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From Meshless Numerical Methods to Blocked Forces for Railway-Induced Vibration Prediction and Experimental Characterization

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Abstract

This paper reviews the evolution of research carried out at the Acoustical and Mechanical Engineering Laboratory of the Universitat Politècnica de Catalunya on the prediction of railway-induced noise and vibration, connecting two topics that have separately marked the group's activity in recent years: meshless boundary-type methods for wave propagation simulation, namely the method of fundamental solutions and the singular boundary method, and the blocked force concept applied to substructuring problems. The paper first revisits a hybrid experimental/numerical methodology, developed in collaboration with the University of Porto, in which a set of virtual point forces distributed over an auxiliary surface is used to reconstruct the incident vibration field measured near an existing railway line and to predict the resulting response of a future nearby building. It is then shown that this virtual-source formulation is closely connected to the blocked force method, with the method of fundamental solutions and the singular boundary method emerging as particular cases depending on whether the collocation points and virtual sources are placed on the auxiliary or the physical boundary. Building on this connection, two recent applications of the blocked force method developed by the group are described: a methodology to account for strong dynamic coupling between the railway track system and the targeted building in where to predict the ground-borne vibration levels, and a track-independent approach to characterise railway rolling stock as a ground-borne vibration source.

Keywords: railway-induced, noise, vibration, meshless methods, method of fundamental solutions, blocked forces.

1 Introduction

This paper traces the evolution of research carried out at the Acoustical and Mechanical Engineering Laboratory (LEAM) of the Universitat Politècnica de Catalunya (UPC) on railway-induced noise and vibration field, particularly regarding its prediction using numerical methods and the hybridization with experimental data to enhance the accuracy of the prognosis. In doing so, it connects two important topics that the group has studied during the last years: meshless methods applied to wave propagation simulation, which marked the group's research in the topic some years ago, and, more recently, the blocked force concept applied to different problems within the field.

Reliable prediction of railway-induced noise and vibration relies on the simulation of elastic and/or acoustic wave propagation since it is a crucial element of the simulation process, specially due to the significance of this element in the overall computational costs of such simulations. Traditional domain-based numerical techniques, such as the finite element method (FEM) and the boundary element method (BEM), can handle simulations of such problems in detail. However, simulations conducted using these methods could become computationally intense in some situations, specially when developing detailed high-fidelity studies often required when conducting research. Thus, back in the previous decade, the group decided to explore other alternatives for the simulation of wave propagation problems. The targeted methodologies were meshless boundary-type methods, particularly the method of fundamental solutions (MFS) [1] and, more recently, the singular boundary method (SBM) [2]. These methods, in contrast of the BEM, does not rely on a mesh to discretize the boundary and, therefore, elements and their connectivity are not longer required. This results in methods with simpler formulations and computational implementations than those usually required by mesh-based approaches. Also, various works has pointed out the potential computational advantages of this type of methods, specially when dealing with high frequency problems.

The MFS employs the fundamental solution of the governing equation of interest as the interpolation basis function. The method mimics the field response induced by the boundary conditions prescribed in the problem with the response at a set of collocation points located on the physical boundary due to the action of a set of virtual sources located outside the computational domain. One of the main drawbacks of the MFS is how sensitive its accuracy is to the location of the assumed distribution of virtual sources. It is known that an incorrect choice of their location may compromise the robustness of the method and lead to large numerical errors [19]. Thus, for many years, researchers have been interested in establishing guidelines to define the position of the virtual sources, which remains an open topic. The SBM has recently emerged as an alternative meshless boundary collocation method to solve boundary value problems. The SBM also considers the fundamental solution of the governing equation as the interpolation basis function but, in contrast to the MFS, does not place the virtual sources outside the domain of interest, but on the physical boundary itself. The SBM inherits the simplicity of the MFS but overcomes one of its main drawback: the lack of robustness due to the sensitivity of the method's accuracy to the location of the virtual sources distribution. However, SBM typically considers the virtual sources to be coincident with the collocation points, which induces a singular behaviour to the fundamental solution when dealing with coincident collocation-source point pairs. This problem is handled by introducing the so-called origin (or source) intensity factor (OIF). Also, the method usually requires to be deployed using the same number of collocation points than sources in order to produce correct results, while the MFS can be employed satisfactorily with different numbers or source and collocation points, a feature that allows for reaching higher computational efficiency.

