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Vertical Wheel-Rail Force Identification Using Frequency-Domain Physics-Based Modeling and Data-Driven Priors

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Abstract

This paper presents a physics-guided frequency-domain framework for the identification of vertical wheel–rail forces from measured rail acceleration responses. A forward operator is constructed based on an Euler–Bernoulli beam model to capture the dynamic relationship between moving wheel–rail forces and rail responses, including frequency coupling and multi-wheelset phase effects. To address the instability of the inverse problem, a learning-based spectral magnitude prior is introduced to constrain the force distribution without directly modelling the complex spectrum. The identification is formulated as a prior-constrained optimization problem that balances data consistency and prior information across frequencies. Field measurements from a metro tunnel are used to validate the proposed approach. The identified forces produce reconstructed responses that agree well with measured data in both frequency and time domains, demonstrating stable and physically consistent performance under real operating conditions. The proposed framework provides a practical solution for vertical wheel–rail force estimation and offers a general approach for solving ill-posed inverse problems in structural dynamics.

Keywords: railway dynamics, wheel-rail force identification, frequency domain analysis, inverse problem, physics guided modelling, deep learning

1 Introduction

Wheel–rail forces are the primary excitation source of railway track systems, governing the dynamic responses of rails, fasteners, sleepers, and substructures, and playing a critical role in structural performance assessment, condition monitoring, and environmental vibration evaluation [1,2]. Accurate characterization of wheel–rail forces is therefore essential for understanding track dynamics and ensuring the safe and reliable operation of railway infrastructure.

Conventional approaches typically rely on vehicle–track coupled dynamic models to estimate wheel–rail forces through forward simulations [3,4]. However, their accuracy strongly depends on detailed system parameters, such as vehicle suspension properties, track characteristics, and track irregularities, which are often difficult to determine precisely in practice and may evolve over time. This introduces significant uncertainty in force estimation under real operating conditions. As an alternative, inverse identification of wheel–rail forces from measured structural responses has attracted increasing attention [5,6].

Nevertheless, such inverse identification constitutes a typical ill-posed problem [7,8]. Under moving train conditions, the relationship between excitation and response is highly complex due to the combined effects of moving loads, multiple wheelsets, and structural dynamics, leading to frequency coupling and phase interactions in the spectral domain. In addition, measurement noise and modelling errors are inevitably amplified during the inversion process, resulting in instability and reduced reliability of the identified forces. Achieving stable and physically consistent wheel–rail force identification thus remains a challenging task.

Existing studies have explored both physics-based and data-driven approaches [9,10]. Physics-based methods provide clear mechanical interpretations but are sensitive to noise and modelling uncertainties. In contrast, data-driven methods offer strong representation capability but often lack physical consistency and generalization under varying conditions. Therefore, effectively integrating physical modelling with data-driven learning remains an open problem.

To address these challenges, this study proposes a frequency-domain framework for wheel–rail force identification by formulating the problem as a physics-guided inverse task. A spectral forward operator is first constructed based on a periodically supported Euler–Bernoulli beam model to describe the mapping between wheel–rail forces and rail responses, while accounting for frequency coupling induced by moving loads and spatial phase effects of multiple wheelsets. In addition, a deep learning-based spectral magnitude prior is introduced to constrain the force amplitude distribution without directly learning the complex spectrum. By integrating data consistency and prior constraints, a unified inversion framework is established, enabling stable and physically consistent identification of wheel–rail forces.

2 Methods

2.1 Overall framework

To achieve stable and physically consistent identification of wheel–rail forces, a physics-guided frequency-domain framework is proposed, as illustrated in Fig. 1. The framework integrates physical modelling and data-driven learning within a unified inverse formulation and consists of three main components: (i) physics-based forward operator construction, (ii) learning-based spectral prior estimation, and (iii) prior-constrained inverse identification.

First, a physics-based forward operator is constructed using a frequency-domain vehicle–track coupled analytical model. The rail is modelled as a periodically supported Euler–Bernoulli beam, which captures the essential dynamic behaviour of the track system. Under moving train excitation, the model accounts for frequency coupling effects and spatial phase differences induced by multiple wheelsets. Based on this formulation, a spectral mapping between the wheel–rail force and the rail acceleration response is established in the frequency domain, forming the forward operator.

