Proceedings of the Eighteenth International Conference on
Civil, Structural and Environmental Engineering Computing
Edited by: P. Iványi, J. Kruis and B.H.V. Topping
Civil-Comp Conferences, Volume 10, Paper 15.1
Civil-Comp Press, Edinburgh, United Kingdom, 2025
ISSN: 2753-3239, doi: 10.4203/ccc.10.15.1
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A Less Complex Wing Theory in Ideal Fluids

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Abstract

In this paper the author suggests a less complex wing theory in ideal fluids that connects lift force and drag force of a finite 3-dimensional wing with mass flow into the generated vortex cores.
Suggesting an induced drag in ideal fluids for 2-dimensional wing sections, leads to a modified version of the equation Prandtl has derived for the relation between lift force F_y and drag force F_x of a 3-dimensional finite wings. This new equation goes over into Prandtl's equation, if we have low aspect ratios of the wings.
The author derives equations that calculates the vortex core radius of the generated vortexes and are connecting the mass flow into the vortex cores with drag and lift of the wing. Additionally, the author shows, that the power to compensate for the drag of a plane in an ideal fluid is consumed by the rotational power of the generated vortex cores.

Keywords: ideal fluids, wing theory, vortex core, lift coefficient, drag coefficient, random walks.

1 Introduction

A model for unsteady potential flow in ideal fluids, called the Random Walk Source Model is presented in [1]. This model generates the shape of a particle front emerging from a line source, creeping towards and past a wing section. The lift coefficient can be predicted by the difference of creeping distance between the two front segments that have been separated by the wing section and an extended separation line which maintains the Kutta condition. The front is a result of a model of unsteady potential flow, and further leads to an unexpected developments of an equation leading to the formulation of an induced drag in 2- dimensions in ideal fluids. We have to account for this part of the drag also in 3- dimensions, which is made in the following description.

Wake vortices behind planes are a typical topic in fluid dynamics. Lanchester [2] and Prandtl [3] described the 3-dimesional flow behind wings with finite span in ideal fluids, resulting in an induced drag out of the ability to generate a circulation, in a relation between lift force F_y and expected drag force F_x:

$$F_x = F_y \cdot \frac{w_\infty}{2u_\infty} \tag{1}$$

Where u_∞ is the velocity of the plane and w_∞ is the induced downward velocity of the fluid flow.

As described in [4] the relation between C_l and C_d is stated as an induced drag in 3-dimensional flow in ideal fluids and now additionally an induced drag in 2-dimensional flow:

$$C_d = \left(\frac{C_l}{2\pi}\right)^2 \cdot \frac{\pi \cdot \ln\left(1+\sqrt{2}\right)}{\sqrt{8}} + \frac{C_l{}^2}{\pi\Lambda} \tag{2}$$

This is leading to the equation:

$$F_x = F_y \cdot \frac{w_\infty}{2u_\infty} \cdot \left[\frac{\Lambda \cdot \ln\,(1+\sqrt{2})}{4 \cdot \sqrt{8}} + 1\right] \tag{3}$$

Where Λ is the aspect ratio of the wing. This is an extension of formular (1). Formular (2) is gradually going over into formular (1) when Λ is close or less 1.

2 Level flight of a wing of finite span in ideal fluids

We calculate C_l out of the lift force F_y and the speed u_∞ of the plane together with the density ρ and the area A of the wing:

$$C_l = \frac{2 \cdot F_y}{\rho \cdot u_\infty \cdot A} \tag{4}$$

C_d the drag coefficient of the wing is then calculated according to equation (2).