Within this context, the research group has, over the last five years, progressively developed and validated a family of meshless and hybrid mesh/meshless methodologies for elastodynamic and acoustic wave propagation problems, with a particular focus on railway-induced vibration applications. This effort began with the formulation of a two-and-a-half-dimensional (2.5D) coupled FEM-BEM-MFS methodology for longitudinally invariant soil-structure interaction (SSI) problems [3], in which the FEM was used for the structure domain, the BEM to determine the soil stiffness at the soil-structure interface, and the MFS to efficiently evaluate the radiated wave field in the soil, together with a control strategy to mitigate the known sensitivity of MFS predictions to the placement of virtual sources. To address this sensitivity in a more fundamental way, the SBM was subsequently incorporated into the group's modelling framework, first through a 2.5D coupled FEM-SBM methodology for SSI problems [4], and, in parallel, through a 2.5D SBM formulation for exterior acoustic radiation and scattering problems [5], later refined to remove spurious eigensolutions that can affect SBM-based exterior acoustic predictions at specific frequencies [6]. The applicability of the SBM to ground-borne vibration problems was further extended through a pile-soil interaction model [7]. More recently, the complementary strengths of the MFS and the SBM were combined into a novel hybrid SBM-MFS methodology, first formulated for acoustic wave propagation problems [8] and subsequently extended to elastodynamic wave propagation [9]. The hybridization allows for employing less sources that collocation points without compromising accuracy, to produce correct results in the presence of non-smooth boundaries and to naturally overcome the issue of the fictitious eigenfrequencies.

In parallel to these developments devoted to the explore more efficient methods for computational acoustics and elastodynamics, the group, in collaboration with researchers at University of Porto, studied other applications of the concept of virtual sources. Section 2 revisits the methodology that arose from that collaboration [10]. More recently, while collaborating in some ideas with some colleagues at the University of Salford, it was found that the blocked force concept can be seen as already implicit in that earlier developments based on virtual sources, revealing an interesting connection between meshless methods for wave propagation problems and dynamic substructuring based on blocked forces. That connection is discussed in Section 3, which also describes two recent novel developments of the group in the field employing blocked forces: the simulation of strongly coupled track-soil-building systems and to the laboratory characterisation of rolling stock as vibration sources.

2 The hybrid experimental/numerical method based on virtual sources

The methodology proposed in [10] targets an important railway-induced vibration assessment case study: the prediction of railway-induced ground-borne vibration levels in a new building to be constructed close to an existing, operational railway line. The idea of this methodology is to combine a theoretical model of the building-soil system with experimental data arising from the operation of the existing railway line towards a methodology with reduced prediction uncertainty with respect to fully numerical models of the track-(tunnel)-soil-building. The methodology is conceptualised through three steps, illustrated in Fig. 1. First, the vibration response (or boundary condition, seen from a computational elastodynamics perspective) induced by railway traffic in the soil domain Ω is evaluated at a set of collocation points distributed over the ground surface area where the building will be constructed, using accelerometers or other vibration sensors. Second, this incident wave field is virtualised by means of a set of point forces (virtual sources) distributed along an auxiliary surface S enclosing the foundations of the future structure, based on the indirect formulation of the boundary integral method for a single-layer potential representation solved using a similar strategy as the MFS. The methodology is constructed in the frequency-domain. The resulting system of equations, defined in matrix form as

$$\mathbf{u}_c = \mathbf{H}_{cf}\mathbf{f}_v, \tag{1}$$

relates the response at the collocation points $\mathbf{u}_c$ to the unknown virtual force amplitudes $\mathbf{f}_v$ through the receptance matrix $\mathbf{H}_{cf}$ of a local subsoil model, without involving the building. Third, these virtual forces are applied to a theoretical building-soil model to obtain the response of the structure, $\mathbf{u}_b$, using $\mathbf{u}_b = \mathbf{H}_{bf}\mathbf{f}_v$, being $\mathbf{H}_{bf}$ the receptance matrix of the building-soil system. Because the virtual surface S can be as simple as a semi-sphere, the whole procedure is meshless and independent of the foundation geometry.

