Proceedings of the Eighteenth International Conference on
Civil, Structural and Environmental Engineering Computing
Edited by: P. Iványi, J. Kruis and B.H.V. Topping
Civil-Comp Conferences, Volume 10, Paper 11.1
Civil-Comp Press, Edinburgh, United Kingdom, 2025
ISSN: 2753-3239, doi: 10.4203/ccc.10.11.1

LACT3: A Fast Tool for Tilting Table Tests Based on Rigid Block Limit Analysis

Y. Hua, M. Buzzetti, N. Pingaro, M. Pourfouladi and G. Milani

Department of Architecture, Built Environment and Construction Engineering, Politecnico di Milano, Italy

Abstract

This study presents a homemade numerical tool, named LACT3, for the limit analysis of masonry structures subjected to in-plane tilting table tests. The method, implemented in MATLAB with a user-friendly interface, allows users to import geometries from CAD files (.dxf format) and assign cohesion, friction angle, and density as mechanical parameters. Based on the principles of rigid block limit analysis, the approach uses a kinematic formulation to determine the collapse multiplier and failure mechanism, while the corresponding static problem provides the internal force distribution. A key feature is the iterative procedure developed to efficiently identify the collapse tilt angle. The tool is tested on three masonry wall configurations: regular, semi-regular, and irregular. Each case is analysed by varying the cohesion parameter to assess its influence on the collapse tilt angle and failure mechanism. Results confirm that higher cohesion increases collapse resistance and that the presence of gaps between units significantly reduces the tilt capacity. Comparisons among the case studies highlight the strong influence of both geometry and contact quality on structural performance. The proposed methodology provides a rapid and insightful evaluation of collapse mechanisms in masonry panels, making it suitable for preliminary assessments under seismic-like loading conditions.

Keywords: limit analysis, LACT3, tilting table tests, kinematic approach, masonry panels, stereotomy.

1 Introduction

Masonry structures, such as arches and walls, have been widely used in construction for centuries [1],[2]. Ensuring their safety and stability, especially under seismic or other dynamic loads, is a crucial challenge in structural engineering [3]. The behaviour of masonry structures is complex due to the heterogeneous nature of the material, the presence of mortar joints, and the interaction between single units [4]. As a result, accurately assessing the collapse mechanisms and internal force distribution of such structures is critical for effective design and evaluation.

Over the past few decades, various approaches have been proposed to model the behaviour of masonry subjected to horizontal loads. Limit analysis, which focuses on determining the loads a structure can withstand before failure, has been widely used for this purpose [3], [5]-[10]. Such an approach, where a rigid plastic behaviour is allowed only at the interfaces between masonry units (assumed infinitely resistant), has proven to be an effective method for evaluating the collapse mechanisms. However, many existing methods suffer from limitations, such as the need for complex computational tools or the inability to accurately handle irregular geometries and material properties.

Recent advancements in numerical methods, including both kinematic and static formulations of limit analysis, have led to more refined models specifically conceived to reproduce better masonry behaviour beyond elasticity [3]. Despite such developments, challenges remain in making the aforementioned methods accessible and efficient for practical use. In particular, many existing tools require substantial computational resources or lack the flexibility to accommodate a wide range of geometric and material variations typically encountered in historical constructions.

Considering the previous issues, this paper presents a simplified, yet effective, limit analysis-based approach implemented in MATLAB to analyse assemblages of blocks subjected to tilting table tests. The method offers a user-friendly tool, by means of a graphical user interface (GUI), for rapidly assessing failure tilting angle, collapse mechanisms and related internal forces, even in presence of complex geometries and patterns. The proposed approach is designed to be accessible to both researchers and practitioners, providing a reliable tool for preliminary evaluations of the load carrying capacity under seismic-like loading conditions. The following sections describe the methodology in detail and present results applying the approach to various brickwork wall configurations.

2 Methods

A homemade limit analysis-based approach, named LACT[3], is developed for analysing masonry structures, such as arches and walls, subjected to tilting table tests. This simple analysis enables a rapid assessment of the effects of horizontal loads on masonry structures (e.g., seismic actions). The proposed approach is implemented in MATLAB and features a user-friendly graphical interface, allowing the operator to import the case study geometry from a .dxf file and define material properties (friction angle, cohesion, and density). The limit analysis yields the tilting table collapse angle, the failure mechanism, and the distribution of internal forces.

