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Nonlinear Earthquake Response Analysis of Unanchored Cylindrical Liquid Storage Tanks Considering Fluid-Structure-Soil Interaction

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Abstract

Considering fluid-structure-soil interaction, a finite element model of an unanchored cylindrical liquid storage tank is presented and a nonlinear earthquake response analysis is formulated. The tank structure is modelled using shell elements, which allow geometric and material nonlinear behaviour to be taken into account. The fluid behaviour is represented by acoustic elements and coupled to the structure by interface elements. To account for soil-structure interaction, the near and far fields of the soil are modelled with solid elements and perfectly matched discrete layers, respectively. The base uplift is considered using nonlinear springs between the tank base and soil. The finite-element model is used to calculate the earthquake response of an unanchored liquid storage tank on flexible soil subjected to an earthquake ground motion.

Keywords: cylindrical liquid storage tank, fluid-structure interaction, soil-structure interaction, earthquake response analysis, nonlinear behaviour, uplift

1 Introduction

Cylindrical liquid storage tanks are an indispensable infrastructure in modern society and industry, storing a wide range of liquids. However, if damaged by earthquakes, they can have a negative impact on the environment and cause serious damage to society and industry. In the worst cases, direct and secondary damage, including fires, can lead to loss of life. It is therefore important to ensure that cylindrical liquid storage tanks are safe in the event of a major earthquake. However, they have been observed to be significantly damaged by ground motions during past earthquakes [1]. Therefore, many studies have been carried out to understand the dynamic behaviour of cylindrical liquid storage tanks and to improve their seismic safety. In the pioneering work of Housner, simplified formulae were proposed for the impact and convective hydrostatic forces of a rigid tank completely anchored on rigid soils and subjected to horizontal ground motion [2]. Theoretical and experimental investigations of the dynamic behaviour of deformable cylindrical liquid storage tanks on flexible soils have also been carried out [3-5]. Based on the studies of fully anchored tanks subjected to earthquake ground motions, a simple mass-spring model has been proposed for the seismic design of the system [6, 7].

Seismic waves from the epicenter propagate to the site where the liquid storage tank is installed and these waves are transmitted through the foundation to the internal liquid, causing it to vibrate. The vibration of the internal fluid in turn affects that of the structure. This phenomenon is known as fluid-structure interaction. If the stiffness of the structure is large, the fluid-structure interaction is not considered because the structure practically behaves as a rigid body, but otherwise the effect of the fluid-structure interaction on the vibrations of the structure and the storage liquid must be considered [8-12].

The flexibility of the ground on which the structure rests will change the dynamic properties of the whole system, which will affect the dynamic behaviour of the superstructure. The affected dynamic behaviour of the superstructure in turn affects the ground vibration, changing the amplitude and frequency characteristics of the earthquake ground motions transmitted through the foundation. This phenomenon, where the dynamic behaviour of the ground and the structure influence each other, is called soil-structure interaction. The effect of soil-structure interaction cannot be ignored when a heavy structure, such as a liquid storage tank, is placed on a flexible ground [13-15].

As described above, the seismic behaviour of liquid storage tanks is complicated by the fluid-structure-soil interaction, and these phenomena must be rigorously considered in order to accurately predict the seismic response and damage of these systems. However, it should be noted that various nonlinear behaviours can be observed in liquid storage tanks resting on flexible ground. These include material and geometrical nonlinear behaviour and buckling of the tank structure, excessive sloshing of the free surface of the storage liquid, nonlinear material behaviour and failure in the subgrade, and nonlinear boundary conditions at the structure-soil interface (partial lifting, sliding, separation of the structure foundation, etc.). Therefore, in order to evaluate the seismic performance of liquid storage tanks, it is necessary to be able to rigorously take into account these nonlinear behaviours.

In this study, the nonlinear seismic response of an unanchored cylindrical liquid storage tank resting on flexible ground is analysed taking into account the uplift of the base plate. For this purpose, a nonlinear finite element model of an unanchored cylindrical liquid storage tank with rigorous consideration of fluid-structure-soil interaction and a nonlinear seismic response analysis is formulated. This method is used to calculate the earthquake response of an unanchored liquid storage tank subjected to an earthquake ground motion.

