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Large Strain Void-Growth Multiplicative Plasticity Preserving the Infinitesimal Framework

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Abstract

The Gurson-Tvergaard-Needleman (GTN) model is used for porous materials with a plastically isochoric matrix and a volume growth due to the plastic flow in the matrix. The model is used extensively for cast and 3D printed metals, and can be used for metal foams and metamaterials. However most simulations are performed with small strain frameworks or using either hypoelastic formulations teamed with objective stress rates, or Green-based elastoplastic decompositions of total strains. These approaches present known inconsistencies. We present herein a new formulation based on the concept of continuum elastic corrector rates. The derived algorithm retains the additive updates typical of the infinitesimal formulation, but it is consistent with the multiplicative decomposition and a general hyperelastic stored energy.

Keywords: large strains, void growth, GTN plasticity, logarithmic strains, damage mechanics, computational plasticity.

1 Introduction

The Gusson-Tvergaard-Needleman (GTN) model [1] is used mainly to model ductile porous materials, where growth of the voids is due to the isochoric deformation of the metal matrix. In theory, the model considers two scales, one of the matrix and one of the continuum. However, in practice, it is treated as a continuum elastoplastic model, where the void growth is indirectly obtained from the volumetric flow from a proposed yield surface. The ideas behind the model go back at least to the work of Rice and Tracey [2]. Rice and Tracey derived the equation for growth of a spherical void in an elastoplastic metal under a mean stress at infinite. This flow motivates the GTN yield function, such that the volumetric rate of the GTN model (from the derivative of the yield function respect to the mean stress) is equivalent to the growth of voids under an infinite mean stress. It has been shown that the Gurson model is equivalent to a damage model [3], where the damage variable is related to the void ratio.

From a computational standpoint, the stress integration in a finite element model is often performed using a small strains framework. However, in most cases, strains in ductile materials reach the large strain regime. The extensions of the GTN model to large strains have followed mostly two paths: the use of hypoelastic formulations with objective stress rates and incrementally objective integration algorithms [4], or the use of Green additive decompositions of logarithmic strains into elastic and plastic parts [5]. The main reason behind the departure from the sound multiplicative decomposition and the hyperelastic framework is the relative simplicity of the resulting theory and computational algorithm. These recover the simpler additive structure of the backward-Euler (predictor-corrector) algorithms [6,7], facilitating both the derivations and the algorithmic implementation. Moreover, these formulations are easily integrated in canned user subroutines in finite element programs by just plugging the small stress integration algorithm.

The cost of the convenience of the previous large strain formulations for GTN models has been the use of questionable assumptions like hypoelasticity (which is not path independent) or the assumption of a plastic strain tensor from an additive decomposition of the total strains. Similar inconveniences were found in anisotropic elastoplasticity or in crystal plasticity.

In recent years we have been developing a new general approach for large strains elastoplasticity and viscoelasticity. This approach is derived from the multiplicative decomposition of the deformation gradient, but also results in an additive update of the elastic deformation tensor due to plastic flow. The formulation is hyperelastic and compatible with any stored energy (hence valid for polymers), and is valid for both elastic and plastic anisotropy [8]. Furthermore, nonlinear kinematic hardening can be obtained without the concept of backstress [9]. For Mises plasticity, the iterative phase of the stress integration is identical to the Krieg and Key algorithm, whereas the consideration of large strains merely consists on the consideration of kinematic (non-iterative, explicit) pre- and post-processors [9]. The approach has been extended to develop a plane stress constrained algorithm (very uncommon at large strains) [10] and to crystal plasticity [11]. All formulations are similar and, in all cases, all the advantages have been preserved (additive updates like in small strains, general hyperelastic relations and consistent with the multiplicative decomposition).

In this work we extend the formulation to pressure-sensitive yield functions, in particular to porous plasticity. It is shown that in this case, all the previous advantages are still preserved. The integration algorithm has the same structure as the infinitesimal one without the need of any approximation. Large strains reduce to the consideration of the same kinematic pre- and post-processors as in the other cases.

2 Volumetric kinematic relations

Consider an infinitesimal reference continuum volume dV composed of voids dV_e and matrix dV_m. The current deformed volumes are, respectively dv, dv_o, $dv_m = dV_m + dv_e + dv_p$, where dv_e is the elastically deformed volume of the matrix and dv_p is the plastic deformation. In void growth GTN-type models, the isochoric-volumetric coupling comes from the growth of the voids during isochoric plastic deformation of the matrix. To simplify the notation, we will omit the differential volume symbol d, unless necessary (we can think of homogeneous deformations). The "rates" $(\dot{\bullet})$ refer to evolution with deformation.