The method was numerically validated in 2D and 2.5D frameworks for tunnel-soil-building systems, for both homogeneous and layered soil profiles [10]. A parametric study established an discretisation rule: the spacing between virtual sources

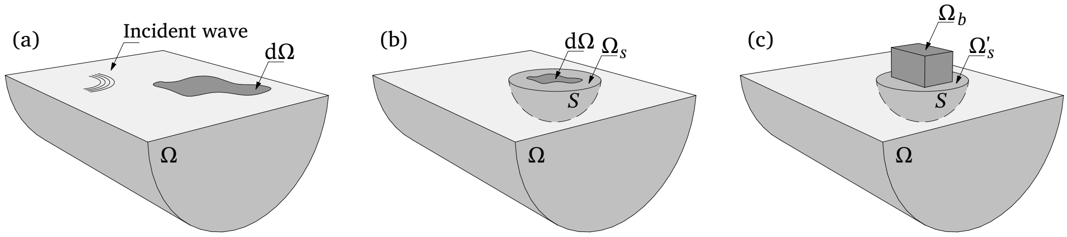

Figure 1: Schematic representation of the proposed experimental/numerical hybrid methodology (extracted from [10]): (a) Incident vibration field evaluated in a set of collocation points within $\mathrm{d}\Omega$; (b) Equivalent source distribution on the auxiliary surface S, generating a equivalent representation of the original incident field within the Ω_s domain; and (c) building response evaluation at any point of the domain consisting of $\Omega_b \cup \Omega'_s$, being $\Omega'_s = \Omega_s - (\Omega_b \cap \Omega_s)$, where $(\Omega_b \cap \Omega_s)$ is the domain occupied by the building foundations embedded in the soil.

should be smaller than the S-wave wavelength of the (upper) soil layer at the highest frequency of interest to guarantee accuracy, while virtual sources placed closer than about half this wavelength to the building foundation lead to ill-conditioning. The methodology was later extended and verified in a fully three-dimensional context in [11], considering one and four rigid shallow foundations embedded in a homogeneous half-space, with virtual sources distributed over a hemispherical surface using a spherical Fibonacci lattice. This study also examined the practically relevant case in which fewer collocation points than virtual sources are available. A significant practical finding was that a single triaxial sensor per foundation system, probably the most detailed practice in vibration assessment studies developed nowadays by engineering consultancy companies, is generally insufficient to reconstruct the incident wave field accurately, particularly at high frequencies.

3 The blocked force concept applied to railway-induced vibration assessment

Consider a mechanical system divided in two subsystems: the source, where the vibration is generated, and the receiver, a coupled structure to the source subsystem which is not subjected to external dynamic loading. Substructuring employing blocked forces [14, 15] characterises the source subsystem through the reaction forces that would appear at the source/receiver interface due to the source excitation if the receiver structure is considered to be infinitely rigid ("blocked") at that interface. Equivalently, blocked forces can be obtained indirectly from the response of coupled system consisting of source subsystem connected to *any* receiver, provided the coupled receptance of the system is known, solving the following system of equations in the frequency domain:

$$\mathbf{u}_{Ce} = \mathbf{H}_{Cec}\mathbf{f}_{Sc}, \tag{2}$$

where $\mathbf{u}_{Ce}$ is the coupled system response at a set of degrees of freedom within the receiver subsystem and $\mathbf{H}_{Cec}$ is the corresponding receptance matrix. For a continuous interface, blocked force is a continuous function along that boundary which is normally discretised in a set of forces collected in the vector $\mathbf{f}_{Sc}$. Once determined, blocked forces provide a receiver-independent description of the source that can be applied to any other receiver model to predict the resulting response, without needing to re-simulate the excitation mechanism.