Alternatively the author suggests to formulate the drag coefficient with help of the mass flow $\dot{m}_{core}$ into the vortex cores generated from the wing and the descent velocity w_∞.

$$C_d = \frac{2 \cdot \dot{m}_{core} \cdot w_\infty}{\rho \cdot u_\infty{}^2 \cdot A} \tag{5}$$

The mass flow $\dot{m}_{core}$ is defined as

$$\dot{m}_{core} = C_d \cdot \frac{\rho}{2} \cdot A \cdot \frac{{u_\infty}^2}{w_\infty} \tag{6}$$

and w_{oo} is defined as

$$w_\infty = \frac{\Gamma_0}{b_1} \tag{7}$$

Where b_1 is the wingspan, and Γ_0 the circulation

$$\Gamma_0 = 4 \cdot T_1 \cdot u_{HK} \tag{8}$$

Where T_1 is the average wing width, the wing area is $A_1 = b_1 \cdot T_1$ with

$$u_{HK} = \frac{C_l \cdot u_\infty}{2 \cdot \pi} \tag{9}$$

We can then write the drag force

$$F_x = \dot{m}_{core} \cdot w_\infty \tag{10}$$

And the lift force

$$F_y = \dot{m}_{core} \cdot \frac{C_l}{C_d} \cdot w_\infty \tag{11}$$

The vertical mass flow generating the lift force is according to Prandtl the flow through a circle area with the wing span as the diameter. Fluid particles are entering this area with u_∞, perpendicular to this area. This mass flow is moving down with w_∞, parallel to the area

$$F_y = \dot{m}_y \cdot w_\infty \tag{12}$$

Where $\dot{m}_y$ is defined (as explained above)

$$\dot{m}_y = \dot{V}_1 \cdot \rho \tag{13}$$

with

$$\dot{V}_1 = A_{01} \cdot u_\infty \tag{14}$$

Here A_{01} is a circle with the wing span as the diameter

$$A_{01} = \frac{\pi \cdot {b_1}^2}{4} \quad (15)$$

The relation between $\dot{m}_y$ and $\dot{m}_{core}$ is

$$\frac{\dot{m}_y}{\dot{m}_{core}} = \frac{C_l}{C_d} \quad (16)$$

3 Comparing calculated core radius with published experiments

In Figure 1 the calculated vortex core

$$r_{core} = \sqrt{\frac{A}{4} \cdot \frac{C_l}{2\pi}} \quad (17)$$

is compared to an experiment published in [5]. The calculated tangential velocity and the core radius is fitting good into the experimental data.

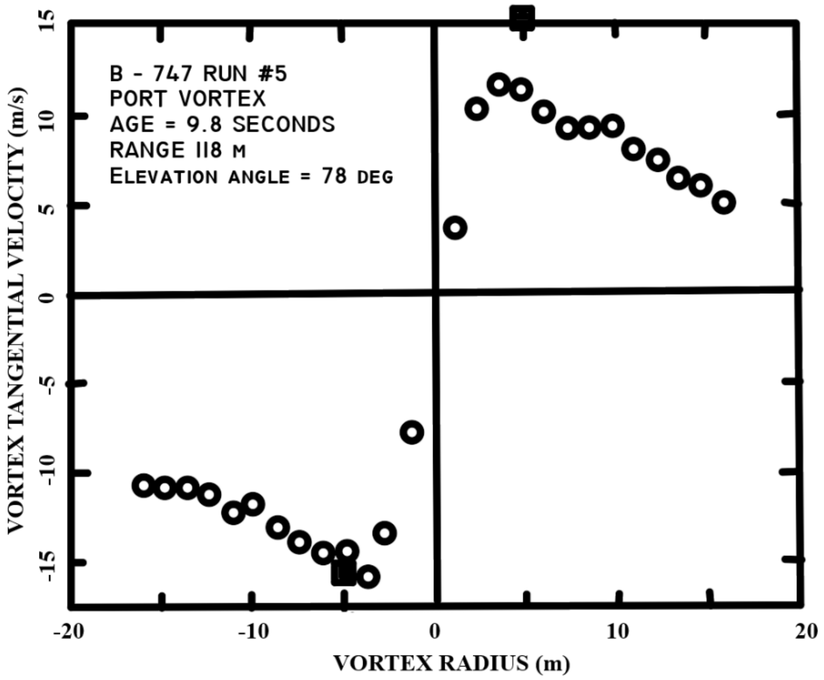

Figure 1: Vortex radius and vortex tangential velocity reproduction from [5] circles are experimental data, squares ■ are calculated according (17)

$$r_{core} = \sqrt{\frac{A}{4} \cdot \frac{C_l}{2\pi}} = 4.99\ m$$

and $u_{HK} = \frac{C_l \cdot u_\infty}{2 \cdot \pi} = 15.26\ m/s$ the vortex tangential velocity. $C_l = 1.23$ is the lift coefficient, u_∞ is the velocity of the plane here 78 m/sec and A is the wing area 510 m^2