The plastic flow in the matrix is isochoric, so $\dot{v}_o \equiv \dot{v}_p$. The deformed continuum volume is

$$v = V_m + v_o + v_e \tag{1}$$

which can be written as

$$v_e = v_e\left(v, v_o\right) = \left(v - V_m\right) - v_o \tag{2}$$

The expression $v_e\left(v, v_o\right)$ is very relevant; we are stating that the elastic volumetric deformation is a function of two state variables, the total volume at the continuum scale and the void content. The rates are $\dot{v}_e = \dot{v} - \dot{v}_o$.

If the deformation gradient is $\mathbf{F}$, and the Jacobian is $J = \det\left(\mathbf{F}\right)$, the change of volume gives

$$J = \frac{v}{V_m} = \left(\frac{V_m + v_o + v_e}{V_m + v_o}\right)\left(\frac{V_m + v_o}{V_m}\right) = J_e J_o \equiv J_e J_p \tag{3}$$

where $J_o \equiv J_p$ because the matrix deformation is isochoric, so the void growth and the volumetric plastic deformation are coincident. Note that this corresponds to the volumetric part of the classical Kröner-Lee decomposition. In GTN formulations, it is customary to use the void ratio, defined as

$$f\left(v_o\right) = \frac{v_o}{V_m + v_o} \tag{4}$$

Then,

$$\frac{J_p - 1}{J_p} = \frac{V_m + v_o}{V_m + v_o} - \frac{V_m}{V_m + v_o} = \frac{v_o}{V_m + v_o} = f \tag{5}$$

The void ratio $\dot{f}$ and $\dot{J}_p$ relate through

$$\frac{\dot{f}}{1 - f} = \frac{\dot{v}_o}{V_m + v_o} = \frac{\dot{J}_p}{J_p} \tag{6}$$

The function dependencies in $v_e\,(v, v_o)$ result into two partial derivatives:

$$\frac{d}{dt}\,(J)|_{\dot{J}_p=\dot{v}_o=0} = \frac{d}{dt}\left(\frac{V_m + v_o + v_e}{V_m}\right)\bigg|_{\dot{J}_p=\dot{v}_o=0} = \frac{\dot{v}_e}{V_m} \tag{7}$$

and

$$\frac{d}{dt}\,(J)|_{\dot{J}_p=\dot{v}_o=0}\,J^{-1} = \frac{\dot{v}_e}{V_m}\frac{V_m}{V_m + v_o + v_e} = \frac{\dot{v}_e}{V_m + v_o + v_e} \tag{8}$$

This partial derivatives correspond conceptually to the trial predictor and elastic corrector terms, but note that we are not dealing here with algorithmic concepts, but with continuum ones. Hence, we preserve that predictor-corrector rate naming because of the parallelism, but the subtle difference is crucial in deriving consistent formulations. Then

$$\frac{\dot{J}_e\big|_{\dot{v}_o=\dot{J}_p=0}}{J_e} = \frac{\dot{J}\big|_{\dot{v}_o=\dot{J}_p=0}}{J} \Leftrightarrow {}^{tr}\dot{E}_e^v = \dot{E}^v\Big|_{\dot{v}_o=\dot{J}_p=0} \tag{9}$$

with $E_e^v = \ln J_e$, and

$$\left[\frac{d}{dt}\,(J_e J_p)\right]_{\dot{v}=0}(J_e J_p)^{-1} = 0 = \frac{\dot{J}_e J_p + J_e \dot{J}_p}{J_e J_p}\bigg|_{\dot{v}=0} = \frac{\dot{J}_e}{J_e}\bigg|_{\dot{v}=0} + \frac{\dot{J}_p}{J_p} \tag{10}$$

so

$${}^{ct}\dot{E}_e^v = \frac{\dot{J}_e}{J_e}\bigg|_{\dot{v}=0} = -\frac{\dot{J}_p}{J_p} \tag{11}$$

and

$$-\,{}^{ct}\dot{E}_e^v = \frac{\dot{J}_p}{J_p} \equiv \frac{\dot{J}_o}{J_o} = \frac{\dot{v}_o}{V_m + v_o} = \frac{\dot{f}}{1 - f} \tag{12}$$

Note that this equation has no approximation and is derived from the multiplicative decomposition. This is not conceptually a plastic strain increment or rate, but a correction of the elastic strain rate.