Now, if one compares Eq. (2) with the expression employed to obtain the virtual source strengths in the hybrid experimental/numerical methodology previously presented, Eq. (1), the resemblance between the two is evident. In particular, Eq. (2), when evaluated at arbitrary points on the receiver outside the interface, can be seen as a form of the MFS where the auxiliary boundary is used to place the collocation points while the virtual sources are placed at the physical boundary. On the other hand, the SBM connects with the direct form of the blocked force method, $\mathbf{u}_{Cc} = \mathbf{H}_{Ccc}\mathbf{f}_{Sc}$, where the response is evaluated at the source/receiver interface. It is also worth noting that a key concept in blocked force substructuring, the completeness of the interface description [20], is closely linked to the studies conducted regarding the discretisation requirements for proper operation of the MFS and SBM numerical schemes.

Recently, the research group has applied the blocked force method to two case studies in the field of railway-induced ground-borne vibration. These cases are described in the next two sections, with the aim of exemplifying the potential of this method in the field and how the complexity of having a moving source subsystem can be treated adequately.

3.1 Simulation of strongly coupled track-soil-building systems

Railway-induced vibration simulations conventionally assume weak dynamic coupling between the track-(tunnel)-soil system and a nearby building, an assumption that breaks down when the two subsystems are in close proximity [16]. The LEAM research group is working nowadays on a blocked-force-based methodology that efficiently accounts for the strong coupling between these subsystems. In this methodology, the train-track-(tunnel)-soil system until the soil-building interface is considered to be the source subsystem, while the building together with its foundations is considered to be the receiver. The blocked forces at the building-soil interface are first obtained from a 2.5D model of the track-(tunnel)-soil system excited by the moving train-track dynamic loads from the expression

$$\mathbf{f}_{bc} = \mathbf{D}^{s}_{bb}\,\mathbf{u}^{f}_{\mathrm{inc}}, \tag{3}$$

where $\mathbf{u}^{f}_{\mathrm{inc}}$ are free-field displacement response of the 2.5D track-(tunnel)-soil model due train traffic at the virtual (since the building is not included yet) building-soil interface, and $\mathbf{D}^{s}_{bb}$ is the dynamic stiffness of the soil at that virtual building-soil interface obtained considering again the same track-(tunnel)-soil model, so that block forces are obtained in the presence of the track and its embankment, for at-grade railway systems, or the track and the tunnel, for underground railway ones. These non-moving blocked forces are then applied to the global track-tunnel-soil-building system to obtain the building response, avoiding the necessity of combining two frames of reference (the one fixed and the one moving with the train) in the same modelling framework. The resulting procedure has a computational cost comparable to that of the classical weak-coupling approach, with the only additional effort being the determination of the matrix $\mathbf{D}_{bb}^{s}$ from the 2.5D track-(tunnel)-soil model. While employed to only numerical simulation purposes in this case, this methodology is conceptually close to the hybrid experimental/numerical method based on virtual sources described in Sec. 2, having only the difference that the virtual sources and collocation points are placed at the same boundary, which in this case is the virtual building-soil interface.

3.2 Track-independent characterisation of rolling stock

While airborne noise emissions of rolling stock are regulated under a standardised, track-independent test procedure (Noise TSI), no equivalent standard exists for ground-borne vibration emissions. Building on the preliminary proposal of [17] and the indirect blocked force method [14, 15], the group investigated in [13] the use of blocked forces as a track-independent quantity to characterise a railway vehicle as a vibration source. In this case, the vehicle is considered to be the source subsystem (including the wheel/rail contact stiffness) while the track system (and eventually its embankment, the tunnel and the surrounding soil) is adopted as the receiver structure, as shown in Fig. 2.