4 Power calculation

Here we write down the power that is consumed to keep the wing flying on level consisting of the drag force multiplicated with the velocity of the wing

$$P^*_{tot} = F_x \cdot u_\infty \tag{18}$$

Alternatively we can see on the descent velocity v_y of the unpowered wing in relation to the horizontal velocity u_{oo} of the wing with the known relation between C_d and C_l. We use the relation

$$\frac{C_d}{C_l} = \frac{v_y}{u_\infty} \tag{19}$$

That leads to the descent velocity

$$v_y = \frac{C_d \cdot u_\infty}{C_l} \tag{20}$$

The lifting force multiplicated with the descent velocity is the power to keep the plane on level according to lift and drag.

$$P^*_{tot} = F_y \cdot v_y = \dot{m}_y \cdot w_\infty \cdot v_y \tag{21}$$

The kinetical energy of the vortex cores (seen as cylinders with radius r_{core}) is calculated

$$E_{kin} = \frac{1}{2} \cdot \Theta \cdot \omega^2 \tag{22}$$

Where Θ is the momentum of inertia of a cylinder (the vortex core) which is growing in length in time leading to $\dot{\Theta}$

The rotational power of the vortex cores (kinetical energy per time) is calculated

$$P^*_{core} = \dot{E}_{kin} = \frac{1}{2} \cdot \dot{\Theta} \cdot \omega^2 \tag{23}$$

Where ω is the angular velocity

$$\omega = \frac{u_{HK}}{r_{core}} \quad (24)$$

Inertial mass Θ of a cylinder (representing the vortex cores)

$$\Theta = \frac{1}{2} m_{core} \cdot {r_{core}}^2 \quad (25)$$

Ongoing flow into the vortex core (the vortex core is a cylinder with constant radius r_{core} increasing the mass with growing length)

$$\dot{\Theta} = \frac{1}{2} \dot{m}_{core} \cdot {r_{core}}^2 \quad (26)$$

Leading to the rotational power of the vortex cores

$$P^*{}_{core} = \frac{1}{4} \cdot \dot{m}_{core} \cdot {u_{HK}}^2 \quad (27)$$

Leading to the relation

$$\frac{P^*{}_{tot}}{P^*{}_{core}} = \frac{c_l}{c_d} \quad (28)$$

3 Results

Figure 2 is showing a survey over the suggested equations.