The proposed approach is based on the principles of rigid block limit analysis, where plasticity is lumped solely at the interfaces between elements, governed by a Mohr-Coulomb failure criterion. Since plastic deformations are lumped at a finite number of interfaces, the lower and upper-bound solutions coincide. By solving the limit analysis problem using the kinematic approach, following the upper-bound theorem, both the load multiplier and the failure mechanism can be determined. The corresponding dual static problem, formulated according to the lower-bound theorem, provides the load multiplier and the distribution of internal forces.

An open-source MATLAB code is employed to extract the geometry of the case study from a .dxf file provided by the user [11], in which wall units must be modelled using polylines. An interface detection algorithm is used to identify them between units. Despite this, it is not compatible with non-conforming nodes. Consequently, if an edge of a block is shared by multiple adjacent units, it must be split by inserting additional nodes, as illustrated in Figure 1. Furthermore, a local reference frame is assigned to each interface, defined by the tangential unit vector $\boldsymbol{t}_i$, oriented in the positive direction toward node N_2, and the normal unit vector $\boldsymbol{n}_i$, obtained by rotating $\boldsymbol{t}_i$ 90° counterclockwise (see Figure 1).

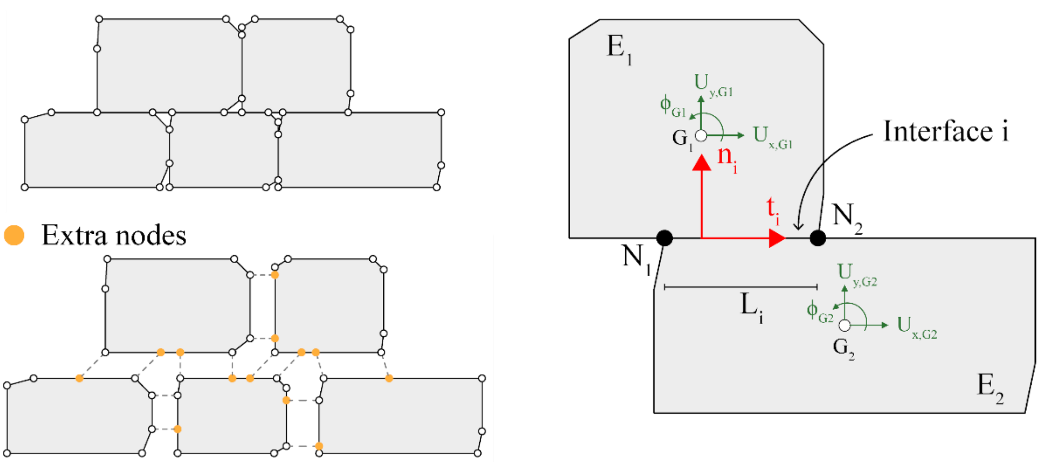

Figure 1: Masonry units modelling and interface local reference frame.

The formulation of the limit analysis problem, based on the upper-bound theorem, is presented in Eq. (1).

$$
\begin{aligned}
&\min(-\boldsymbol{f}_D^T\boldsymbol{u} + \boldsymbol{c}_0^T\boldsymbol{\lambda}) \\
&s.t.\begin{cases} \boldsymbol{f}_L^T\boldsymbol{u} = 1 \\ \boldsymbol{A}^T\boldsymbol{u} = \boldsymbol{N}^T\boldsymbol{\lambda}, \quad \boldsymbol{\lambda} \geq 0 \end{cases}
\end{aligned}
\tag{1}
$$

The variables describing the rigid body motion of the elements are assembled into the vector $\boldsymbol{u}$. For each block, the three unknowns are the horizontal and vertical velocities of the centroid, $U_{x,GE}$ and $U_{y,GE}$, and the rotation rate around the centroid,

ϕ_{GE}. According to the kinematic approach, the collapse load is determined by minimizing the objective function, which represents the balance of power between external and internal forces. Internal power, resulting from plastic dissipation at the interfaces, is computed using the cohesion vector $\boldsymbol{c}_0$ and plastic multiplier $\boldsymbol{\lambda}$. External power is associated with dead and live loads, represented by the vectors $\boldsymbol{f}_D$ and $\boldsymbol{f}_L$, respectively. To simulate the effect of a tilt load, the gravity force acting on each element is resolved into components parallel and perpendicular to the tilt direction (see Eq. (2)). This decomposition allows the load to be interpreted as a combination of diminished vertical weight and an induced horizontal force.