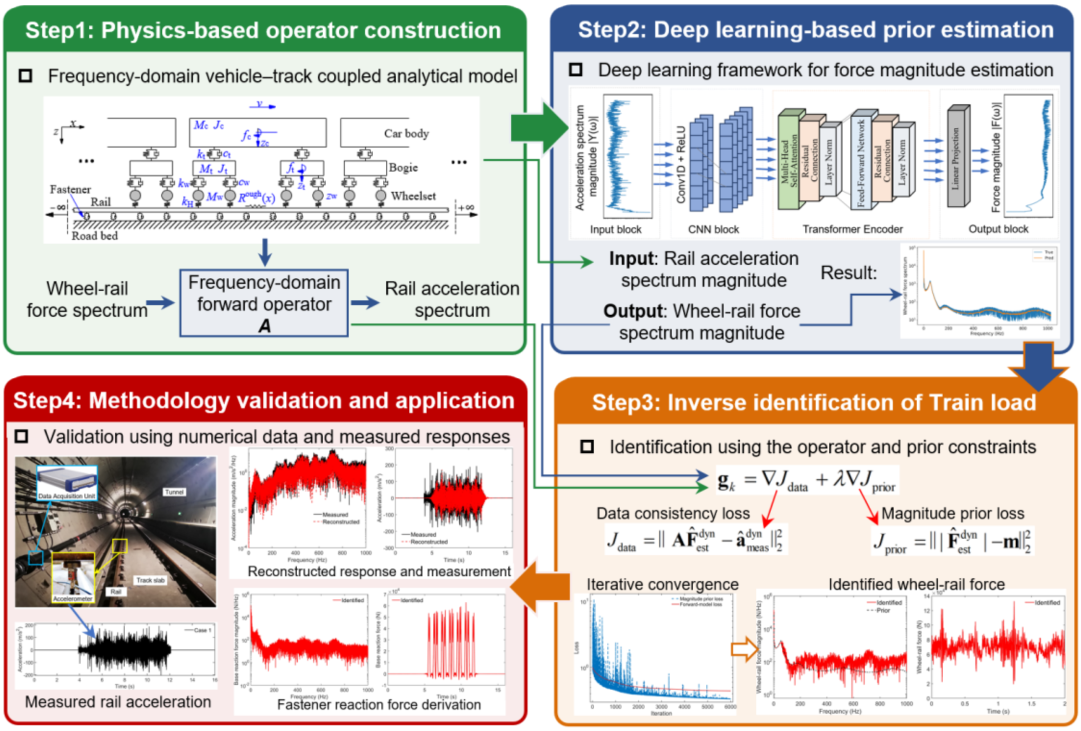

Figure 1: Overall framework of the proposed wheel-rail force identification approach.

Second, a deep learning model is introduced to estimate the spectral magnitude prior of the wheel–rail force from measured rail acceleration spectra. The model takes the magnitude of the rail acceleration spectrum as input and predicts the corresponding force spectrum magnitude. By learning the underlying spectral patterns from data, the prior provides frequency-dependent constraints on the solution space without directly modelling the complex-valued spectrum.

Finally, the forward operator and the learned prior are integrated into a unified inverse identification framework. The wheel–rail force is reconstructed by solving a prior-constrained optimization problem, in which a data consistency term ensures agreement between predicted and measured responses, while a magnitude prior term enforces physically plausible spectral distributions. The two components are adaptively balanced across frequencies, enabling stable and robust identification under noisy and uncertain conditions.

The effectiveness of the proposed framework is subsequently validated using both numerical simulations and field measurements, demonstrating its capability to provide reliable and physically interpretable wheel–rail force estimation.

2.2 Physics-based forward model

A physics-based forward model is developed to describe the dynamic relationship between wheel–rail forces and rail responses in the frequency domain. The rail is modelled as an Euler–Bernoulli beam subjected to moving loads, providing a physically consistent representation of the track dynamics.

The vertical dynamic equilibrium of the rail can be expressed as

$$EI\frac{\partial^4 w(x,t)}{\partial x^4}+m\frac{\partial^2 w(x,t)}{\partial t^2}=p(x,t) \tag{1}$$

where $w(x,t)$ is the rail displacement, EI is the bending stiffness, m is the mass per unit length, and $p(x,t)$ denotes the wheel–rail force.

For a train moving at a constant speed v, a harmonic component of the wheel–rail force can be written as:

$$p(x,t)=\hat{f}(\omega_f)\delta(x-vt-x_0)e^{i\omega_f t} \tag{2}$$

where $\hat{f}(\omega_f)$ is the force amplitude at excitation frequency ω_f, and x_0 is the initial position. This formulation indicates that the excitation travels along the rail and introduces space–time coupling.

