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Thermal Buckling Analysis of FG Porous Thin-Walled Beam

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Abstract

This study presents a numerical model for predicting the thermal buckling behaviour of thin-walled porous functionally graded beams. A geometric nonlinear algorithm, utilizing a 1D numerical model with a spatial beam finite element, is employed. Small strains are defined using the Green-Lagrange tensor. The finite element model is developed based on Euler-Bernoulli theory for bending and Vlasov theory for torsion. Nonlinear analysis is conducted using the updated Lagrangian incremental formulation and the principle of virtual work. The displacement field accounts for large rotations and torsion with warping. Material properties are assumed to vary continuously through the wall thickness, following a power-law distribution. The proposed beam model analyzes buckling under uniform, linear, and nonlinear temperature distributions across the thickness of the cross-sectional walls, while also considering the temperature-dependent mechanical material properties. Numerical results explore critical buckling temperatures and post-buckling behaviour for various thin-walled sections, with different configurations including boundary conditions, geometry, FG skin-core-skin ratios, and power-law indices. Numerical results investigate critical buckling temperatures and post-buckling behaviour for various thin-walled beam cross-sections, boundary conditions, geometry, FG skin-core-skin ratios, and power-law exponent. The algorithm is validated with commercial software 2D finite element results. An acceptable agreement is recognized comparing to those obtained by shell models.

Keywords: finite element analysis, thermal buckling, thin-walled beam, porous FG material, temperature distribution, large rotations.

1 Introduction

Functionally graded (FG) materials with porosity are a class of advanced composite materials that exhibit spatially varying properties across their structure. Usually, FG materials are composed of ceramic and metal, and their material properties varies continuously over the thickness of the cross-section. The inclusion of porosity, which can be controlled to varying degrees, adds another layer of complexity to these materials, allowing for tailored mechanical, thermal, and acoustic properties.

The thermal buckling and vibration of FG thin-walled beams and structures have gained significant attention in recent research, primarily due to the intricate behavior of these lightweight materials with enhanced thermomechanical properties. However, only a limited number of studies are referenced here [1–5]. On the other hand, research concerning the thermal buckling of FG porous beams in thermal environments remains relatively limited [6–11].

The primary goal of this paper is to introduce the developed beam model for analyzing thermal buckling in FG thin-walled beam structures and to explore the impact of the porous volume fraction on buckling behavior. The analysis relies on the numerical model created by the authors [12–15], which has been validated through benchmark shell examples.

2 Theoretical background

Since this paper is an extension of the previous ones [12-15], in which all the theoretical background is described including: the description of the basic beam model, beam kinematics, finite element formulation, nonlinear stability algorithm and the author's software *Thinwall FG*, this chapter will just shortly present the distribution of FG material mechanical and temperature properties, temperature distribution as well and introduce the constitutive equations themselves.

The material properties vary continuously through the wall thickness according to the power law distribution [7]:

$$P\,(n,T) = [P_o\,(T) - P_i\,(T)] \cdot V_c\,(n) + P_i\,(T) - 0{,}5\rho \cdot [P_o\,(T) + P_i\,(T)]. \tag{1}$$

Where P.representes the material property as Young's modulus E, shear modulus G, Poisson's ratios v or coefficient of thermal expansion α and conductivity K. The subscripts i and o indicates the inner and outer surface constituents respectively. In addition, V_c is the volume fraction of the ceramic phase. The small imperfection of the material is presented by scalar coefficient ρ<<1. Imperfect porous FG material is shown in Fig. 1.

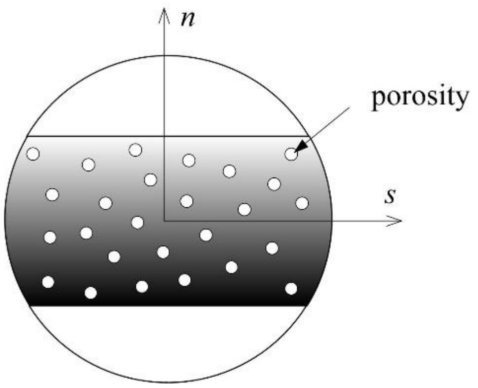


Figure 1: Imperfect porous FG material.