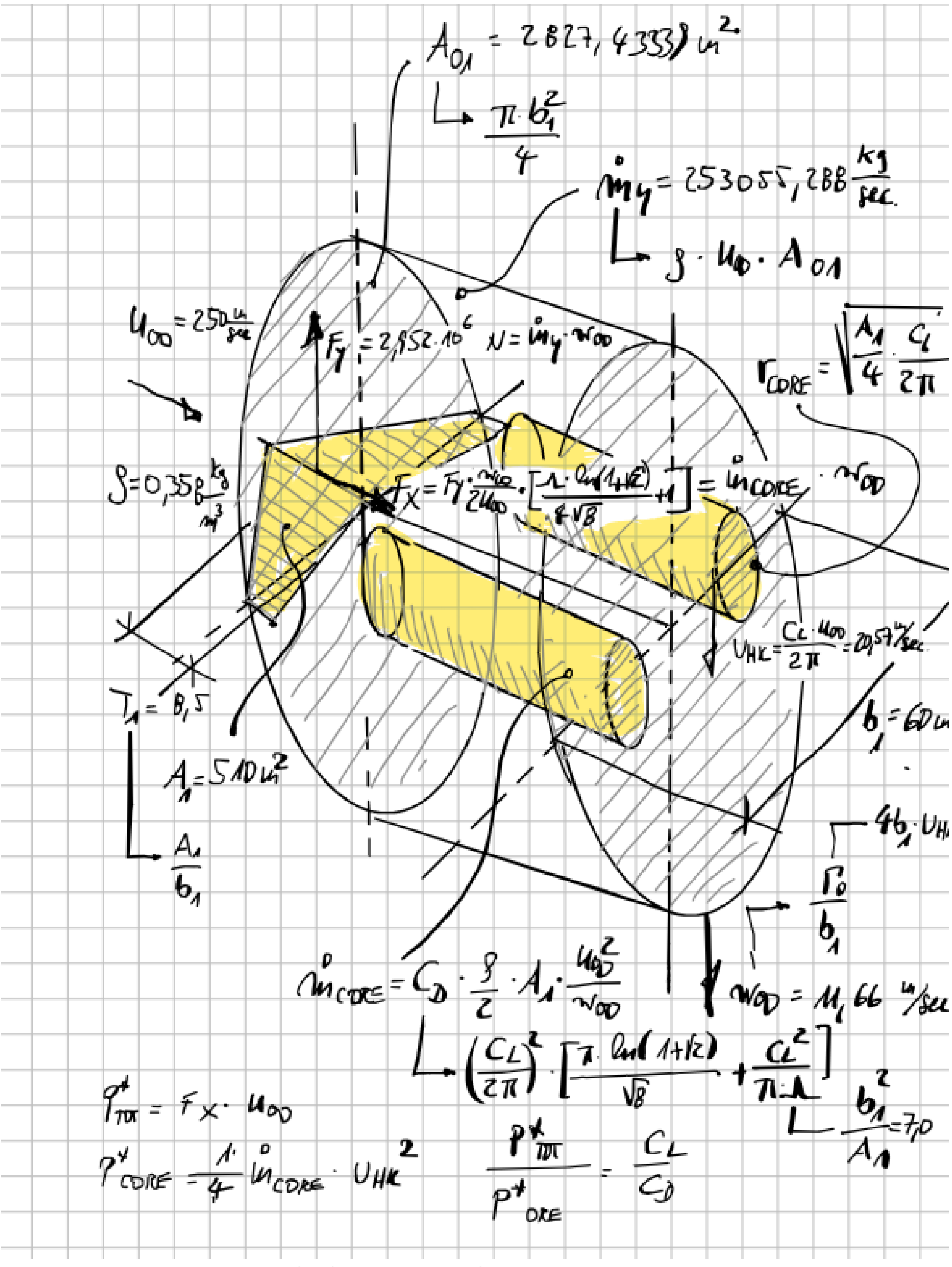

Figure 2: Visualization of lift and drag of a Boeing 747 according to the theory suggested in this paper. Plane parameter from [6] calculated values in Table 1

Description	Parameter	Value	
Lift force	F_y	2952000 N	
Air density	ϱ	0.358 kg/m^3	
Plane velocity	u_∞	250 m/s	
Wing span	b_1	60 m	
Wing area	A_1	510 m^2	
Circulation	Γ	699.92 $\frac{m^2}{s}$	
Aspect ratio	Λ	7.058	
	Calculated values		Used equation
Drag force	F_x	106746,16 N	(3)
Mass descent velocity	w_∞	11,6654 m/s	(7)
Area of influenced mass	A_{01}	2827,433 m^2	(15)
Downward accelerated volume	$\dot{V}_1$	706858.347 $\frac{m^3}{s}$	(14)
Lift coefficient	C_l	0.517384	(4)
Average wing section	T_1	8.5 m	
Trailing edge velocity	u_{HK}	20.586 m/s	(9)
Drag coefficient	C_d	0.01870893	(2)
Mass flux into the cores	$\dot{m}_{core}$	9150.6369 $\frac{m^3}{s}$	(6)
Mass flow vertical	$\dot{m}_y$	253055.2882 $\frac{m^3}{s}$	(13)
Core radius	r_{core}	3.240199 m	(17)
Total power used	${P^*}_{tot}$	26686539.8 Nm/s	(18)
Power consumed from the vortex cores	${P^*}_{core}$	969477.8 Nm/s	(27)
Power relation	$\frac{{P^*}_{tot}}{{P^*}_{core}} = \frac{C_l}{C_d}$	27.65	(28)

Table 1: Input parameter Boeing 747 [6] and from the author calculated values.