Applying the Fourier transform with respect to time leads to the frequency-domain governing equation:

$$EI\frac{\partial^4 \hat{w}(x,\omega)}{\partial x^4}-m\omega^2\hat{w}(x,\omega)=\hat{f}(\omega_f)\delta(x-x_0)e^{-i(\omega-\omega_f)x/v} \tag{3}$$

which shows that the rail response at frequency ω is influenced by the excitation at frequency ω_f. Therefore, moving loads introduce a frequency-coupled mapping between excitation and response.

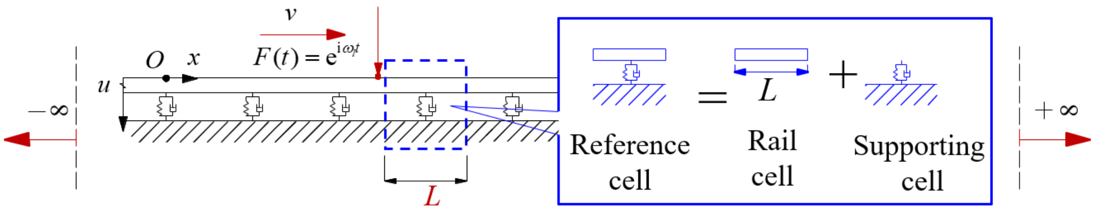

Figure 2: Schematic of the periodically supported rail model and reference cell decomposition under moving harmonic load

Due to the spatial periodicity of the track structure, the response satisfies a phase-consistency condition between adjacent locations,

$$\hat{w}(x+L,\omega)=\hat{w}(x,\omega)e^{-i(\omega-\omega_f)L/v} \tag{4}$$

and the same relation applies to its spatial derivatives. Based on this property, the infinite system can be reduced to a representative segment, within which the displacement solution is obtained by enforcing continuity and phase conditions.

For a unit excitation, the frequency-domain transfer function between the wheel–rail force and rail displacement can be expressed as

$$H_w(\omega,\omega_f)=\frac{\hat{w}(\omega,\omega_f)}{\hat{f}(\omega_f)} \tag{5}$$

The rail acceleration is related to the displacement through

$$\hat{a}(\omega,\omega_f)=-\omega^2\hat{w}(\omega,\omega_f) \tag{6}$$

and the corresponding transfer function between wheel–rail force and acceleration is given by

$$H_a(\omega,\omega_f)=-\omega^2 H_w(\omega,\omega_f) \tag{7}$$

When multiple wheelsets are considered, their longitudinal offsets introduce additional phase shifts, and the total response is obtained by superposition of individual contributions. After discretization over the excitation and response frequency grids, the forward model can be written in matrix form as:

$$\mathbf{y}=\mathbf{A}\mathbf{f} \tag{8}$$

where $\mathbf{f}$ denotes the wheel–rail force spectrum, $\mathbf{y}$ is the rail acceleration spectrum, and $\mathbf{A}$ is the forward operator.

The constructed operator captures the essential effects of moving loads, frequency coupling, and multi-wheelset phase interactions, and serves as the physical foundation for the subsequent inverse identification. Based on the identified wheel–rail force, other structural responses can be further derived through the same physical model.

2.3 Learning-based spectral prior

The inverse identification of wheel–rail forces from measured rail responses is inherently ill-posed, as small perturbations in the measured data may lead to large deviations in the reconstructed forces. To improve the stability and physical plausibility of the solution, a learning-based spectral prior is introduced to constrain the inverse problem.

Instead of directly learning the complex-valued mapping between the force and response spectra, which is difficult to model and sensitive to phase uncertainty, the proposed approach focuses on learning the spectral magnitude of the wheel–rail force. Specifically, a data-driven model is developed to estimate the magnitude distribution of the force spectrum from the measured rail acceleration spectrum.

Let $\mathbf{y}$ denote the measured rail acceleration spectrum and $\mathbf{f}$ the corresponding wheel–rail force spectrum. The learning objective is defined as:

$$\hat{\mathbf{m}}=\mathrm{F}_\theta(|\mathbf{y}|) \tag{9}$$

where $|\mathbf{y}|$ represents the magnitude of the acceleration spectrum, $\hat{\mathbf{m}}$ is the predicted magnitude prior of the force spectrum, and F_θ denotes the nonlinear mapping parameterized by a neural network.