To more accurately predict the buckling behavior of functionally graded (FG) beams under thermal loads, the material properties are considered temperature-dependent. The nonlinear relationship between material properties and temperature T can be expressed as:

$$P = P_0\,(P_{-1}\,T^{-1} + 1 + P_1\,T + P_2\,T^2 + P_3\,T^3), \tag{2}$$

where temperature coefficients P_0, P_{-1}, P_1, P_2 and P_3 are specific to each material[16].

The stress and strain relation can be defined in generalized form of Hooke's law, given by:

$$\sigma_z = E\,(n,T) \cdot [\varepsilon_z - \alpha\,(n,T) \cdot \Delta T],\ \tau_{zs} = G\,(n,T) \cdot \gamma_{zs}. \tag{3}$$

The normal and shear stress components are represented by σ_z and τ_{zs}, corresponding strain components are denoted as ε_z, and γ_{zs}. The directions *n* and *s* correspond to the normal and transverse directions of the flange, respectively, whereas the *z*-direction runs along the beam's axis. The term ΔT signifies the change in temperature.

Temperature distribution through the thickness of the beam may be uniform, linear, or nonlinear. When there is no internal heat generation, the steady-state one-dimensional heat conduction equation takes the form[17]:

$$\mathrm{d}(K(n,T)\mathrm{d}T/\mathrm{d}n)/\mathrm{d}n = 0, \tag{4}$$

and the temperature variation through the wall thickness is given by:

$$T(n) = T_i(z) + C(T) \cdot [T_o(z) - T_i(z)] \,/\, D(n,T), \tag{5}$$

The constants *C* and *D* can be found in Ref [2]. By assuming equal thermal conductivity coefficients, $K_i = K_o$, equation (5) can be used to derive a linear temperature distribution. On the other hand, to obtain a uniform temperature distribution, it is necessary to assume that the temperature is constant throughout the beam, i.e., $T_o = T_i$.

3 Example

Fig. 2 shows a three-story spatial frame subjected to four vertical forces of the same intensity F. The frame is made of I-beams. The height of the frame columns is H = 10 m, and the length of the horizontal beams is L = 10 m. In addition at the supports, the warping is prevented as well as at all joints from E to P. The frame and the flanges of the profile are made from FG materials, SUS304 and Al_2O_3, in the FG sendwich form with a pure metal core in the middle and FG skins that vary from pure metal to pure ceramic on the outer surface. Three cases of FG material distribution were considered for the exponent p = 0, p = 0.3, and p = 3.

