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Optimisation of Asymmetric Single-Tower Cable-Stayed Concrete Bridges

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Abstract

An optimisation-based approach is presented for the optimal design of asymmetric single-tower cable-stayed concrete bridges. The original non-convex optimisation problem is addressed using a gradient-based algorithm combined with a multi-start procedure that automates the definition of initial designs. The finite element method is used for the three-dimensional analysis considering geometric nonlinearities, dead load and road traffic live load, and the time-dependent effects of concrete. The design is formulated as a cost minimisation problem subject to constraints on the displacements and stresses, considering service and strength criteria defined in accordance with the Eurocodes' provisions. This problem is addressed using a constraint aggregation approach by minimising a convex scalar function derived from an entropy-based formulation. The structural response to variations in the design variables is obtained using the discrete direct sensitivity analysis method. The design variables are the tower height, the deck and tower cross-sectional sizes, and the prestressing forces of cable-stays' and deck's internal prestressing tendons' and their cross-sectional areas. The optimisation of a 220 m bridge, with a main span of 128 m, illustrates the features and applicability of the proposed approach. The optimum design features a tower height-to-main span ratio of 1/1.82 and a deck slenderness of 1/80.

Keywords: cable-stayed bridges, optimisation, optimal design, asymmetric, single-tower, concrete, cable tensioning forces, sizing design variables, shape design variables.

1 Introduction

Cable-stayed bridges are widely used worldwide, from pedestrian crossings to medium- and long-span roadway and railway applications, owing to their structural and construction efficiency, as well as their economic and aesthetic advantages. Modern cable-stayed bridges are characterised by multiple inclined cable-stays, which provide continuous support and introduce prestressing effect in the deck, enabling long spans with slender decks. The static and dynamic response of these highly redundant structures is strongly influenced by the distribution of cable tensioning forces, and by the stiffness and mass distributions of their load-bearing members (deck, tower, and cable-stays). These bridges may adopt different structural arrangements, including single-span, two-span, three-span or multi-span solutions, with either symmetric or asymmetric layouts [1, 2].

In addition to the conventional symmetrical three-span arrangement, asymmetric configurations often adopt a single-pylon solution with two spans of unequal length. This solution is primarily used where site constraints (e.g., urban layouts, riverbanks, or obstacles) require asymmetry, and where aesthetic considerations promote the structure as a visual landmark. Self-anchored or externally anchored staying systems may be adopted, with either central (axial) or lateral cable arrangements. Some notable examples of the externally anchored solution are the Sancho el Mayor and Alamillo bridges in Spain, the Erasmus bridge in Rotterdam, and the Rainha Santa Isabel bridge in the authors' city of Coimbra (Portugal). The self-anchored scheme was adopted in some relevant examples with concrete superstructure, such as the East Huntington bridge in the USA, the Langreo bridge in Asturias (Spain), and the Ben-Ahin and Wandre bridges in Belgium. The same scheme was also utilised in the Franjo Tuđman Bridge in Croatia, the Wando Grand Bridge in Korea and the Elbe Bridge Schönebeck in Germany, which adopted steel-concrete composite solutions for the deck design. It is also worth mentioning the recently completed Danjiang bridge in Taiwan, which has a total length of 920 m, a main span of 450 m, and a tower height of 200 m; it is currently the single-tower asymmetric cable-stayed bridge with the longest main span in the world [4]. In single-tower asymmetric bridges, decks are most commonly constructed using prestressed concrete or steel-concrete composite solutions, which may be adopted either independently or in combination to accommodate the unbalanced load effects inherent to asymmetric configurations. Towers are almost exclusively constructed in concrete [1–3, 5].

The design of cable-stayed bridges is a challenging task that involves defining the structural system, determining the members' cross-sectional sizes, and calculating the distribution of cable tensioning forces. The geometrical nonlinearities, erection stages and the time-dependent behaviour of concrete should be considered when evaluating the structural response. Specifically, in single-tower asymmetric bridges, the lack of longitudinal equilibrium due to permanent load imbalances results in significant bending moments in the pylon and increased horizontal reactions at the supports. This condition is further exacerbated under live loading, leading to significant variations in cable forces and internal force distribution. These effects are influenced by the degree of asymmetry, typically defined by the ratio of the side span length (L_2) to the main span length (L_1), as well as by the contribution of the unsupported length (L_3) in the main span. Therefore, asymmetric configurations increase the complexity of cable-stayed bridge design, requiring an appropriate balance of weight and stiffness distribution among load-bearing members (deck, pylon, and stay cables) and careful tuning of cable forces. Additionally, variable deck cross-sections may be required, and in some cases, the use of counterweights may be necessary [1−3, 5].