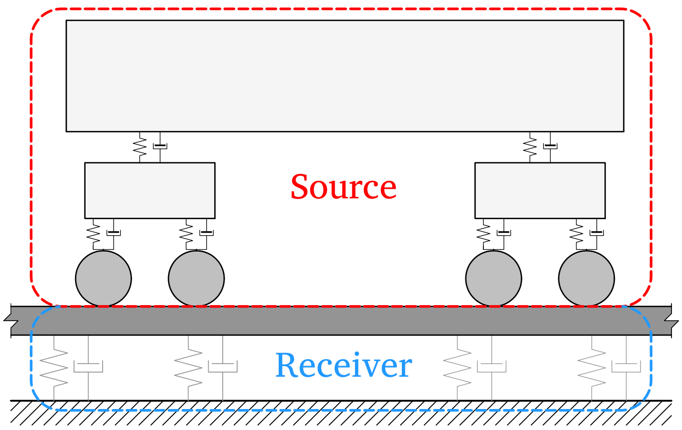

Figure 2: Source (vehicle) and receiver (track-embankment system) definitions adopted in the blocked force framework to characterise railway vehicle as ground-borne vibration sources. Source: [13].

A vehicle-track model is used to demonstrate the approach numerically, combining a rigid multibody representation of the vehicle (carbody and two bogies with vertical and pitch degrees of freedom, wheelsets with vertical motion only, primary and secondary suspensions, and Hertzian contact springs) with an Euler-Bernoulli beam on a Winkler foundation representing the track. Blocked forces are determined at the wheel-rail contact points via Eq. (2), using both a moving-roughness formulation (fixed vehicle, roughness convected under the contact points) and a moving-train formulation (vehicle travelling at constant speed in a moving frame of reference). Two key feature are verified numerically: the blocked forces obtained are independent of the track support condition (results for stiff and soft supports coincide, confirming their suitability as a track-independent emission indicator; and the blocked forces computed under the moving-roughness and moving-train formulations are identical, confirming the consistency of both approaches. Based on this results, the paper proposes a practical implementation of the method, in where blocked forces of a real vehicle prototype are measured at a manufacturer's own facilities using only a short track section and a set of excitation devices reproducing the wheel/rail roughness excitation, without requiring access to an operational railway line.

4 Concluding remarks

This paper has traced the link of two lines of work initially pursued somewhat independently: meshless boundary-type numerical methods for elastodynamic and acoustic wave propagation, and dynamic substructuring based on blocked forces. By comparing the th expressions employed to determine the virtual forces and the blocked forces, it has been shown that the two approaches are formally related: the method of fundamental solutions and the singular boundary method can be recovered as particular instances of the blocked force method depending on whether the auxiliary or the physical boundary is used to locate the collocation points and the virtual sources. Thus, evidences are shown that the blocked force method, beyond its established role in dynamic substructuring, is conceptually closely related to meshless boundary methods such as the MFS and the SBM. This a surprising coincide that may lead to interesting findings when testing, for example, how special features of the substructuring based on blocked forces are translated to the framework of boundary-based numerical methods and vice versa.

Building on this connection, two recent applications of the blocked force method to railway-induced ground-borne vibration have been presented. The first addresses the simulation of strongly coupled track-soil-building systems, using blocked forces obtained from the track-(tunnel)-soil system to avoid combining the fixed and train-moving reference frames within the same model, at a computational cost comparable to that of the conventional weak-coupling approach. The second concerns the track-independent characterisation of railway vehicles as ground-borne vibration sources, in which blocked forces computed at the wheel-rail contact points were shown to be independent of the track support condition and consistent between moving-roughness and moving-train formulations, opening the way to a practical test procedure that could be conducted at a manufacturer's facilities without access to an operational line. These examples exemplify the potential of blocked forces in railway-induced vibration assessment, and how the complexity of having a moving source subsystem can be treated adequately.

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