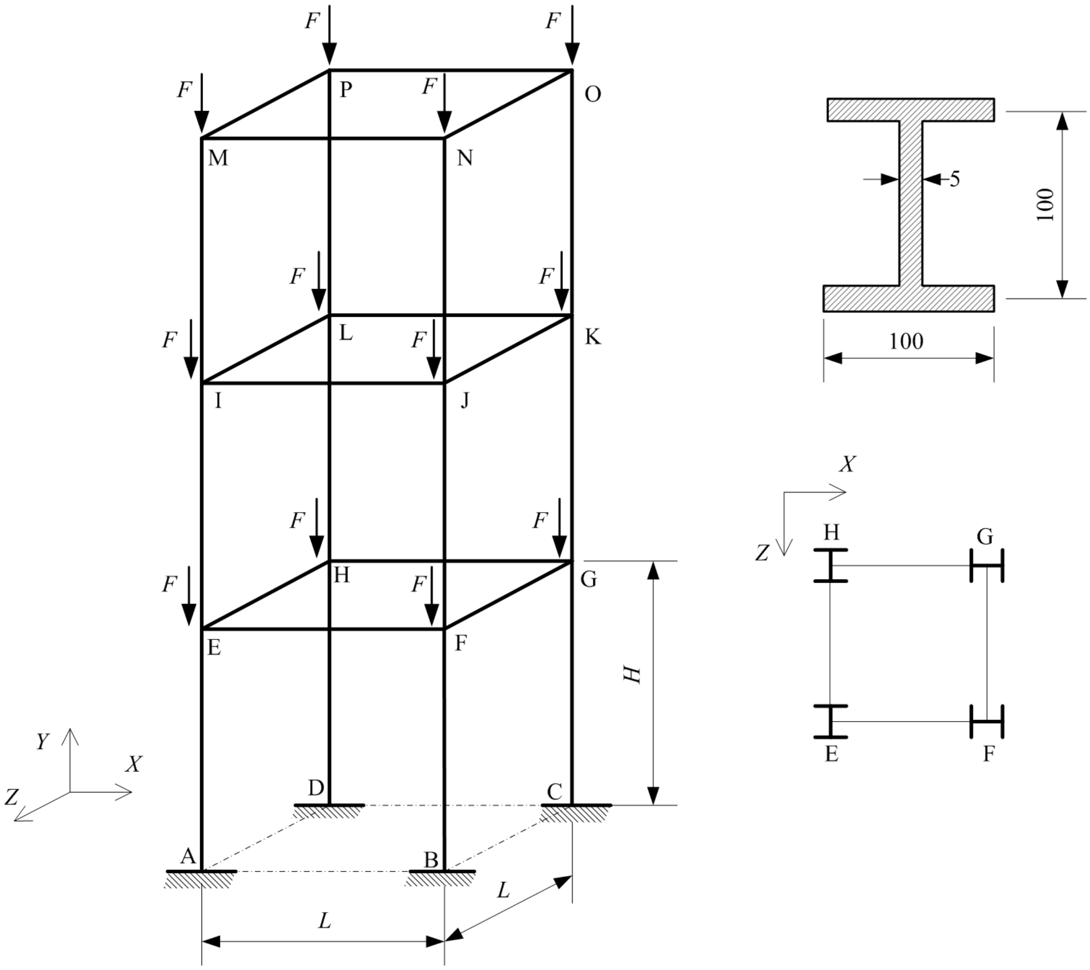
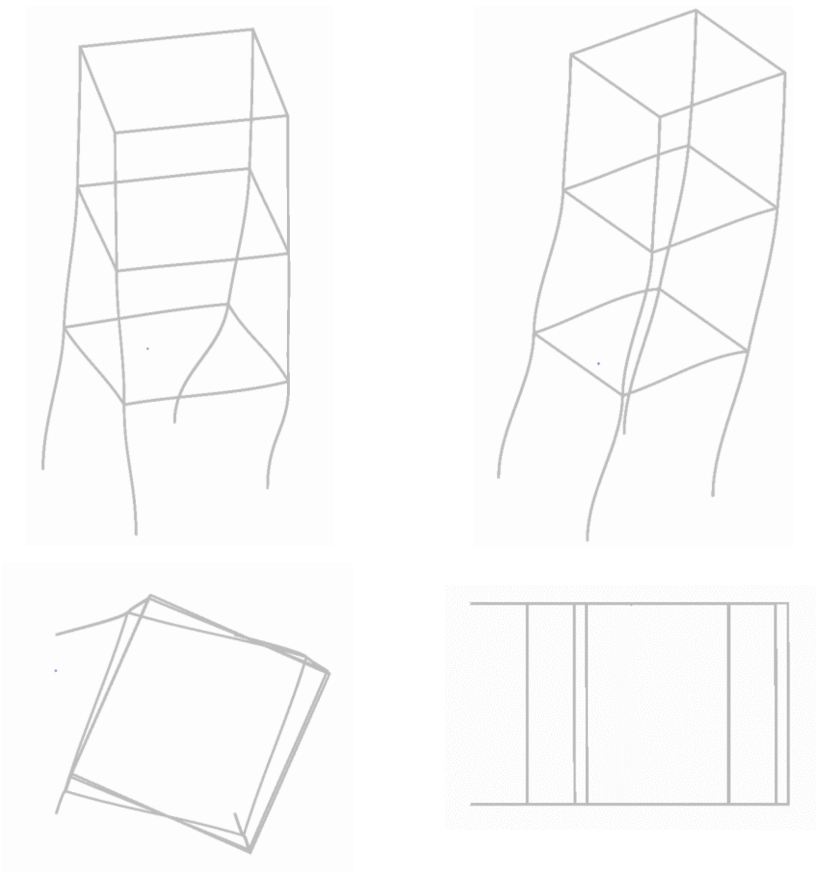


Figure 2: The spatial frame subjected to vertical forces of intensity F.