This design problem comprises complex and time-consuming tasks, relying on a trial-and-error process and mostly guided by designers' experience, thus an optimised design may not be attained. Although not usually utilised in civil engineering design practice, optimisation techniques are particularly well-suited to this problem, favouring efficient use of materials and promoting cost-effective, sustainable structural solutions. Moreover, with the increasing availability of computational resources, implementing these techniques in practice has become more relevant, contributing to the automation of design procedures. The optimisation of cable-stayed bridges remains a relevant research topic, as highlighted in a recent literature survey by Martins et al. [6]. The optimal design considering responses to wind and earthquakes, the reliability-based optimum design, and the robust design, including, for example, cable loss scenarios, were identified as relevant topics for future research and are already addressed in recent works [7]. Furthermore, the optimisation of footbridges and curved bridges, innovative cable arrangements such as crossing cables, as well as the application of metaheuristic algorithms, artificial neural networks, and surrogate models, are also being explored by researchers [7]. The vast majority of works on the optimisation of cable-stayed bridges concern symmetrical configurations, with only a few examples addressing asymmetric solutions. Therefore, it is considered relevant to investigate the optimisation of non-conventional cable-stayed bridges, such as asymmetric and curved bridges, under-deck cable-stayed bridges, and combined cable-stayed bridges [8].

Some previous works have addressed the optimisation of asymmetric cable-stayed bridges, with a particular focus on determining optimal cable tensioning forces. Sung et al. [9] presented a constrained optimisation method based on the minimisation of total strain energy to determine optimal cable forces in asymmetric cable-stayed bridges. Lee et al. [10] proposed a two-step unit load method to optimise the cable tensioning strategies of asymmetric bridges, including the effects of construction stages. The Wando Grand Bridge was used as an application example. Wang et al. [11] introduced an integrated multi-objective framework that simultaneously optimises cable forces and counterweights to achieve a reasonable dead-load equilibrium state of asymmetric bridges. The minimum weighted total bending energy and the sum of the counterweights were considered as two objective functions. The dynamic weighted coefficient method was employed to solve the problem, accounting for constraints on displacements and internal forces. Guo and Guan [12] studied the application of the differential evolution (DE) algorithm to optimise cable forces, considering counterweights, geometric nonlinearities, and multiple constraints in an asymmetric three-span bridge with towers of different heights. Xiong et al. [13] proposed an improved particle swarm optimisation (PSO) algorithm for cable force optimisation in asymmetric single-tower bridges. A quadratic programming model was formulated to minimise the total bending energy of the deck and tower, subject to constraints on displacements, stresses, cable forces, and their uniformity.

From the literature review and to the best of the authors' knowledge, the optimal design of asymmetric single-tower cable-stayed concrete bridges has not been reported previously. Moreover, the specific characteristics of this bridge typology and of structural concrete design, including prestressing, do not allow the direct application of existing algorithms developed for steel or steel-concrete composite symmetric cable-stayed bridges.

The main objective of this work is to present an optimisation-based approach for the design of asymmetric single-tower cable-stayed concrete bridges. To this end, a MATLAB program was developed to implement the proposed optimal design strategy. This program combines a gradient-based optimisation algorithm with a multi-start procedure to automate the definition of the initial designs. Local optima are obtained from various initial designs, and the minimum-cost solution is selected as the optimal design. This work contributes to the conceptual and preliminary design of this bridge typology and also aims to offer additional insight into its optimal design.

In Section 2, the proposed optimal design strategy is described. In Section 3, the optimisation of a 220 m two-span asymmetric single-tower cable-stayed concrete bridge illustrates the features of the proposed approach. The main conclusions and prospective research developments are presented in Section 4.

2 Optimal Design Strategy

The design of asymmetric single-tower cable-stayed bridges is formulated as a cost minimisation problem subject to constraints in accordance with the provisions of the Eurocodes. This can be written as

$$\begin{array}{lll} \min & C(\underline{x}) & \\ s.t. & g_j(\underline{x}) \le 0 & j = 1,2,\cdots,M \\ & \underline{x}_{\min} \le \underline{x} \le \underline{x}_{\max} & \underline{x} = \{x_1, x_2, \ldots, x_N\}^T \end{array} \tag{1}$$

where $C(\underline{x})$ is the total cost of the structure, $g_j(\underline{x})$ represents the design constraints, $\underline{x}$ is the global design variable vector, M is the number of design constraints and N is the number of design variables, $\underline{x}_{min}$ and $\underline{x}_{max}$ are the vectors with the lower and upper bound values of the design variables, respectively.

