the search space explored at each generation. This reduces the 352 force densities to only 15 independent variables in this specific case.

The dome is inscribed in a circle of radius $r = 6\,\mathrm{m}$. The admissible design space prescribes that the vertical coordinate of any free node must lie between $z_{min} = 3.00\,\mathrm{m}$ and $z_{max} = 8.00\,\mathrm{m}$, while each branch is required to have a length between $l_{min} = 0.75\,\mathrm{m}$ and $l_{max} = 2.00\,\mathrm{m}$. The aim of the analysis is to determine the dome geometry that minimizes the Maxwell number defined in Eqn. (5), while complying with the bounds on member lengths and on nodal elevations. Self-weight is the only load considered: the bars are assigned a square cross-section of $0.3 \times 0.3\,\mathrm{m}^2$ and a specific weight of $25\,\mathrm{kN/m}^3$.

The Galapagos evolutionary solver is run with a population of 100 individuals, an initial boost factor of 5, a maintain rate of 2% and an inbreeding factor of 96%. Convergence is declared when the best individual has not improved for 150 consecutive generations, which also defines the stopping criterion.

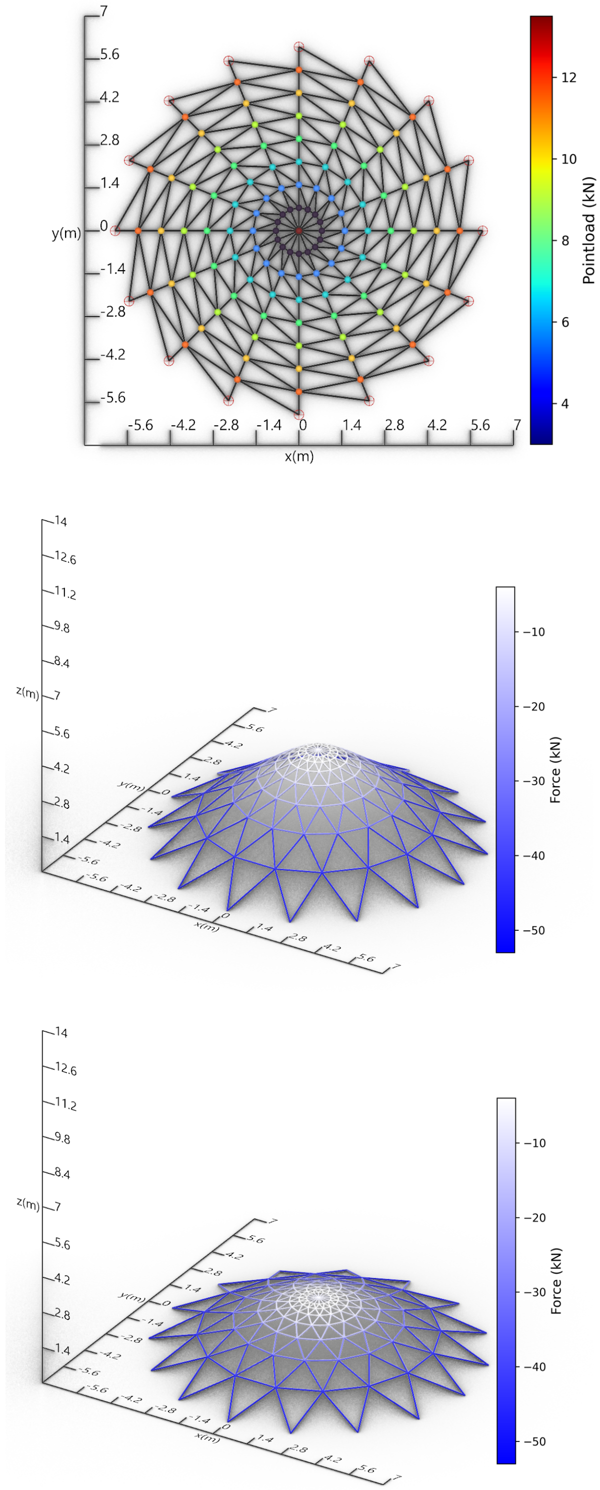
Figure 1(a) shows the nodal loads acting on the initial topology, while Figure 1(b) depicts the starting guess obtained by assigning the same force density ($25\,\mathrm{kN/m}$) to every branch. Figure 1(c) shows the initial guess after the inner self-weight loop has converged, which is the configuration actually seen by the evolutionary solver. It can be observed that this configuration does not fulfill the geometric requirements and already departs noticeably from the original grid.