The loss of the stable deformation form of the frame when the vertical forces F reach the critical value is manifested in the twist-sway mode, where the upper floors move along the X-axis with rotation around the Y-axis, and the sway mode, where the upper floors move along the X-axis without rotation around the Y-axis. To initiate the twist-sway mode, the horizontal forces in the X-axis direction were applied at points E, I, and M, as well as , the horizontal forces in the negative Z-axis direction at points F, J, and N with intensity ΔF = 0.001F were applied. To initiate the sway mode, horizontal forces in the X-axis direction with intensity ΔF = 0.001F were applied at E, H, I, L, M, and P. A comparison of beam model critical buckling results with the shell model is given in Table 1, and the buckling modes are shown in Fig. 3.

Exponent p	Buckling mode	Thinwall FG	Shell model
$p = 0$	*Twist-sway*	6075,67	6148,38
	Sway	8727,68	8838,96
$p = 0,3$	*Twist-sway*	6557,62	6666,67
	Sway	9419,38	9579,07
$p = 3$	*Twist-sway*	7555,59	7704,73
	Sway	10852,40	11073,03

Table 1: Comparison of eigenvalue results.



a) b)

Figure 3: Buckling modes a) twist-sway; b) sway

Furthermore, a nonlinear stability analysis of the frame was performed for both buckling modes. Fig. 4 shows the nonlinear response for the twist-sway buckling mode, where the displacement of point E along the X-axis was observed, and Fig. 5 shows the sway mode, where the displacement of point H along the X-axis was observed. Both analyses were carried out for all three values of exponent p.