The network is constructed based on a hybrid architecture combining convolutional layers and Transformer blocks, which enables effective extraction of both local spectral features and long-range dependencies across frequencies. The convolutional layers capture local patterns such as spectral peaks and banded structures, while the self-attention mechanism models global interactions along the frequency axis.

The model is trained using supervised data generated from the analytical vehicle–track coupled model. The loss function is defined as the mean squared error between the predicted and reference magnitude spectra:

$$\mathrm{L}=\frac{1}{N}\sum_{i=1}^{N}\left(\hat{m}_i-m_i\right)^2 \tag{10}$$

where $\mathbf{m}=|\mathbf{f}|$ denotes the true magnitude of the force spectrum at the i-th frequency point.

It is important to emphasize that the learning-based model does not directly produce the final identification result. Instead, the predicted magnitude prior serves as a structural constraint in the inverse problem, guiding the solution toward physically plausible spectral distributions while preserving consistency with the physics-based forward model. This formulation effectively separates the roles of data-driven learning and physical modelling, improving both stability and interpretability of the identified wheel–rail forces.

2.4 Inverse identification

Based on the physics-based forward model and the learning-based spectral prior, the wheel–rail force identification is formulated as a prior-constrained inverse problem in the frequency domain.

The observation model can be written as:

$$\mathbf{y}=\mathbf{A}\mathbf{f}+\boldsymbol{\varepsilon} \tag{11}$$

where $\mathbf{y}$ is the measured rail acceleration spectrum, $\mathbf{f}$ is the unknown wheel–rail force spectrum, $\mathbf{A}$ is the forward operator, and $\boldsymbol{\varepsilon}$ represents measurement noise and modelling errors.

Due to the ill-posed nature of the problem, direct inversion of the forward model is unstable and highly sensitive to noise. To address this issue, a prior-constrained optimization framework is adopted, in which both data consistency and spectral prior information are incorporated.

The data consistency term is defined as:

$$\mathrm{L}_{\mathrm{data}}=\left\|\mathbf{A}\mathbf{f}-\mathbf{y}\right\|_2^2 \tag{12}$$

which enforces agreement between the predicted and measured responses.

The prior constraint is constructed based on the learned spectral magnitude prior. Specifically, the magnitude of the reconstructed force is encouraged to be consistent with the predicted prior:

$$\mathrm{L}_{\mathrm{prior}}=\left\||\mathbf{f}|-\hat{\mathbf{m}}\right\|_2^2 \tag{13}$$

where $\hat{\mathbf{m}}$ is the predicted magnitude prior obtained from the learning-based model.

The overall objective function is defined as a weighted combination of the two terms:

$$\mathrm{L}=\left\|\mathbf{A}\mathbf{f}-\mathbf{y}\right\|_2^2+\lambda\left\||\mathbf{f}|-\mathbf{m}\right\|_2^2 \tag{14}$$

where λ is a regularization parameter that controls the balance between data fidelity and prior constraint.

The inverse problem is solved iteratively in the frequency domain. At each iteration, the update direction is determined by the gradient of the objective function, combining contributions from both the data consistency term and the prior constraint. This formulation enables a frequency-dependent balance between the two components: the prior plays a dominant role in stabilizing the solution in ill-conditioned frequency regions, while the data consistency term ensures accurate reconstruction where the forward model is reliable.

In addition, the forward operator exhibits a banded structure due to the localized frequency coupling induced by moving loads. This property is exploited to improve computational efficiency by restricting the interactions to a limited frequency range, significantly reducing computational cost while preserving essential dynamic characteristics.

Through the integration of physics-based modelling and learning-based prior constraints, the proposed inverse framework achieves stable and physically consistent reconstruction of wheel–rail forces under noisy and uncertain conditions.

3 Results

To evaluate the effectiveness of the proposed method under real operating conditions, field measurements were conducted in a metro tunnel, as shown in Figure 3(a). Accelerometers were installed beneath the rail to record vertical vibration responses during train passages. Two representative cases with different train configurations were considered, and the corresponding measured acceleration signals are presented in Figure 3(b)–(c).

The recorded signals exhibit typical characteristics of train-induced vibration, where the response is localized in time and dominated by broadband frequency components. These measured acceleration spectra serve as the input for the proposed wheel–rail force identification framework.