$$\boldsymbol{f}_D^i = \boldsymbol{f}_L^i = [W_i sin\theta \quad -W_i cos\theta \quad 0]^T \tag{2}$$

The first constraint in Eq. (1) represents the normalization condition, which is necessary to identify one failure mechanism among the infinite set of homothetic deformed shapes. The second constraint enforces both system compatibility and the interfacial flow rule. System compatibility is ensured by constraining the jump of velocity at the interfaces. For each overlapping node defining an interface, two discontinuous variables are introduced, namely the normal displacement ($\Delta u_{n,k}$) and the tangential displacement ($\Delta u_{t,k}$), with $k = 1,2$. These jumps of velocity are linked to the plastic multipliers $\boldsymbol{\lambda}$ through the associated flow rule. For a detailed description of the Mohr-Coulomb frictional slide criterion with an associated plastic flow rule, the reader is referred to [12].

The problem defined by Eq. (1) is solved using linear programming for a given tilt angle θ. The collapse angle is determined through an iterative procedure. Specifically, the tilt angle will be incrementally increased until the solved collapse multiplier is close enough to zero. The implementation of the iterative kernel is a key innovation of the proposed approach, enhancing both computational speed and reliability. For more details on the iterative algorithm, the reader is referred to [13].

Internal actions are obtained from the dual static problem formulation (Eq. (3)), which follows the lower bound theorem. In the static approach, the external load multiplier is maximized while satisfying the constraints imposed by element equilibrium and the interfacial constitutive relationships. For further details on the limit analysis problem formulation from both static and kinematic perspectives, the reader is referred to [14],[15].

$$\max(\alpha)$$
$$s.t.\begin{cases} \boldsymbol{A}\boldsymbol{x} = \alpha \boldsymbol{f}_L + \boldsymbol{f}_D \\ \boldsymbol{N}\boldsymbol{x} - \boldsymbol{z}_0 = \boldsymbol{c}_0, \ \ \boldsymbol{z}_0 \leq 0 \end{cases} \tag{3}$$

3 Results

The methodology proposed in Section 2 is applied to three case studies, each consisting of a wall with a regular, semi-regular, or irregular masonry pattern. The collapse tilt angle is computed assuming a friction angle of 30° and a density of 2500 kg/m^3 for all the walls. Additionally, a sensitivity analysis is carried out by varying the cohesion value, considering three scenarios: 0.01, 0.005, and 0.001 MPa.

The first case study (Wall A) features a wall with a regular arrangement of masonry units, including a door with a lintel. The wall dimensions are 3.5 m in height and 4 m in width. The results in terms of collapse tilt angle, failure mechanism, and distribution of internal forces are shown in Figure 2 and Table 1 for cohesion values of 0.01, 0.005, and 0.001 MPa. As expected, higher cohesion values result in a higher collapse tilt angle, with the collapse mechanism being less influenced by stair-stepped cracking at the element interfaces.

The second case study (Wall B) has the same dimensions as Wall A but features a semi-regular arrangement of stones. The masonry pattern is not characterized by fully contiguous interfaces, with some gaps between the masonry units, as shown in Figure 1. The results, presented in terms of collapse tilt angle, failure mechanism, and distribution of internal forces, are shown in Figure 3 and Table 1 for cohesion values of 0.01, 0.005, and 0.001 MPa. For Wall B, lower collapse tilt angles are obtained with lower cohesion values, similar to Wall A. Furthermore, the collapse tilt angles for Wall B are consistently lower than those for Wall A due to the irregular pattern.