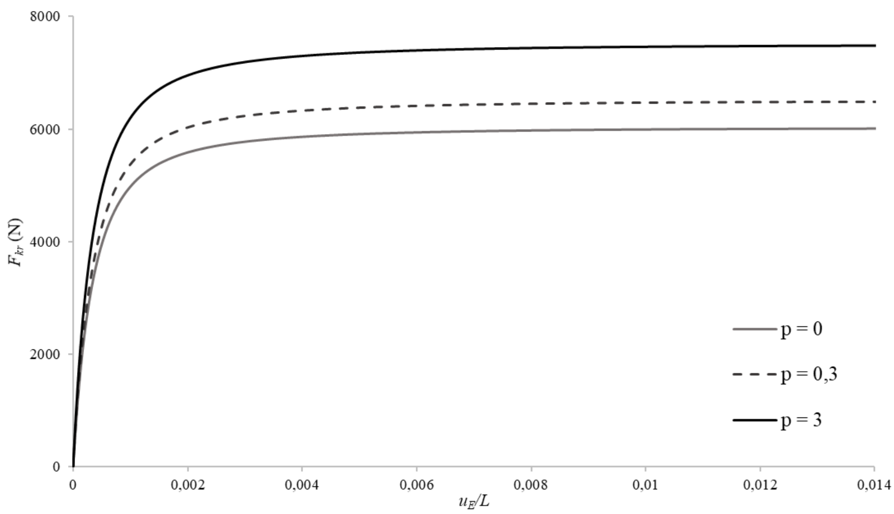


Figure 4: Nonlinear response for the twist-sway buckling mode

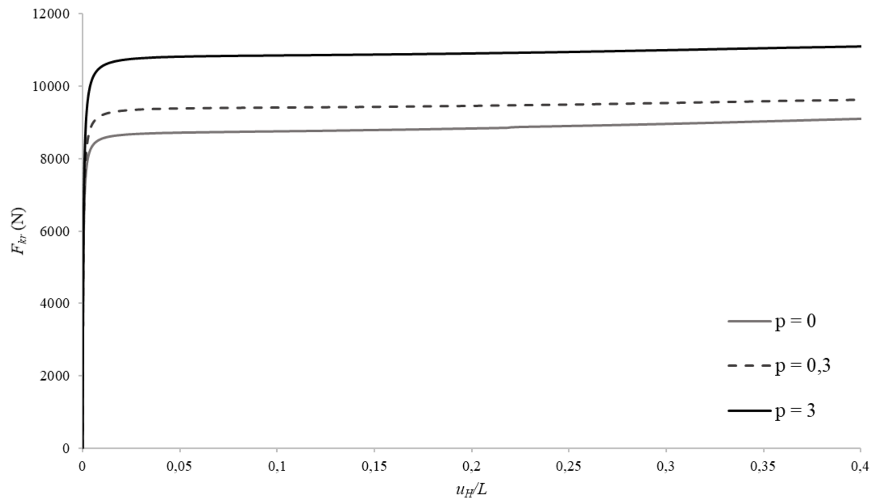


Figure 5: Nonlinear response for the sway buckling mode

The nonlinear response of the frame was observed when the columns are uniformly heated. It is assumed that the material properties change with temperature. Fig. 6 shows the displacement of point H along the X-axis as a function of temperature for three levels of critical buckling force and the exponent p=0 (isotropic metal). The higher critical buckling temperature coresponds to lower the force.

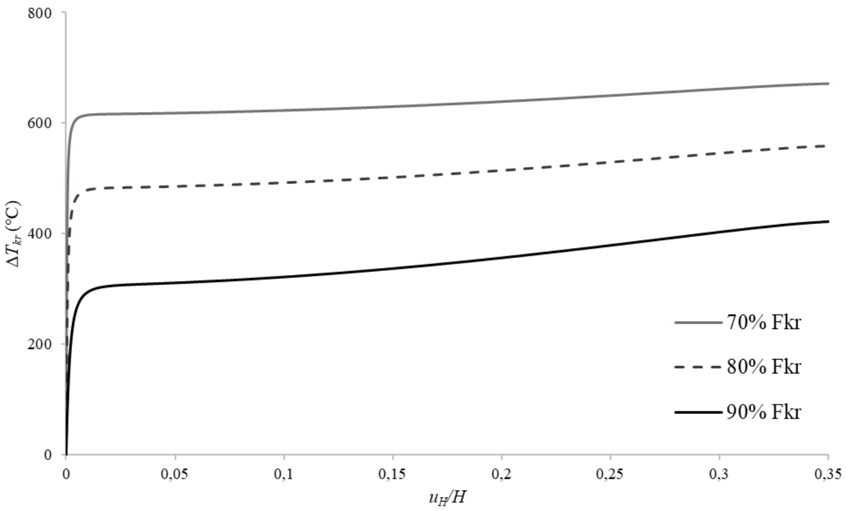


Figure 6: Temperature vs displacement for three levels of critical force F for p=0

The frame's was then analyzed for the exponent p=0.3 at a force of 80% Fkr and for the exponent p=3 at a force of 70% Fkr. A comparison was also made with a porous material with a porosity coefficient ρ=0.1. As the exponent p increases, the ceramic content in the composite material increases, resulting in higher critical buckling temperatures for the exponent p=3. As expected, porous materials conduct heat less efficiently, and therefore the critical temperature is higher than for perfect materials. A comparison of the nonlinear responses of perfect and porous materials is shown in Fig. 7, and Fig. 8.