If the yield stress in the matrix is κ, according to Rice and Tracey, the growth of a spherical void under a mean stress p is

$$\frac{\dot{r}}{r} = \dot{\epsilon}_m^p a \sinh\left(\frac{3}{2}\frac{p}{\kappa}\right) \tag{13}$$

where a is a constant, p/κ is the triaxiality factor and $\dot{\epsilon}_m^p$ is the rate of effective plastic strain in the matrix (uniaxial equivalent). To transfer this equation to large strains, we interpret κ and p as Kirchhoff measures, and $\dot{\epsilon}_m^p$ as the rate of a logarithmic equivalent plastic strain in the matrix. Then, we can write

$$-\ {}^{ct,g}\dot{E}_e^v = 3\dot{\epsilon}_m^p af \sinh\left(\frac{3}{2}\frac{p}{\kappa}\right) \ \Rightarrow\ -\ {}^{ct,g}\dot{\mathbf{E}}_e^v = \tfrac{1}{3}\ {}^{ct,g}\dot{E}_e^v \mathbf{I} = \dot{\epsilon}_m^p af \sinh\left(\frac{3}{2}\frac{p}{\kappa}\right)\mathbf{I} \tag{14}$$

3 Matrix deformations

Let $\Psi(\mathbf{E}_e)$ be the stored energy (any suitable function can be employed; there is no restriction), where $\mathbf{E}_e = \frac{1}{2}\ln\left(\mathbf{F}_e^T\mathbf{F}_e\right)$. The plastic dissipation in a reference volume is [8]

$$\begin{aligned}\dot{\mathcal{D}}^p = \mathcal{P} - \dot{\Psi} &= \mathbf{T} : \dot{\mathbf{E}} - \frac{d\Psi}{d\mathbf{E}_e} : \dot{\mathbf{E}}_e \\ &= \mathbf{T} : \dot{\mathbf{E}} - \frac{d\Psi}{d\mathbf{E}_e} : \left({}^{tr}\dot{\mathbf{E}}_e + {}^{ct}\dot{\mathbf{E}}_e\right)\end{aligned} \tag{15}$$

Since

$$\mathbf{E}_e(\mathbf{E}, \mathbf{F}_p) \ \Rightarrow\ \dot{\mathbf{E}}_e = \frac{\partial \dot{\mathbf{E}}_e}{\partial \mathbf{E}} : \dot{\mathbf{E}} + \frac{\partial \dot{\mathbf{E}}_e}{\partial \mathbf{F}_p} : \dot{\mathbf{F}}_p = \dot{\mathbf{E}}_e\Big|_{\dot{\mathbf{F}}_p=\mathbf{0}} + \dot{\mathbf{E}}_e\Big|_{\dot{\mathbf{E}}=\mathbf{0}} = {}^{ct}\dot{\mathbf{E}}_e + {}^{tr}\dot{\mathbf{E}}_e \tag{16}$$

following the standard Colemann arguments

$$\mathbf{T} = \frac{d\Psi}{d\mathbf{E}_e} : \frac{\partial \mathbf{E}_e}{\partial \mathbf{E}}\Big|_{\dot{\mathbf{F}}_p=\mathbf{0}} = \mathbf{T}^{|e} : \frac{\partial \mathbf{E}_e}{\partial \mathbf{E}}\Big|_{\dot{\mathbf{F}}_p=\mathbf{0}} \tag{17}$$

where $\mathbf{T}^{|e}$ are the stresses in the intermediate configuration (for logarithmic strains) and the latter fourth order tensor is the mapping to the reference one. Recall that $\mathbf{T}$ is work-conjugate of $\mathbf{E}$ in the most general anisotropic case..