This problem presents a large number of design variables of different types (shape, sizing, and mechanical) and numerous nonlinear, conflicting constraints arising from structural discretisation and several load cases. As commonly encountered in structural optimisation, this problem may be non-convex. Gradient-based and gradient-free optimisation strategies can be adopted to address this problem. Gradient-free approaches do not rely on derivative information to minimise the objective function. These encompass techniques such as random search, branch-and-bound and metaheuristic algorithms, including evolutionary algorithms, genetic algorithms, simulated annealing, and particle swarm optimisation. While these strategies are generally versatile and straightforward to implement, they may be less efficient due to their exponential convergence time as the number of design variables increases. Furthermore, the inherently stochastic nature of metaheuristic algorithms can hinder the identification of suitable cable tensioning force distributions in highly redundant cable-supported structures. In contrast, gradient-based strategies require evaluating the gradients of the objective function and constraints with respect to the design variables. This information is used to define search directions and iteratively update the design towards improved solutions. These strategies exhibit polynomial convergence time and are particularly efficient for problems with a large number of design variables, such as multiple cable tensioning forces combined with sizing and shape design variables. However, their performance depends on the choice of an appropriate initial design, and convergence to a local (not necessarily global) optimum is generally expected.

A gradient-based strategy is proposed here to efficiently address the optimum design of asymmetric single-tower cable-stayed bridges with concrete superstructure. This strategy integrates a gradient-based optimisation algorithm with a multi-start procedure. Within this framework, the multi-start procedure automatically determines the cable tensioning forces and corresponding cross-sectional areas, while the remaining design variables are defined in accordance with usual design practice. This approach alleviates the time-consuming task of defining suitable starting designs. In addition, considering multiple starting points reduces the likelihood of converging to suboptimal local minima while simultaneously enhancing exploration of the design space. The analysis and optimisation process is conducted for each starting design, yielding local optimum solutions. The least-cost design is chosen as the optimum design.

Constraint aggregation approaches are particularly suited to addressing optimisation problems with a large number of design variables and constraints when solved using gradient-based nonlinear programming algorithms. In this context, both classical formulations, such as the p-norm and the Kreisselmeier-Steinhauser (KS) function, and induced aggregation methods, including induced exponential and induced power methods, can be used [14]. Consequently, the problem in Equation (1) can be solved as a single-objective optimisation to minimise the cost subject to aggregated constraints. Alternatively, by aggregating the cost and all the design goals (defined by the constraints) in a single objective function, the problem can be addressed as a multi-objective optimisation [15]. Here, this latter approach was adopted. The multi-objective problem is solved indirectly by the minimisation of the convex scalar function expressed by Equation (2), obtained through an entropy-based approach [16]

$$\min F(\underline{x}) = \min \frac{1}{\rho} \ln\left[\sum_{j=1}^{M} e^{\rho\left(g_j(\underline{x})\right)}\right] \tag{2}$$

where ρ is the aggregation parameter that should be tuned for each problem (usual values range from 100 to 2000) and must not be decreased during the optimisation process. This function corresponds to the KS function and aggregates all design goals with different probabilities of becoming active, yielding a convex approximation that is close to the boundaries of the original non-convex domain. As the iterations progress, the uncertainty regarding the most relevant constraints for finding the optimum decreases. Throughout the iterative process, the cost objective is reduced relative to previous iterations while maintaining all constraints within their prescribed limits.

The design goals, $g_j(\underline{x})$, do not have an explicit algebraic form. Therefore, the problem is addressed through an explicit approximation based on a first-order Taylor series expansion about the current design vector, yielding:

$$\min F(\underline{x}) = \min \frac{1}{\rho} \ln \left[\sum_{j=1}^{M} e^{\rho \left(g_j(\underline{x}) + \sum_{i=1}^{N} \frac{dg_j(\underline{x})}{dx_i} \Delta x_i \right)} \right] \tag{3}$$

where $g_j(\underline{x})$ is the *j-th* design goal and $dg_j(\underline{x})/dx_i$ is the sensitivity of the *j-th* design goal with respect to *i-th* design variable. To ensure the accuracy of the explicit approximation, bound constraints with move limits were adopted. The MATLAB function *fmincon*, which minimises a scalar function of several variables subject to bound constraints using an interior-point algorithm, was selected to minimise the objective function given by Equation (3).