The optimized layout returned by the evolutionary solver is reported in Figure 2(b): the solution is fully feasible with respect to all local constraints and differs substantially from the initial guess. In particular, only the meridional ribs and the diagonal members remain structurally active, whereas the rings are essentially negligible, as confirmed by the force intensity map in the same picture.

For reference, the same benchmark has been solved in [10] by means of a gradient-based sequential convex programming scheme, under identical topology, loading and geometric bounds, yielding an optimal Maxwell number of 7576 kNm. The value of 7645 kNm returned by the evolutionary solver is within less than 1% of this reference, which supports the claim that the derivative-free workflow is competitive with gradient-based techniques on this class of problems, while requiring no sensitivity analysis.

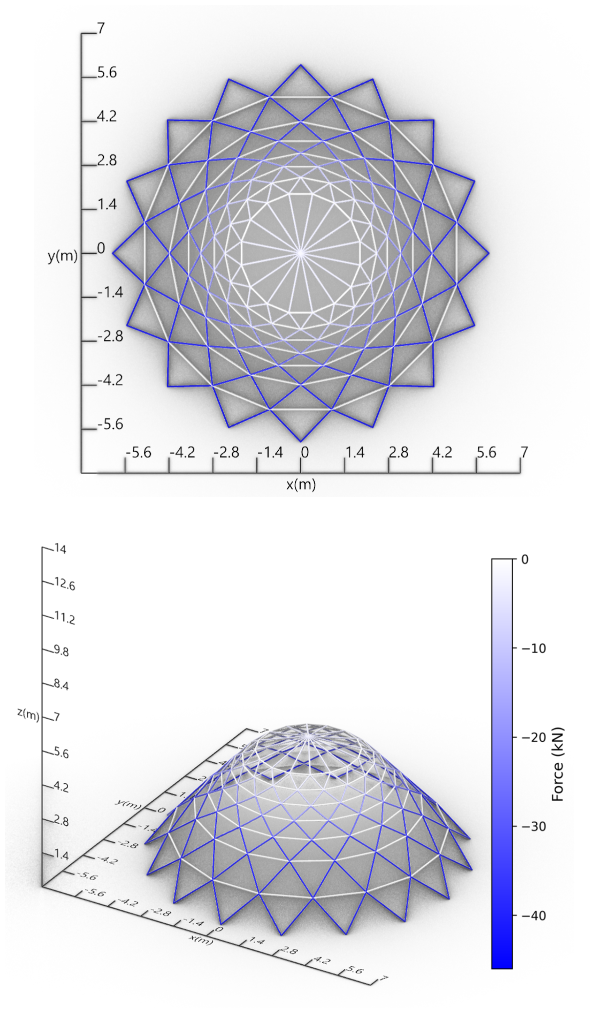
5 Concluding remarks

This work has presented a numerical framework for the form-finding and optimal design of reticulated domes based on funicular analysis. Building on recent contributions in the literature, the Force Density Method has been adopted as the equilibrium kernel for spatial truss networks, owing to its compactness and to the ease with which it can be embedded inside an optimization loop. The Maxwell number, defined as the sum of the products between axial force and length over all the branches of the network, has been chosen as the quantity to be minimized. The feasible design space has been



(a) (b) (c)

Figure 1: Topology of the starting grid and nodal loads in kN (a), initial guess (b), initial guess after converged self weight (c).



(a)

(b)

Figure 2: Top view of the optimal network and element forces (a), 3D view of the structure (b). Objective function at convergence 7645 kNm.

delimited by geometric constraints acting both on the vertical position of the nodes and on the admissible range of member lengths.

Differently from previous works relying on sequential convex programming, the resulting multi-constrained problem has here been solved through an evolutionary algorithm, namely the Galapagos solver available in the Rhino-Grasshopper environment, with the geometric requirements introduced into the objective by means of penalty terms. Such a choice removes the need for any sensitivity analysis and allows the whole workflow to be deployed within a widely used parametric modelling platform with only a minimal amount of scripting.

On the Schwedler benchmark, the Maxwell number returned by the evolutionary solver is within less than 1% of the value obtained by a gradient-based sequential convex programming scheme on the same problem, which supports the viability of the derivative-free route.