Obviously the voids cannot carry stresses, so by homogenization:

$$\mathbf{T}^{|e} = \frac{1}{v_0 + V_m}\left[0 + \int_{V_m} \mathbf{T}^{|e}_{\text{micro}} dV_m\right] = \frac{V_m}{v_0 + V_m}\mathbf{T}^{|e}_m \tag{18}$$

Tvergaard added a stress concentration constant q_1 (typically $q_1 = 1.5$), as

$$\mathbf{T}^{|e}_m = \frac{\mathbf{T}^{|e}}{(1 - q_1 f)} \tag{19}$$

Then, the dissipation taking place in the matrix is purely isochoric, and is

$$\dot{\mathcal{D}}_m^p = -\mathbf{T}^{|e}_m : {}^{ct}\dot{\mathbf{E}}_e \equiv -\mathbf{T}^{d|e}_m : {}^{ct}\dot{\mathbf{E}}_e^d =: \kappa\dot{\epsilon}_m^p > 0 \tag{20}$$

where by $(\bullet)^d$ we denote isochiric/deviatoric parts and by $(\bullet)^v$ the volumetric ones. In von Mises plasticity, we write the flow rule as

$$-\,{}^{ct}\dot{\mathbf{E}}_e^d = \dot{\gamma}\hat{\mathbf{N}} = \frac{3}{2}\dot{\epsilon}_m^p \frac{\mathbf{T}_m^{d|e}}{\kappa} \tag{21}$$

where $\hat{\mathbf{N}}$ is the unit tensor in the isochoric matrix flow direction. Then, we can write

$$\dot{D}_m^p - \kappa\dot{\epsilon}_m^p = \left(\frac{1}{\kappa}\frac{3}{2}\mathbf{T}_m^{d|e} : \mathbf{T}_m^{d|e} - \kappa\right)\dot{\epsilon}_m^p = 0 \tag{22}$$

or using Eq. (19)

$$\dot{D}_m^p - \kappa\dot{\epsilon}_m^p = \left(\frac{1}{\kappa}\frac{3}{2}\frac{\mathbf{T}^{d|e}}{(1-q_1 f)} : \frac{\mathbf{T}^{d|e}}{(1-q_1 f)} - \kappa\right)\dot{\epsilon}_m^p = 0 \tag{23}$$

which for $\dot{\epsilon}_m^p \neq 0$ is equivalent to consider

$$F^d = \frac{\frac{3}{2}\left\|\mathbf{T}^{d|e}\right\|^2}{\kappa^2} - (1-q_1 f)^2 = 0 \tag{24}$$

which is the Gurson yield function (without the triaxiality influence).

4 Triaxiality: pressure influence

The GTN yield function adds the influence of the triaxiality (ratio between the pressure and the yield stress in the matrix) in the void growth:

$$F(\mathbf{T}^{d|e}, p) = \frac{\frac{3}{2}\left\|\mathbf{T}^{d|e}\right\|^2}{\kappa^2} - (1-q_1 f)^2 + 2q_1 f\left[\cosh\left(q_2\frac{3}{2}\frac{p}{\kappa}\right) - 1\right] = 0 \tag{25}$$

where q_2 is another parameter. Associativity of the flow gives the flow rule

$$\begin{aligned} -\,{}^{ct}\dot{\mathbf{E}}_e &= \dot{\lambda}\frac{dF}{d\mathbf{T}^{|e}} = \dot{\lambda}\frac{dF}{d\mathbf{T}^{d|e}} : \frac{d\mathbf{T}^{d|e}}{d\mathbf{T}^{|e}} + \dot{\lambda}\frac{dF}{dp}\frac{dp}{d\mathbf{T}^{|e}} \\ &= 3\dot{\lambda}\frac{\mathbf{T}^{d|e}}{\kappa^2} + \dot{\lambda}\frac{q_2\,(q_1 f)}{\kappa}\sinh\left(\frac{3}{2}\frac{p}{\kappa}q_2\right)\mathbf{I} \\ &= -\,{}^{ct}\dot{\mathbf{E}}_e^d - {}^{ct}\dot{\mathbf{E}}_e^v = -\,{}^{ct}\dot{\mathbf{E}}_e^d - \tfrac{1}{3}\,{}^{ct,g}\dot{E}_e^v\mathbf{I} \end{aligned} \tag{26}$$

After some algebra, using the previous equations, we can derive the relation

$$\dot{f} = \frac{3}{2}\frac{1-f}{1-q_1 f}q_2 q_1 f\sinh\left(\frac{3}{2}\frac{p}{\kappa}q_2\right)\dot{\epsilon}_m^p \tag{27}$$