Figure 1 depicts the flowchart of the proposed optimal design strategy. The computer program begins by reading the problem data and generating a finite element model based on the design variables that describe the bridge geometry and the cross-sectional sizes of the deck and tower. These are defined based on the designer's experience and the values typically observed in design practice. An automated preliminary design procedure is used to determine the prestressing forces and cross-sectional areas of the cable-stays and internal prestressing tendons for an initial design. This is an iterative procedure comprising structural analysis, an influence matrix approach, and preliminary design calculations. A feasible or slightly unfeasible design, thus defined, enters the analysis and optimisation process. Using sensitivity information, the gradient-based algorithm iteratively updates the design variables to reduce the objective function and improve the current design. The analysis and optimisation process is repeated until both the variations in the design variables and the cost of the structure become negligible. The optimal design is selected as the minimum-cost solution among the local optima (in the Pareto sense) obtained for each initial design N_{id}.

Different types of design variables (shape, sizing, and mechanical) were considered in this design problem. The shape design variable concerns the tower height, which characterises the cable-supporting system and affects the structure's mass, stiffness, and cost. The sizing variables correspond to the cross-sectional dimensions of the deck and tower, as well as the cross-sectional areas of the cable-stays and tendons. The mechanical design variables correspond to the prestressing forces of the cable-stays and tendons. Although these variables do not directly influence the cost, they are fundamental in controlling the structure's behaviour. Figure 2 depicts the 87 design variables considered. To describe the variation of the cross-sectional sizes along the tower height, three zones were defined. Three zones were also considered in the deck to address the specificities of this bridge typology, which require different cross-sections due to the need for a counterweight near the abutment on the side span (Zone 2) and internal prestressing in the unsupported length of the main span (Zone 3). Zone 1 refers to the remaining length of the deck. Therefore, different main girder widths were considered across these zones. Nevertheless, the depth of the main girders was kept constant, which is considered aesthetically superior. In Zone 2, concrete cross-beams were adopted instead of steel.

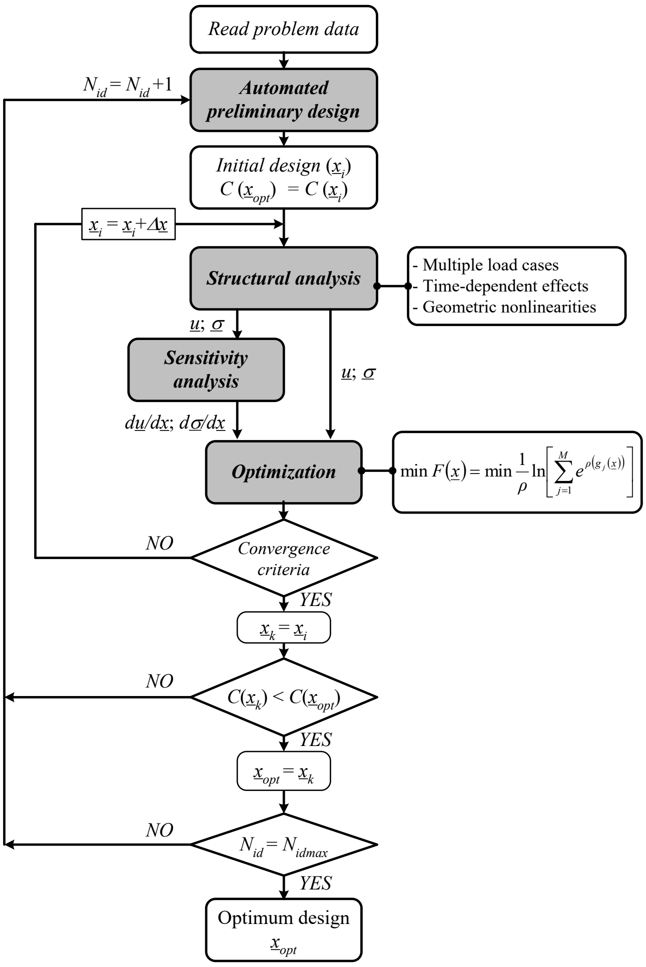

Figure 1: Flowchart of the optimal design strategy.

In addition to cost minimisation, the design must satisfy a set of serviceability and safety requirements. In the constraint aggregation approach adopted, the cost is considered the first design goal and can be expressed as

$$g_1(\underline{x}) = \frac{C}{C_0} - 1 \le 0 \tag{4}$$

where C represents the current cost of the structure, and C_0 is the initial cost of each analysis and optimisation cycle. This approach ensures that cost is always a priority goal for optimisation. The second set of objectives aims to limit the vertical displacements of the deck and the horizontal displacements of the tower under service conditions, accounting for time-dependent effects

$$g_2(\underline{x}) = \frac{|\delta|}{\delta_0} - 1 \le 0 \quad (5)$$

where δ and δ_0 are the value of the displacement and the corresponding allowable value, respectively. For the long-term analysis of the bridge, the $L_1/1000$ and $H/1000$ values were considered as limits for vertical and horizontal displacements, respectively [17]. L_1 and H correspond to the main span length and the height of the tower, respectively.