Current research activities are devoted to the assessment of the proposed workflow on a broader set of initial topologies, taking inspiration from [18], with the aim of comparing the optimal configurations obtained for different starting grids in terms of the adopted performance measure, i.e. the Maxwell number and possibly other objectives (i.e. compliance). Future developments will also address the extension of the methodology to the case of multiple loading conditions.

Acknowledgements

PROJECT NO. 2024-2.1.2-EKÖP-KDP-2024-00005 HAS BEEN IMPLEMENTED WITH THE SUPPORT PROVIDED BY THE MINISTRY OF CULTURE AND INNOVATION OF HUNGARY FROM THE NATIONAL RESEARCH, DEVELOPMENT AND INNOVATION FUND, FINANCED UNDER THE EKÖP KDP-24-1-BME-24 FUNDING SCHEME.

References

[1] Rozvany GIN, Prager W. A new class of structural optimization problems: Optimal archgrids. Comput Methods Appl Mech Eng. 1979;19(1):127-50.

[2] Dzierzanowski G, Czubacki R. Optimal archgrids spanning rectangular domains. Comput Struct. 2021;242.

[3] O'Dwyer D. Funicular analysis of masonry vaults. Comput Struct. 1999;73(1-5):187-97.

[4] Adriaenssens S, Block P, Veenendaal D, Williams C. Shell structures for architecture: Form finding and optimization. Routledge; 2014.

[5] Block P, Ochsendorf J. Thrust network analysis: A new methodology for three-dimensional equilibrium. J Int Assoc Shell Spat Struct. 2007;48(155):167-73.

[6] Schek H. The force density method for form finding and computation of general networks. Comput Methods Appl Mech Eng. 1974;3(1):115-34.

[7] Bruggi M. A constrained force density method for the funicular analysis and design of arches, domes and vaults. Int J Solids Struct. 2020;193-194:251-69.

[8] Svanberg K. The method of moving asymptotes—a new method for structural optimization. Int J Numer Methods Eng. 1987;24(2):359-73.

[9] Christensen PW, Klarbring A. An introduction to structural optimization. vol. 153 of Solid Mechanics and its Applications. Dordrecht, Netherlands: Springer; 2008.

[10] Bruggi M, Tóth B, Lógó J. Optimal Design of Lattice Domes by Means of a Constrained Force Density Method. In: Proceedings of the Fifteenth International Conference on Computational Structures Technology. Edinburgh, UK: Civil-Comp Press; 2024. Online volume: CCC 9, Paper 5.1.

[11] Rutten D. Galapagos: On the Logic and Limitations of Generic Solvers. Architectural Design. 2013;83(2):132-5. Available from: https://onlinelibrary.wiley.com/doi/abs/10.1002/ad.1568.

[12] Nasır CA, Tan Bayram F, Güzelçoban Mayuk S. A Comparison of Galapagos and Wallacei Optimization Solvers in High-Rise Building Design. Journal of Innovations in Civil Engineering and Technology. 2025;7(2):131–153. Available from: https://izlik.org/JA95XE23YH.

[13] Guillén JFG, Chao YH, Reich S, Buchmann C. A computational workflow for crafting large-scale coral inspired modules through additive manufacturing. Construction Robotics. 2025;9(1):7. Available from: https://doi.org/10.1007/s41693-025-00150-4.

[14] Maxwell JC. The Scientific Papers of James Clerk Maxwell. vol. 2 of Cambridge Library Collection - Physical Sciences. Niven WDE, editor. Cambridge University Press; 2011.

[15] Liew A, Avelino R, Moosavi V, Van Mele T, Block P. Optimising the load path of compression-only thrust networks through independent sets. Struct Multidiscip Opt. 2019;60(1):231-44.

[16] Jiang Y, Zegard T, Baker WF, Paulino GH. Form-finding of grid-shells using the ground structure and potential energy methods: a comparative study and assessment. Struct Multidiscip Opt. 2018;57(3).

[17] Tarnai T. Simultaneous static and kinematic indeterminacy of space trusses with cyclic symmetry. Int J Solids Struct. 1980;16(4):347–359.

[18] Nooshin H. Space structures and configuration processing. Prog Struct Eng Mat. 1998;1(3):329–336.