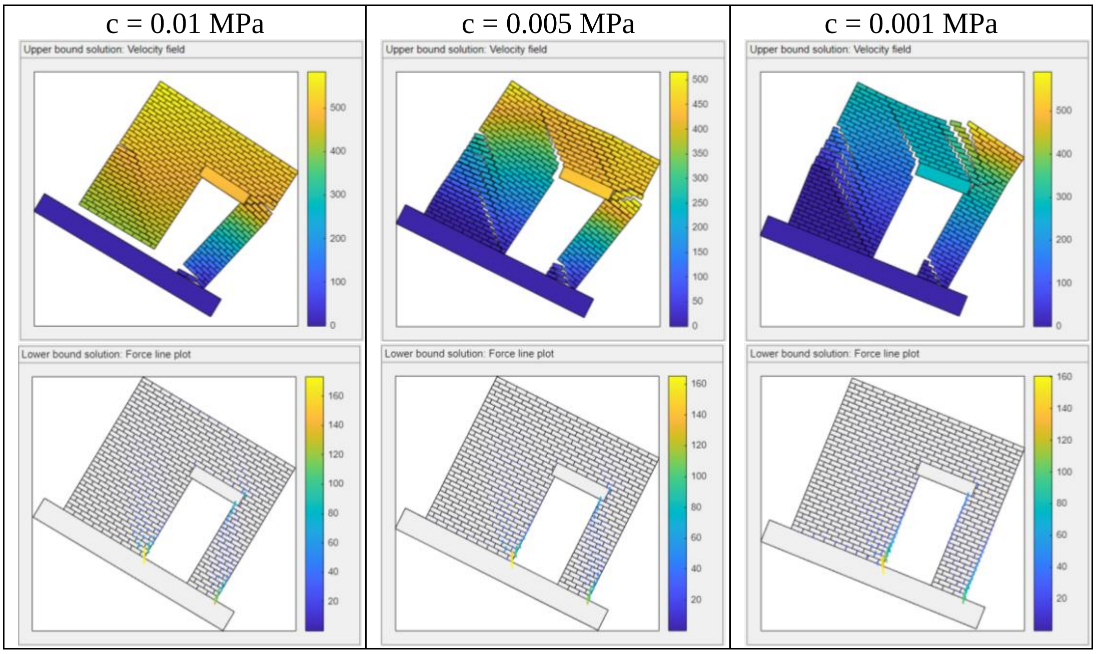

Figure 2: Failure mechanism and distribution of internal forces for Wall A assuming different cohesion values.

The last case study (Wall C), which has the same spatial dimensions as the other two walls, features an irregular masonry pattern. In this case, the interfaces are fully in contact with one another, without any gaps. As observed in the previous examples, lower collapse tilt angles are obtained for lower cohesion values, as shown in Table 1. Failure mechanisms and distribution of internal actions are represented in Figure 4. Higher collapse tilt angles are observed for Wall C compared to Wall B, indicating that the presence of gaps significantly influences the in-plane behaviour of masonry panels. Despite this, the irregular pattern still plays a significant role, as the collapse angles of Wall C are lower than those of Wall A (Table 1).

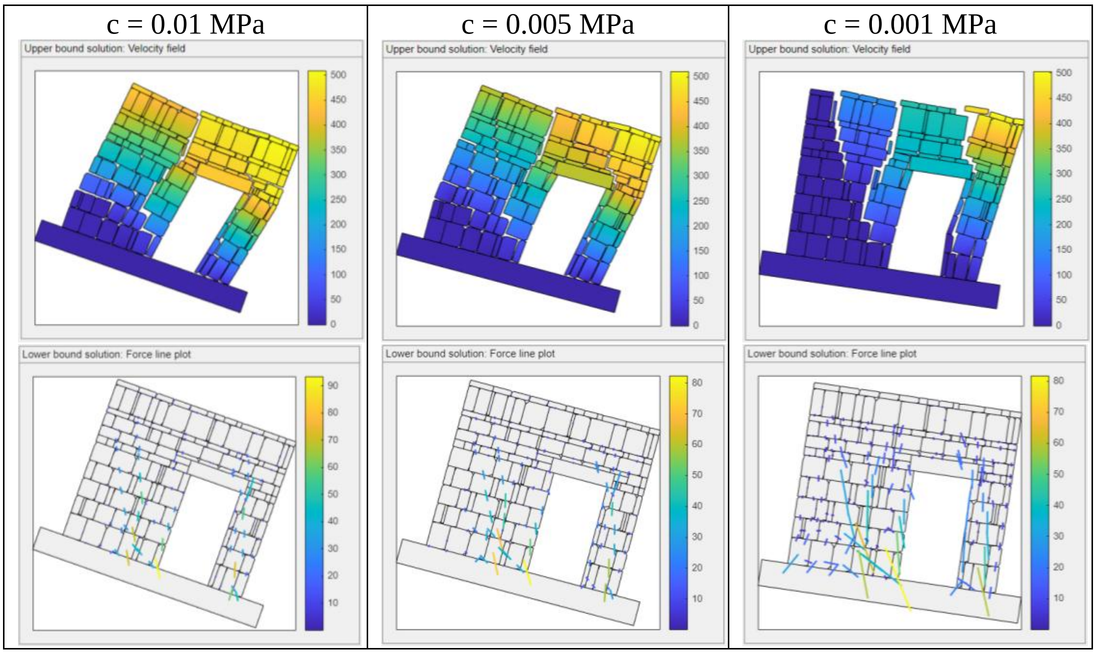

Figure 3: Failure mechanism and distribution of internal forces for Wall B assuming different cohesion values.