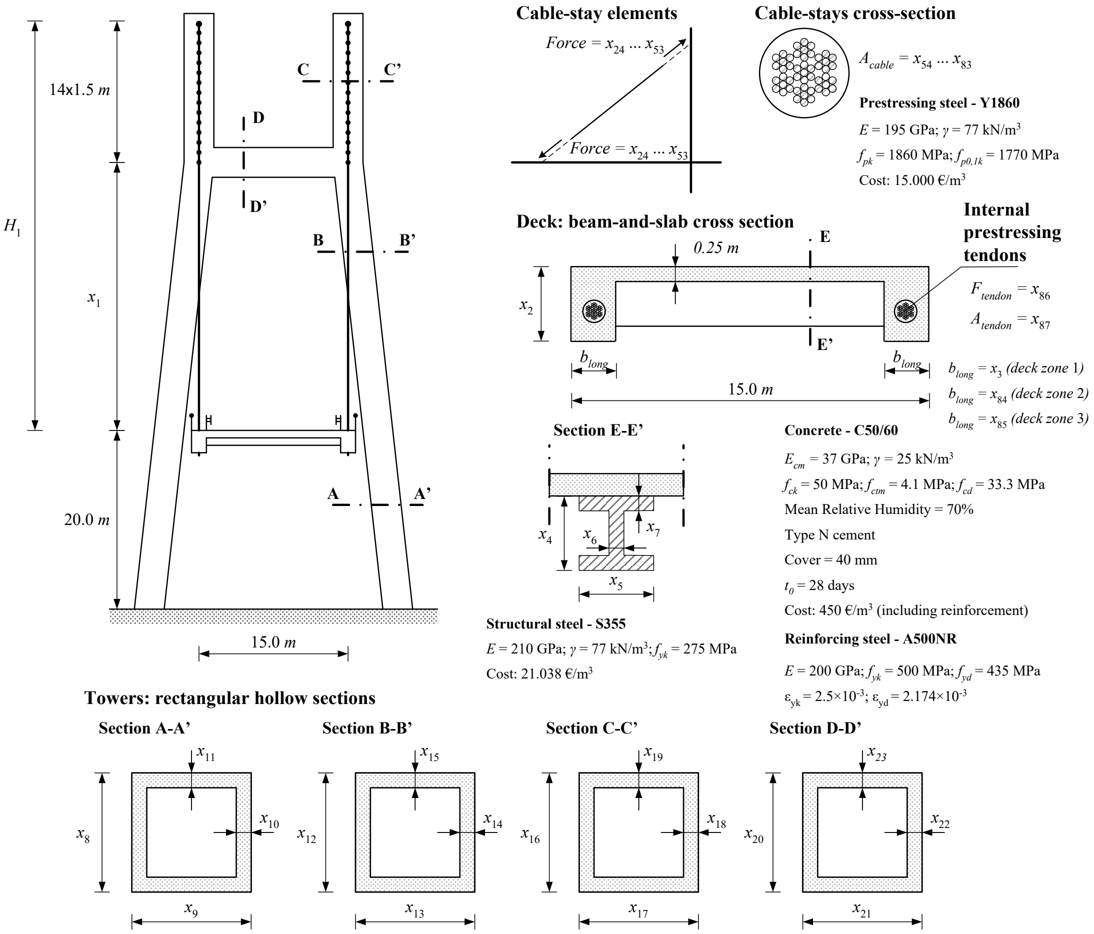

Figure 2: Design variables, material properties and unit costs.