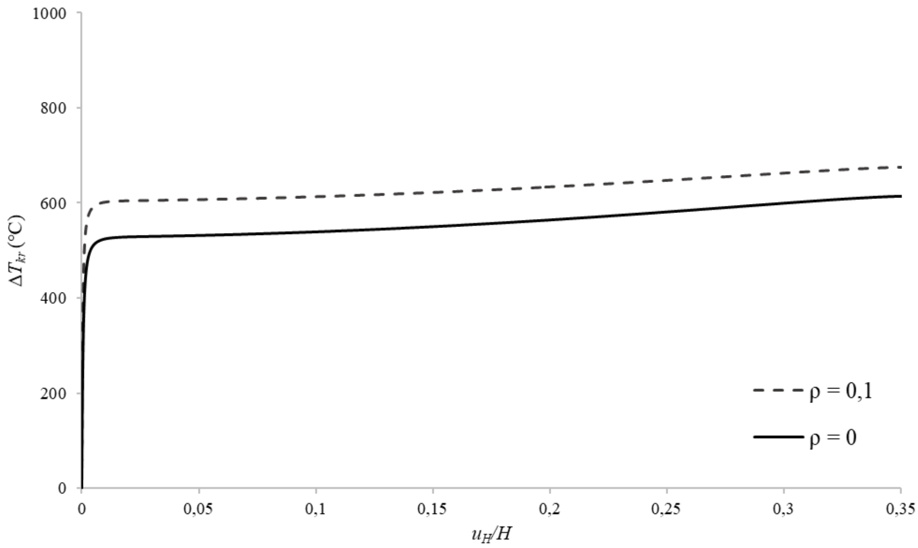


Figure 7: Comparison of perfect and porous materials for p=0.3 at 80% Fkr

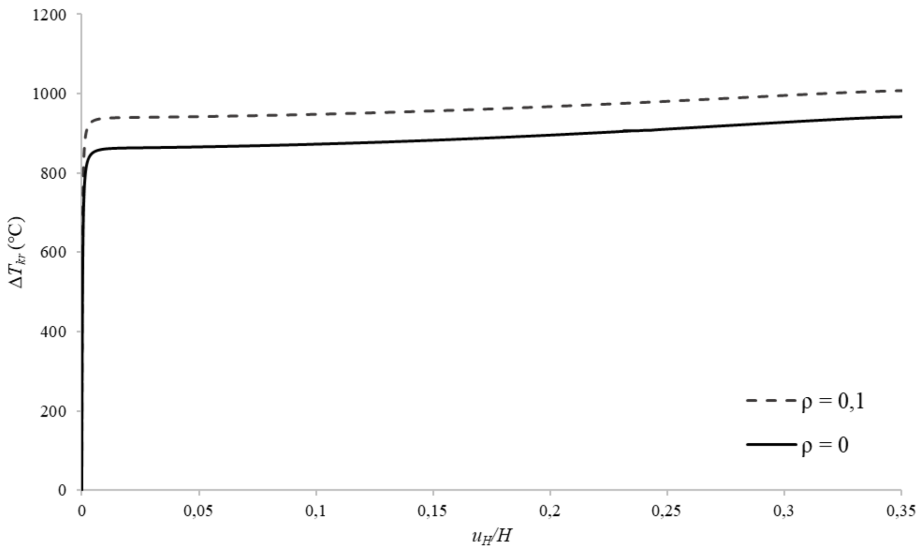


Figure 8: Comparison of perfect and porous materials for p=3 at 70% Fkr

4 Conclusion

A finite element beam model has been developed to investigate the thermal buckling behavior of thin-walled FG beam type structure with porosities. The model utilizes finite element incremental equilibrium equations derived from the updated Lagrangian (UL) formulation, including a nonlinear displacement field of cross-section that accounts for large spatial rotations. The analysis examines the influence of the power-law index, coefficient of porosity, and skin-core-skin thickness ratios on the critical buckling temperatures. The post-buckling bahaviour is also considered. The model's accuracy has been validated comparing to comercial software shell finite element results.

The critical buckling temperature decreases with an increase in the power-law index p due to a higher metal content in the cross-section. . In contrast, the critical buckling temperature increases with the porosity coefficient ρ as an imperfect beam, which is less thermally conductive, offers greater thermal stability compared to a perfect beam. This trend is observed for all boundary conditions examinated. As anticipated, beams with temperature-dependent material properties show a reduced resistance to thermal buckling.

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References

[1] Y. Kiani, M.R. Eslami, "Thermomechanical Buckling of temperature-dependent FGM Beams", Lat. Am. J. Solids Struct. 10, 223–246, 2013.

[2] L.C. Trinh, T.P. Vo, H.T. Thai, T.K. Nguyen, "An analytical method for the vibration and buckling of functionally graded beams under mechanical and thermal loads", Compos. Part B Eng. 100, 152–163, 2016, https://doi.org/10.1016/j.compositesb.2016.06.067.