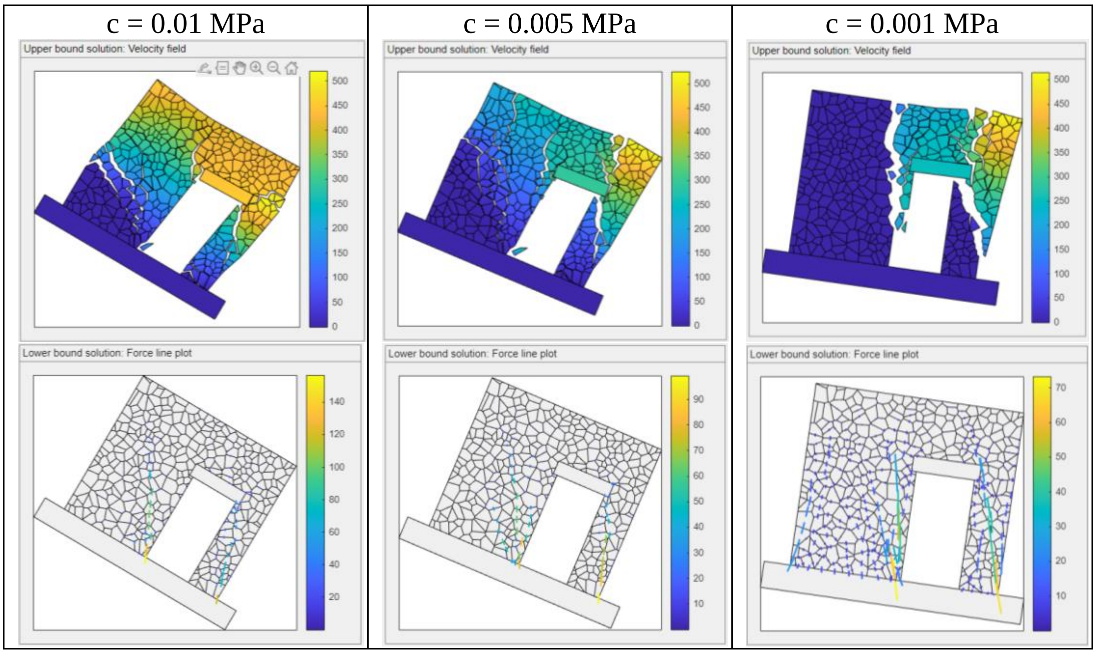

Figure 4: Failure mechanism and distribution of internal forces for Wall C assuming different cohesion values.

Case study	c = 0.01 MPa	c = 0.005 MPa	c = 0.001 MPa
Wall A	30.98°	26.69°	22.11°
Wall B	19.43°	14.94°	8.31°
Wall C	30.65°	23.05°	8.14°

Table 1: Collapse tilt angles for Wall A, B, and C for different cohesion values.

4 Conclusions

This paper presented a homemade limit analysis-based approach for assessing the in-plane behaviour of masonry walls subjected to tilting table tests. The method, grounded in rigid block kinematics and governed by a Mohr-Coulomb failure criterion at the interfaces, enables the identification of failure mechanisms, internal force distributions, and collapse angles through a computationally efficient and user-friendly framework implemented in MATLAB.

The results obtained from the application to three case studies—with regular, semi-regular, and irregular masonry patterns—highlight the method ability to capture the influence of geometric arrangement and material cohesion on structural performance. In particular, the presence of gaps between masonry units significantly reduces the collapse capacity, as seen in the comparison between semi-regular and irregular configurations. The sensitivity analysis on cohesion values further confirms the strong dependency of collapse angles and failure modes on interfacial strength parameters.

The proposed methodology offers a valuable tool for the rapid assessment of masonry structures under seismic-like loading conditions, especially in preliminary design or vulnerability evaluation phases. Future developments will aim at extending the method applicability to three-dimensional and out-of-plane problems, further enhancing its potential as a practical and flexible solution for the structural assessment of masonry heritage and contemporary brick-made constructions.