For the particular case of a tensile test, we can obtain the following approximate relation between the yield stress in the continuum and the yield stress in the matrix as a function of the void ratio:

$$T_Y \approx \frac{(1-q_1 f)}{\sqrt{1+\frac{1}{4}q_1 q_2^2 f}}\,\kappa \tag{28}$$

In a similar way, it also brings the relation between the logarithmic plastic strain rate in the matrix $\dot{\epsilon}_m^p$ and in the continuum $\dot{\epsilon}^p$

$$\dot{D}^p = (1-q_1 f)\kappa\dot{\epsilon}_m^p \approx \sqrt{1+\tfrac{1}{4}q_1 q_2^2 f}\; T_Y\dot{\epsilon}_m^p = T_Y\dot{\epsilon}^p \;\Rightarrow\; \dot{\epsilon}^p = \sqrt{1+\tfrac{1}{4}q_1 q_2^2 f}\;\dot{\epsilon}_m^p \tag{29}$$

$\dot{\epsilon}_m^p$ and $\dot{\epsilon}^p$ are only different for large void ratios, so in practice we can take $\epsilon^p \simeq \epsilon_m^p$.

5 Stress integration algorithm

With the previous framework, it is straightforward to establish a fully implicit Backward-Euler stress integration algorithm for the formulation. The two nonlinear equations to solve iteratively are the yield function and the flow rule, where $\mathbf{E}_e$ and $\Delta\epsilon_m^p$ are the iterative variables

$$F(\mathbf{E}_e) \equiv \frac{\frac{3}{2}\left\|\mathbf{T}^{d|e}(\mathbf{E}_e)\right\|^2}{\kappa^2} - (1-q_1 f)^2 + 2q_1 f\left[\cosh\left(q_2\frac{3}{2}\frac{p}{\kappa}\right)-1\right] \to 0 \tag{30}$$

$$G(\mathbf{E}_e,\Delta\epsilon_m^p) \equiv {}^{ct}\mathbf{E}_e + 3\Delta\bar{\lambda}(\Delta\epsilon_m^p)\frac{\mathbf{T}^{d|e}(\mathbf{E}_e)}{\kappa} + \Delta\bar{\lambda}(\Delta\epsilon_m^p)\, q_2 q_1 f \sinh\left(\frac{3}{2}\frac{p}{\kappa}q_2\right)\mathbf{I} \to \mathbf{0} \tag{31}$$

with

$${}^{ct}\mathbf{E}_e = \mathbf{E}_e - {}^{tr}\mathbf{E}_e \tag{32}$$

$$\mathbf{T}^{|e}(\mathbf{E}_e) = d\Psi(\mathbf{E}_e)/d\mathbf{E}_e \tag{33}$$

$$p(\mathbf{E}_e) = \tfrac{1}{3}\mathbf{T}^{|e} : \mathbf{I} \tag{34}$$

$$\mathbf{T}^{d|e}(\mathbf{E}_e) = \mathbf{T}^{|e} - p\mathbf{I} \tag{35}$$

$$\Delta\bar{\lambda}(\Delta\epsilon_m^p) = \frac{\Delta\epsilon_m^p}{2(1-q_1 f)} \tag{36}$$

and $\kappa = \kappa(\Delta\epsilon_m^p)$ in the case of hardening.

Remarkably, this iterative algorithm is the same as that of infinitesimal strains, where $\mathbf{T}^{|e}$ plays the role of the Cauchy stresses and $\mathbf{E}_e$ plays the role of the infinitesimal elastic strains.

The development of the Newton algorithm follows the typical steps, including the consistent tangent for the global iterations.

The formulation has been developed in terms of Generalized Kirchhoff stresses and referential logarithmic strains. Since finite element codes are usually formulated in terms of second Piola-Kirchhoff stresses and Green-Lagrange strains, the proper mappings need to be performed, both in the final stresses and in the corresponding tangent. Since these steps are the same as those for the other models using this framework, we refer to [8–11] for further details.

6 Example

We have implemented the present GTN model in our in-house finite element program Dulcinea. In this example we show the comparison of the present formulation with the predictions by Abaqus. The chosen example is the stretch of the plate with a hole, found in many other works [6, 8, 9]. The plate has been modelled with symmetry conditions. The dimensions are given in Fig. 1. The stored energy is linear in logaritmic strains, with elastic constants $C_{11} = 269.4$ GPa, $C_{12} = 115.4$ GPa, $C_{44} = 77$ GPa, and initial yield stress for the matrix of $\kappa_0 = 96$ MPa. The Tvergaard constants are $q_1 = 1.25$ and $q_2 = 1.25$. The hardening law (needed to avoid early localization) is $\kappa = \kappa_0 + 53\epsilon_m^p$ for Abaqus, and equivalent values for our model. The finite elements used are C3D20R and the Dulcinea equivalent.