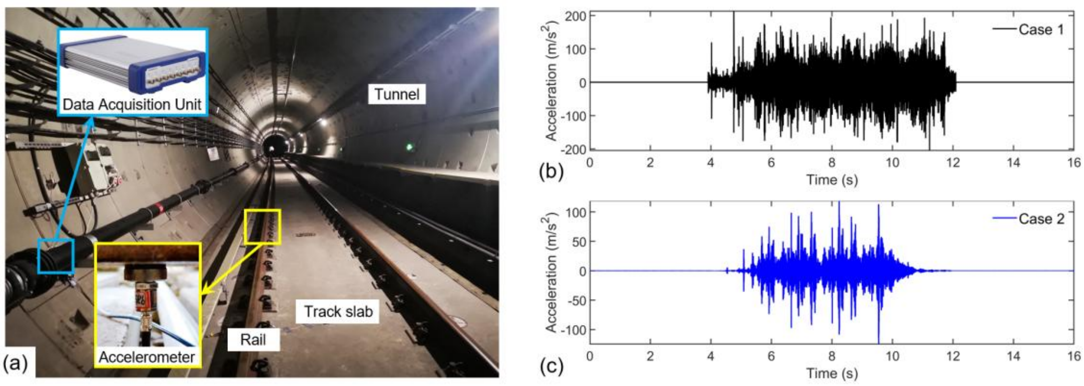

Figure 3: Field measurement setup and measured rail acceleration signals: (a) schematic of rail acceleration measurement; (b) measured rail acceleration in Case 1; (c) measured rail acceleration in Case 2.

The identification results for Case 1 are shown in Figure 4. In the frequency domain, the identified wheel–rail force spectrum demonstrates a smooth distribution without noticeable non-physical oscillations, while maintaining consistency with the learned spectral prior, as shown in Figure 4(a). The reconstructed rail acceleration agrees well with the measured response over a wide frequency range (Figure 4(b)), indicating that the identified force captures the essential dynamic characteristics of the system.

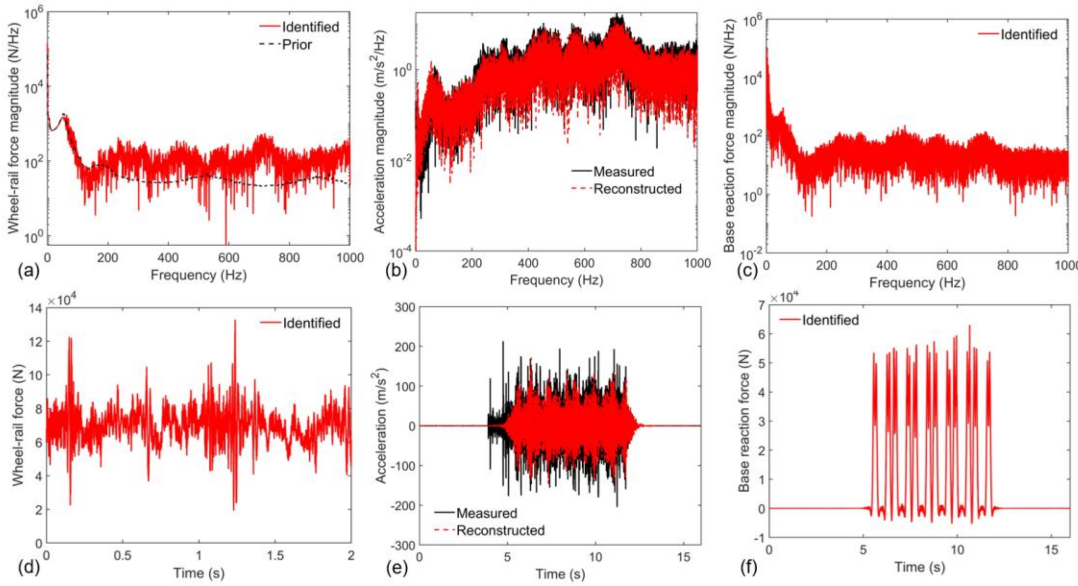

Figure 4: Identification results of the proposed method for Case 1: (a) magnitude spectrum and (d) time history of the identified train load; (b) magnitude spectrum and (e) time history of the measured and reconstructed rail acceleration; (c) magnitude spectrum and (f) time history of the computed fastener reaction force.