Acknowledgements

The study was partially developed within the research activities carried out in the frame of 2024-2026 ReLUIS Project – WP10 masonry structures (Coordinator – Prof. Guido Ma-genes), specifically by means of the research carried out in WP10.1.1 and WP10.2 Masonry Structures (Task Coordinators – respectively Prof. Serena Cattari and Prof. Sergio Lagomarsino). The Italian Department of Civil Protection has funded such a project. Additionally, the study was partially developed with the financial support of the Italian Ministry of Scientific Research MUR within the research project PRIN-2022 (https://www.dabc.po li-mi.it/en/progetto/advanced-mechanical-models-and-computational-methods-for-large-scale 3d-printing-of-innovative-concrete-structures/) titled “Advanced mechanical models and computational methods for large-scale 3D printing of innovative concrete structures (COM^3D CREATE)” (National PI: Prof. Andrea Chiozzi, Local PI: Prof. Gabriele Milani). Finanziato dall’Unione europea- Next Generation EU, Missione 4 Componente 1 CUP

D53D23004070006. Finally, Yiwei Hua would like to thank the financial support from China Scholarship Council (CSC) under the grant CSC No. 202108320019.

References

[1] A. Gandolfi, N. Pingaro, G. Milani, “Elastic Body Spring Method (EBSM) for the stability analysis of the Global Vipassana Pagoda in Mumbai, India”, Buildings, 15, 653, 2025.

[2] N. Pingaro, M. Buzzetti, G. Milani, “Advanced FE nonlinear numerical modeling to predict historical masonry vaults failure: Assessment of risk collapse for a long span cloister vault heavily loaded at the crown by means of a general-purpose numerical protocol”, Engineering Failure Analysis, 167, 109070, 2025.

[3] M. Buzzetti, N. Pingaro, G. Milani, “Automatic detection of local collapse mechanisms in historical masonry buildings: Fast and robust FE upper bound limit analysis”, Engineering Failure Analysis, 170, 109310, 2025.

[4] M. Pourfouladi, N. Pingaro, M. Valente, “PoliBrick Plugin as A Parametric Tool for Digital Stereotomy Modelling”, Computers and Structures, 311, 107722, 2025.

[5] G. Milani, “Simple lower bound limit analysis homogenization model for in- and out-of-plane loaded masonry walls”, Constr. Build. Mater., 25 (12), 4426-4443, 2011.

[6] F. P. A. Portioli, “DynABlock_2D: An optimization-based MATLAB application for rocking dynamics, nonlinear static and limit analysis of masonry block structures”, SoftwareX, 23, 101454, 2023.

[7] A. Iannuzzo, A. Dell’Endice, T. Van Mele, P. Block, “Numerical limit analysis-based modelling of masonry structures subjected to large displacements”, Computers and Structures, 242, 106372, 2021.

[8] M. F. Funari, L. C. Silva, E. Mousavian E., P. B. Lourenço, “Real-time Structural Stability of Domes through Limit Analysis: Application to St. Peter’s Dome”, International Journal of Architectural Heritage, 17, 915-937, 2023.

[9] T. E. Boothby, D. Coronelli, “The Stone Skeleton: A Reappraisal”, Heritage, 7, 2265-2276, 2024.

[10] A. Iannuzzo, A. Montanino, “A limit analysis-based CASS approach for the in-plane seismic capacity of masonry façades”, International Journal of Solids and Structures, 289, 112633, 2024.

[11] L. Wischounig. dxf2coord 2.0 (https://www.mathworks.com/matlabcetral/fileexchange/28791-dxf2coord-2-0), MATLAB Central File Exchange, 2025.

[12] D.C. Drucker, “Coulomb Friction, Plasticity, and Limit Loads”, Journal of Applied Mechanics, 21, 71–74, 1954.

[13] H. Yiwei, M. Buzzetti, N. Pingaro, L. C. M. da Silva, G. Milani, “A computerized tool for the kinematic limit analysis of 2D masonry structures failing on a tilting table”, in press.

[14] Y. Hua, G. Milani, “Rigid Block Limit Analysis of Masonry Arches with Associated and Non-associated Slides”, Research for Development, 169-203, 2022.
[15] M. C. Ferris, F. Tin-Loi, “Limit analysis of frictional block assemblies as a mathematical program with complementarity constraints”, International Journal of Mechanical Sciences, 43, 209–224, 2001.