The design provisions of Eurocode 2 [18] and Eurocode 3 [19] were considered to define the stress constraints concerning the deck and tower members. These constraints can be generally written as follows

$$g_3(\underline{x}) = \frac{\sigma}{\sigma_{allow}} - 1 \le 0 \quad (6)$$

where σ and σ_{allow} are the acting stress and the corresponding allowable stress, respectively. The allowable stress assumes different values for either service conditions or strength verification. For concrete members under service conditions, the allowable stresses of 4.1 MPa for tension and 22.5 MPa for compression were used. For strength verification, the allowable value corresponds to the concrete member's structural resistance (including its reinforcement). This is computed based on the acting internal forces, such as bending and axial force, biaxial bending and axial force, or shear force. For the deck cross-beams made of structural steel under service conditions, the acting stress is evaluated using the von Mises criterion, and the allowable stress is set at the steel yield strength. For strength verification, the acting stress represents the acting internal force (bending moment or shear force), and the allowable value is the corresponding resistance evaluated assuming a Class 2 cross-section. The set of constraints concerning the stresses in the cable-stays and internal prestressing tendons can be written as

$$g_4(\underline{x}) = \frac{\sigma}{k \cdot f_{pk}} - 1 \leq 0 \tag{7}$$

$$g_5(\underline{x}) = 1 - \frac{\sigma}{0.10 \cdot f_{pk}} \leq 0 \tag{8}$$

where σ and f_{pk} are the acting stress and the characteristic value of the prestressing steel tensile strength, respectively. The value of k in Equation (7) was considered equal to 0.50 for service conditions and 0.74 for strength verification of the cable-stays. For the prestressing tendons, k is set to 0.75 for service conditions and 0.88 for strength verification. To ensure the structural efficiency of the cable-stays, a lower limit for tension is imposed through the constraint in Equation (8).

The finite element method was used for the three-dimensional analysis under dead load and live load due to road traffic, including geometric nonlinearities and time-dependent effects. The deck and towers were modelled using 2-node 12-degrees-of-freedom Euler-Bernoulli beam elements. To consider second-order effects, the stiffness matrix of the beam elements was evaluated, including elastic and geometric contributions, and the cable-stays were modelled as 2-node bar elements with an equivalent modulus of elasticity as given by the Ernst formulation. Consequently, the structural analysis was conducted iteratively to perform a second-order elastic analysis. The deck internal bonded prestressing was modelled using 2-node tendon elements with a linear profile. These elements are defined as connected to the 2-node and 12-degrees-of-freedom Euler-Bernoulli beam elements and share the same nodal displacements.

Structural concrete was modelled as a linear viscoelastic material, and the time-dependent effects of ageing, creep and shrinkage of concrete, and relaxation of prestressing steel were evaluated in accordance with the Eurocode 2 [18] formulations. Equivalent nodal forces were employed to model the time-dependent effects. A previous work by Martins et al. [20] provides detailed information on the modelling of time-dependent effects. A linear elastic behaviour of the materials (concrete, reinforcing steel, structural steel, and prestressing steel) and homogeneous concrete cross-sections were assumed in the structural analysis. Materials nonlinearity and the reinforcing steel were considered only in formulating stress constraints for the different structural members. The sensitivity analysis provides the optimisation algorithm with the derivatives of the objective function and all constraints with respect to the design variables. From the different available approaches, the discrete direct method was adopted, combining analytical and semi-analytical derivatives. Analytical derivatives were used for sizing and mechanical design variables, whereas semi-analytical derivatives were used for the shape design variable.

3 Numerical Example and Results

The numerical example concerns the optimisation of a single-tower asymmetric cable-stayed bridge with a total length of 220 m, a main span of 128 m (L_2/L_1 of 0.72) and a deck width of 15 m (Figure 3). An "H-shaped" tower was considered, with the deck placed 20 m above the foundation, and the total height depending on the design variable x_1. A semi-harp cable arrangement with lateral suspension and 60 cables was considered. The cable spacing is 8 m on the deck and 1.5 m on the tower. A full-suspension solution was adopted with the deck simply supported at the abutments, continuously supported by the inclined cable-stays along the bridge length and without vertical support at the towers. The deck-tower connection was considered in both longitudinal and transverse directions and was modelled with link elements. The deck features a slab-girder cross-section. The slab is supported by longitudinal concrete beams and steel cross-beams (at 4 m spacing), except in the side span near the abutment, where concrete cross-beams were used. The use of steel beams reduces deck weight, which favours both static and seismic behaviour. A similar solution was adopted in the East Huntington and Vasco da Gama bridges. Beam elements were used to model the tower and the deck's longitudinal and transverse beams. The bridge finite element model is depicted in Figure 3 and comprises 164 nodes and 287 finite elements. The properties and unit costs of the materials considered are presented in Figure 2.