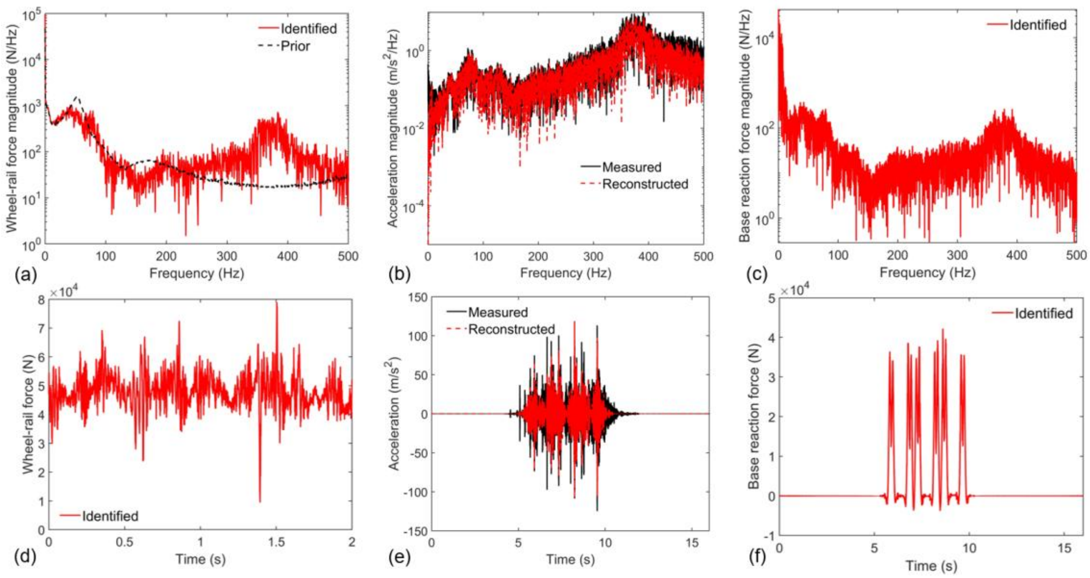

Figure 5: Identification results of the proposed method for Case 2: (a) magnitude spectrum and (d) time history of the identified train load; (b) magnitude spectrum and (e) time history of the measured and reconstructed rail acceleration; (c) magnitude spectrum and (f) time history of the computed fastener reaction force.

In the time domain, the identified wheel–rail force exhibits a stable dynamic pattern (Figure 4(d)), and the reconstructed acceleration signal closely matches the measured waveform (Figure 4(e)). These results confirm that the proposed method can reliably reconstruct both spectral and temporal features of the measured responses.

Figure 5 presents the identification results for Case 2. Similar trends are observed, where the identified force spectrum preserves the main frequency characteristics without introducing spurious fluctuations (Figure 5(a)). The reconstructed acceleration again shows good agreement with the measured data in both frequency and time domains (Figure 5(b) and Figure 5(e)), demonstrating the robustness of the method under different operating conditions.

4 Conclusions and Contributions

This study presents a physics-guided frequency-domain framework for the identification of vertical wheel–rail forces from measured rail acceleration responses. By integrating a physics-based forward operator with a learning-based spectral magnitude prior, the proposed approach provides a structured solution to the ill-posed inverse problem inherent in force identification.

The physics-based model, derived from an Euler–Bernoulli beam representation of the track, explicitly captures the effects of moving loads, frequency coupling, and multi-wheelset phase interactions. In parallel, a data-driven spectral prior is introduced to constrain the magnitude distribution of the force spectrum, avoiding direct learning of complex-valued mappings while improving numerical stability and physical plausibility.

The inverse identification is formulated as a prior-constrained optimization problem, where data consistency and prior information are adaptively balanced across frequencies. This enables stable reconstruction of vertical wheel–rail forces under noisy and uncertain conditions, while preserving the essential dynamic characteristics of the system.

Field validation results demonstrate that the identified forces lead to reconstructed rail responses that are in close agreement with measured data in both frequency and time domains. The smooth spectral distribution and absence of non-physical oscillations further confirm the robustness and reliability of the proposed method.

Overall, the proposed framework provides a practical and physically consistent approach for vertical wheel–rail force identification under real operating conditions. The identified forces can serve as reliable excitation inputs for subsequent track dynamics analysis, structural performance assessment, and vibration source characterization, and the methodology offers a general paradigm for solving ill-posed inverse problems in railway engineering and related fields.

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