[3] N. Wattanasakulpong, B. Gangadhara Prusty, D.W. Kelly, "Thermal buckling and elastic vibration of third-order shear deformable functionally graded beams", Int. J. Mech. Sci. 53, 734–743, 2011, https://doi.org/10.1016/j.ijmecsci.2011.06.005.

[4] A. Paul, D. Das, "Non-linear thermal post-buckling analysis of FGM Timoshenko beam under non-uniform temperature rise across thickness", Eng. Sci. Technol. an Int. J. 19, 1608–1625, 2016, https://doi.org/10.1016/j.jestch.2016.05.014.

[5] A.A. Daikh, M. Guerroudj, M. El Adjrami, A. Megueni, "Thermal Buckling of Functionally Graded Sandwich Beams", Adv. Mater. Res. 1156, 43–59, 2019, https://doi.org/10.4028/www.scientific.net/amr.1156.43.

[6] K. Gao, Q. Huang, S. Kitipornchai, J. Yang, "Nonlinear dynamic buckling of functionally graded porous beams", Mech. Adv. Mater. Struct. 28, 418–429, 2021, https://doi.org/10.1080/15376494.2019.1567888.

[7] N. Wattanasakulpong, V. Ungbhakorn, "Linear and nonlinear vibration analysis of elastically restrained ends FGM beams with porosities", Aerosp. Sci. Technol. 32, 111–120, 2014, https://doi.org/10.1016/j.ast.2013.12.002.

[8] F. Ebrahimi, E. Salari, "Thermal buckling and free vibration analysis of size dependent Timoshenko FG nanobeams in thermal environments", Compos. Struct., 2015, https://doi.org/10.1016/j.compstruct.2015.03.023.

[9] H. Babaei, M.R. Eslami, A.R. Khorshidvand, "Thermal buckling and postbuckling responses of geometrically imperfect FG porous beams based on physical neutral plane", J. Therm. Stress. 43, 109–131, 2020, https://doi.org/10.1080/01495739.2019.1660600.

[10] S. Mesmoudi, M. Rammane, Y. Hilali, O. Askour, O. Bourihane, "Efficient buckling and post-buckling analysis of porous FG sandwich beams by new RPIM-HOCM mesh-free approach", Eng. Struct. 293, 2023.

[11] N. Ziane, S.A. Meftah, G. Ruta, A. Tounsi, "Thermal effects on the instabilities of porous FGM box beams", Eng. Struct. 134, 150–158, 2017, https://doi.org/10.1016/j.engstruct.2016.12.039.

[12] G. Turkalj, J. Brnic, J. Prpic-Orsic, "Large rotation analysis of elastic thin-walled beam-type structures using ESA approach", Comput. Struct. 81, 1851–1864, 2003, https://doi.org/10.1016/S0045-7949(03)00206-2.

[13] D. Lanc, G. Turkalj, I. Pesic, "Global buckling analysis model for thin-walled composite laminated beam type structures", Compos. Struct. 111, 371–380, 2014, https://doi.org/10.1016/j.compstruct.2014.01.020.

[14] S. Kvaternik Simonetti, G. Turkalj, D. Lanc, "Thin-Walled Structures Thermal buckling analysis of thin-walled closed section FG beam-type

structures", Thin-Walled Struct. 181, 110075, 2022, https://doi.org/10.1016/j.tws.2022.110075.

[15] S. Kvaternik Simonetti, G. Turkalj, D. Lanc, D. Banic, "A Numerical Model for Thermal Buckling Analysis of Functionally Graded Porous Thin-Walled Structures", Proceedings of Fifteenth International Conference on Computational Structures Technology, Edinburgh: Civil-Comp Press, 2024. pp. 1-8, doi: 10.4203/ccc.9.8.1.

[16] J.N. Reddy, C.D. Chin, "Thermomechanical Analysis of Functionally Graded Cylinders and Plates", J. Therm. Stress. 21, 593–626, 1998, https://doi.org/10.1080/01495739808956165.

[17] P. Malekzadeh, S.M. Monajjemzadeh, "Dynamic response of functionally graded beams in a thermal environment under a moving load, Mech". Adv. Mater. Struct. 23, 248–258, 2016, https://doi.org/10.1080/15376494.2014.949930.