The specimen deformation is 7.5% of the length of the plate. Results are shown in Figure 1. It can be observed that the differences between the predictions from both models are very small. This should be expected given that loading is monotonic and the level of deformation is moderately large.

As it is well known, given that localization of deformations is expected, the results can be mesh dependent, and suitable formulations (as phase field) which include a length scale should be used for non-academic examples. However, we note that both meshes (Abaqus and Dulcinea) are identical, so a one-to-one comparison of predictions from both models, which is the focus of this work, can be made.

7 Conclusion

In this paper we have presented a continuum formulation for large strain porous plasticity that is consistent with the multiplicative decomposition and with any hyperelastic stored energy function. The formulation facilitates a very simple, additive, stress integration algorithm, mimicking the infinitesimal one. In practice, the large strains enter the algorithm as a standard pre- and post-processor, the same one as those used by other models based on strain rate corrector concept.

The formulation has been compared to the built-in formulation in Abaqus, which is hypoelastic, based on objective stress rates and the incrementally objective Hughes-Winget integration algorithm, obtaining very similar results for the moderately large range of ductile metals.

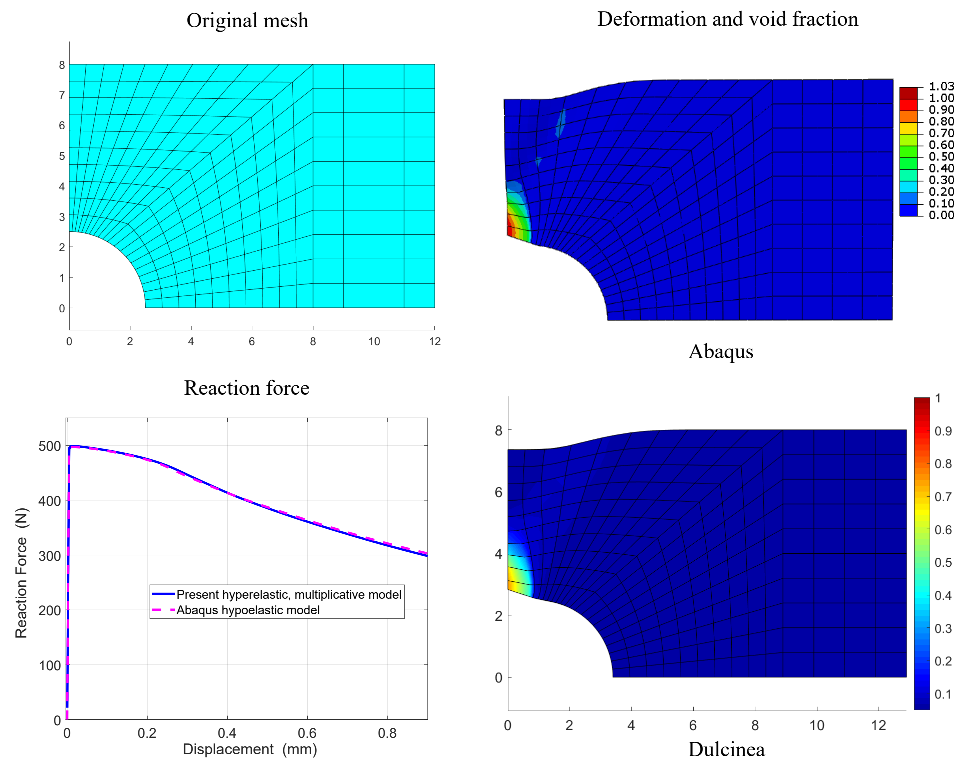

Figure 1: Plate with a hole. Comparative example of the present formulation (hyperelastic, based in the multiplicative decomposition) with the built-in formulation in Abaqus (hypoelastic, using objective stress rates). Left: Original, undeformed mesh and reaction force. Right: deformed meshes and void ratio predictions using both models. Slight differences are due to the different nodal extrapolation of results for plotting employed by both programs.

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