4 Conclusions

A wing section has no drag in ideal fluids according to the potential theory. We can calculate the circulation which is leading to the lift force. The start vortex is moved to vicinity and has lost the influence in the theoretical model. In reality we have a vortex pair in 2- dimensions which is continuously fed with attracted fluid particles. In the random walk source model [1],[4] it is shown, that you can identify an induced drag from wing sections in ideal fluids in 2- dimensions which is resulting in an equation that connects lift coefficient and drag coefficient. This is unexpected. In [4] it is shown that you can calculate the relation of this coefficients for a rotating cylinder with endplates. Also this is unexpected because we assume, that we have a highly turbulent flow behind the cylinder. In this paper the author transfers the knowledge of induced drag in 2- dimensions to wings in ideal fluids in 3- dimensions, se Figure 3. It is shown, that the core radius of vortex cores is depending on the lift coefficient and the wing area. Attracted fluid particles are feeding the vortex core generating the drag through the rotational power in the vortex cores. It is presented an equation (3) which is a modification of Prandl's relation (1) between lift force and drag force of wings in ideal fluids. In the presented equations the relation between lift coefficient and drag coefficient is the same relation as between the mass that is moved downward from the plane (to generate the lift) compared to the mass that is flowing into the vortex cores (16). The total power to keep the plane on level (18) divided by the power to keep the vortex cores rotating (27) is $\frac{C_l}{C_d}$ (28).

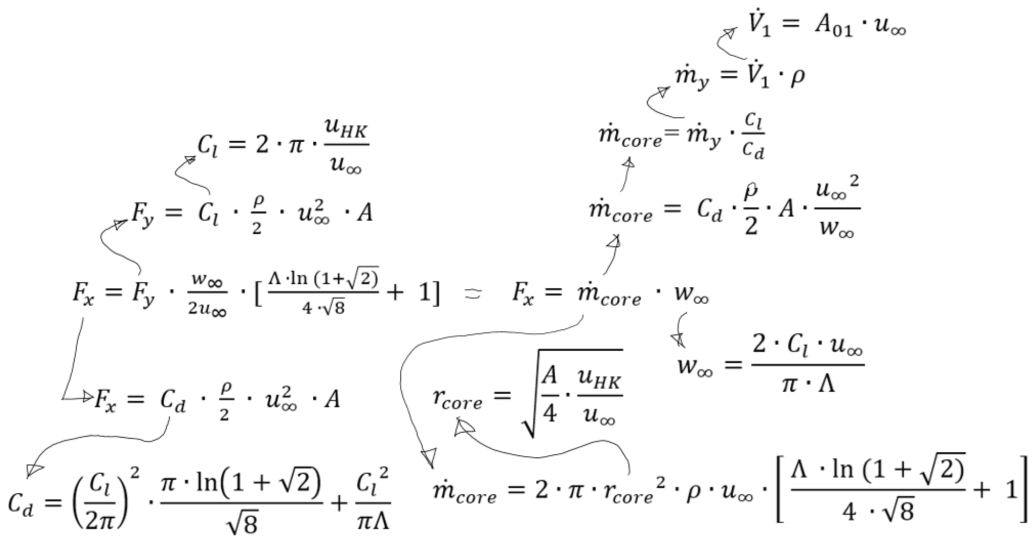

Figure 3: Equations connected with arrows, showing the calculation from generated lift force to drag force of a finite wing in ideal fluids. Calculation of lift and drag of a Boeing 747 according to the theory suggested in this paper taking into account the volume flux into the vortex cores and the vertical vortex flux generated by the plane. Plane parameter from [6] calculated values explained in Table 1.

Acknowledgements

This work is funded by the Western Norway University of Applied Sciences, Haugesund, Norway, by funding the research group "Random walk source model in fluid dynamics".

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