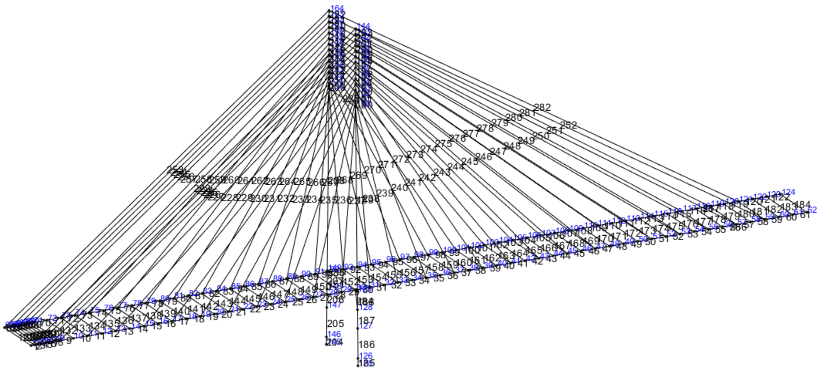

Figure 3: Finite element mesh of the bridge example.

To check the relevant service and strength design constraints, five load cases were considered. The first case concerns the bridge at the end of construction, subjected to dead load, including self-weight and an additional dead load of 2.5 kN/m^2 (flooring, walkways, safety barriers, and guardrails). The second case corresponds to the long-term analysis (18,250 days) of the bridge under the quasi-permanent load combination (dead load plus 20% of road traffic live load). To account for the most unfavourable effects of the road traffic live load (5 kN/m^2), three additional load cases were considered. These correspond to the live load applied over the entire deck length, or only over the main span or the side span. The problem features 2160 design constraints for the five load cases. This type of bridge is usually built using the balanced cantilever method or its combination with temporary supports on the side span. Although the construction stages may be relevant in the design of these bridges, this paper focuses on the static response of the complete bridge; thus, these were not directly considered. Given the concrete superstructure and span length, and assuming the bridge is built in a low-seismicity zone, wind- and earthquake-induced vibrations are not expected to govern the design. Nevertheless, the response to wind and seismic actions, as well as the construction stages, should be considered in a detailed design stage. Eight initial designs were considered and optimised to explore the design space. These were defined by varying the tower height and the cross-sectional sizes of the deck and tower. This aimed to achieve: $H_1 = 0.4\ L_1$ and $H_1 = 0.5\ L_1$; deck slenderness of 1/100 and 1/120; depth of the tower cross-sectional in Zone 1 equal to $H_1/10$ and $H_1/15$.

The evolution of the bridge cost throughout the optimisation process is shown in Figure 4. The optimised solutions are obtained after around 60 to 70 iterations. A desktop personal computer (3.20 GHz processor, 64.0 GB of RAM, and Windows 10 Pro operating system) was used to run the example. The average time of optimising each starting design is approximately 57.7 minutes, corresponding to 80 iterations. Similar optimised solutions are obtained with the different initial designs. The least-cost solution is obtained with the starting design 50_100_10 characterized by x_1 = 43 m ($H_1 = 0.50\ L_1$), ($h_{long} = x_2 = L_1/100$) and ($h_{tower\_Z1} = x_8 = H_1/10$) (red curve in Figure 4).

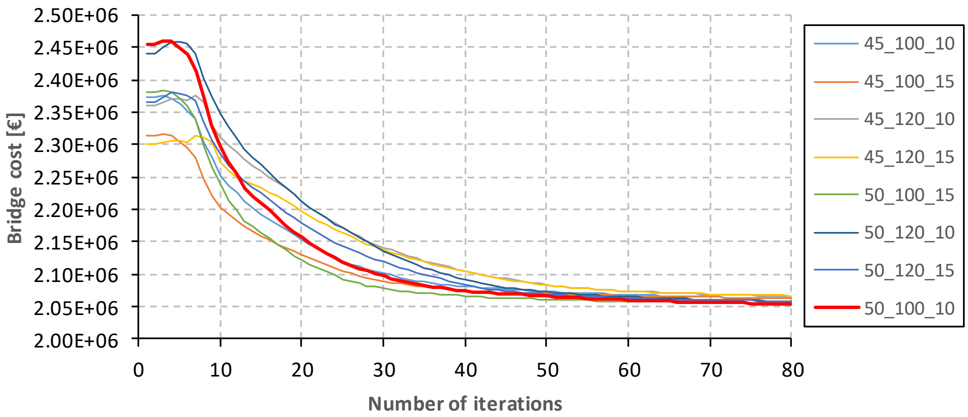

Figure 4: Bridge cost *vs.* number of iterations – multiple starting points.

The least-cost solution represents a 16.4% cost reduction relative to the corresponding initial solution. This is mainly due to a reduction in the cross-sectional sizes of the tower and of the width of the main girders (Table 1).

Figure 5 depicts the initial and final values of the cables' cross-sectional areas and prestressing forces for the least-cost solution. The cables are numbered from the abutment of the side span to the main span. The least-cost solution yields a maximum deck vertical displacement of 12.8 cm and a maximum tower horizontal displacement of 2.9 cm, considering time-dependent effects.

Design variable	Initial value	Final value	Design variable	Initial value	Final value
x_1 [m]	43.000	49.356	x_{14} [m]	0.427	0.250
x_2 [m]	1.280	1.594	x_{15} [m]	0.320	0.203
x_3 [m]	1.024	0.701	x_{16} [m]	3.200	2.833
x_4 [m]	0.768	0.835	x_{17} [m]	2.560	2.030
x_5 [m]	0.400	0.300	x_{18} [m]	0.320	0.201
x_6 [m]	0.012	0.012	x_{19} [m]	0.256	0.201
x_7 [m]	0.027	0.018	x_{20} [m]	4.267	2.623
x_8 [m]	6.400	3.529	x_{21} [m]	3.200	2.623
x_9 [m]	4.267	2.949	x_{22} [m]	0.427	0.202
x_{10} [m]	0.640	0.202	x_{23} [m]	0.320	0.202
x_{11} [m]	0.427	0.202	x_{23} [m]	0.427	0.250
x_{12} [m]	4.267	2.846	x_{84} [m]	1.024	0.707
x_{13} [m]	3.200	2.833	x_{85} [m]	1.024	0.826
Cost	Initial value	Final value	Cost	Initial value	Final value
Deck	808,997 €	614,912 €	Cables	1,247,399 €	1,255,112 €
Tower	377,581 €	170,775 €	Tendons	20,589 €	12,322 €
Total cost	2,454,566 €	2,053,122 €			

Table 1: Initial and final values of the cost and design variables – least-cost solution.

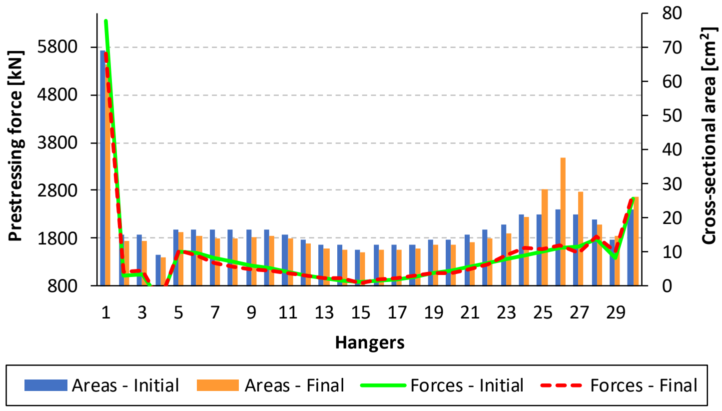

Figure 5: Initial and final values of the cable-stays' prestressing forces and cross-sectional areas – least-cost solution.

4 Conclusions and Contributions

The following conclusions can be drawn:

- An optimisation-based approach was proposed to assist in the conceptual and preliminary design of single-tower asymmetric cable-stayed concrete bridges. The design is posed as a cost minimisation problem subject to constraints on displacements and stresses, with service and strength criteria considered.

- The original non-convex problem is addressed employing an efficient convex optimisation strategy that combines a gradient-based algorithm with an automated procedure for defining multiple starting designs. This strategy provides minimum cost and structurally efficient solutions that balance the weight and stiffness distribution among load-bearing members (deck, pylon, and stay cables) and the suspension effect of the cable-staying system. The optimised solutions satisfy all stress constraints for both serviceability and strength, while also controlling deck vertical and tower horizontal displacements under permanent dead loads and long-term time-dependent effects.
- In the least-cost solution, the deck, tower, cables, and tendons represent 30.0%, 8.3%, 61.1% and 0.6% of the total cost, respectively. The design is governed by the deck's vertical displacements and normal stresses under service conditions, as well as the cables' stresses. The optimum solution features a deck slenderness of 1/80 and a tower height-to-main span ratio of 1/1.82.
- Future developments should investigate how parameters, such as L_2/L_1, H_1/L_1, and L_3/L_1, that describe the overall bridge geometry, affect the optimal designs. Different solutions for the deck design (prestressed concrete, steel, steel-concrete composite) and their combinations, and the seismic action should also be considered in this optimal design problem.
- The development of metaheuristic algorithms for addressing this design problem should also be investigated, with upcoming research also focusing on improving the exploration of the design space by combining gradient-based algorithms with global search procedures.

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