2 Nonlinear Fluid-Structure-Soil Interaction

A governing equation for the system of fluid-structure-soil interaction is derived for an unanchored liquid storage tank resting on flexible ground. Typically, a tank is located in a layered half-space. In this case, the soil can be divided into two parts, the near- and the far-field regions, as shown in Figure 1. The near-field region can have irregular geometry and inhomogeneous material properties, while the far-field region is assumed to have regular, homogeneous, linear material with regular geometry in an infinite direction. The behaviour in the near-field region can be simulated using finite elements. Therefore, using the finite element technique, the discretized equation of motion for the tank structure can be obtained as follows [16]:

$$\begin{bmatrix} \mathbf{M}_{ss} & \mathbf{M}_{sh} & \mathbf{M}_{sv} \\ \mathbf{M}_{hs} & \mathbf{M}_{hh} & \mathbf{M}_{hv} \\ \mathbf{M}_{vs} & \mathbf{M}_{vh} & \mathbf{M}_{vv} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{U}}_s \\ \ddot{\mathbf{U}}_h \\ \ddot{\mathbf{U}}_v \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{ss} & \mathbf{C}_{sh} & \mathbf{C}_{sv} \\ \mathbf{C}_{hs} & \mathbf{C}_{hh} & \mathbf{C}_{hv} \\ \mathbf{C}_{vs} & \mathbf{C}_{vh} & \mathbf{C}_{vv} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{U}}_s \\ \dot{\mathbf{U}}_h \\ \dot{\mathbf{U}}_v \end{Bmatrix} + \begin{Bmatrix} \mathbf{F}_s^{int}\left(\mathbf{U},\dot{\mathbf{U}}\right) \\ \mathbf{F}_h^{int}\left(\mathbf{U},\dot{\mathbf{U}}\right) \\ \mathbf{F}_v^{int}\left(\mathbf{U},\dot{\mathbf{U}}\right) \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_s^{liquid} \\ \mathbf{0} \\ \mathbf{F}_v^{liquid} \end{Bmatrix} + \begin{Bmatrix} \mathbf{0} \\ \mathbf{F}_h^{soil} \\ \mathbf{F}_v^{soil} \end{Bmatrix} \tag{1}$$

where $\mathbf{M}$ and $\mathbf{C}$ are the mass and damping matrices of the structure, respectively; $\mathbf{F}^{int}\left(\mathbf{U},\dot{\mathbf{U}}\right)$ is the internal force taking into account material and geometric nonlinearities; $\mathbf{U}(t)$ is the total displacement of the structure; $\mathbf{F}^{liquid}(t)$ and $\mathbf{F}^{soil}(t)$ are the hydrodynamic force for the liquid and the interaction force between the structure and soil, respectively. The subscript *s* denotes the degrees of freedom of the structure that are not in contact with soil, and the subscripts *h* and *v* denote those on the tank base in the horizontal and vertical directions, respectively.

To determine the hydrodynamic force $\mathbf{F}^{liquid}(t)$ exerted by the liquid, the finite-element technique is used to obtain the solution of the equation of motion for fluid [17]. Assuming that the liquid in the tank is an ideal incompressible non-viscous fluid, the hydrodynamic pressure exerted by the liquid is obtained as follows:

$$\mathbf{G}^{fs}\ddot{\mathbf{P}} + \mathbf{HP} = \mathbf{Q} \tag{2a}$$

$$\mathbf{G}^{fs} = \frac{1}{g}\int \mathbf{N}^T \mathbf{N} dS \tag{2b}$$

$$\mathbf{H} = \int \mathbf{B}^T \mathbf{B} dV \tag{2c}$$

$$\mathbf{B} = \begin{bmatrix} \dfrac{\partial \mathbf{N}}{\partial x} & \dfrac{\partial \mathbf{N}}{\partial y} & \dfrac{\partial \mathbf{N}}{\partial z} \end{bmatrix}^T \tag{2d}$$

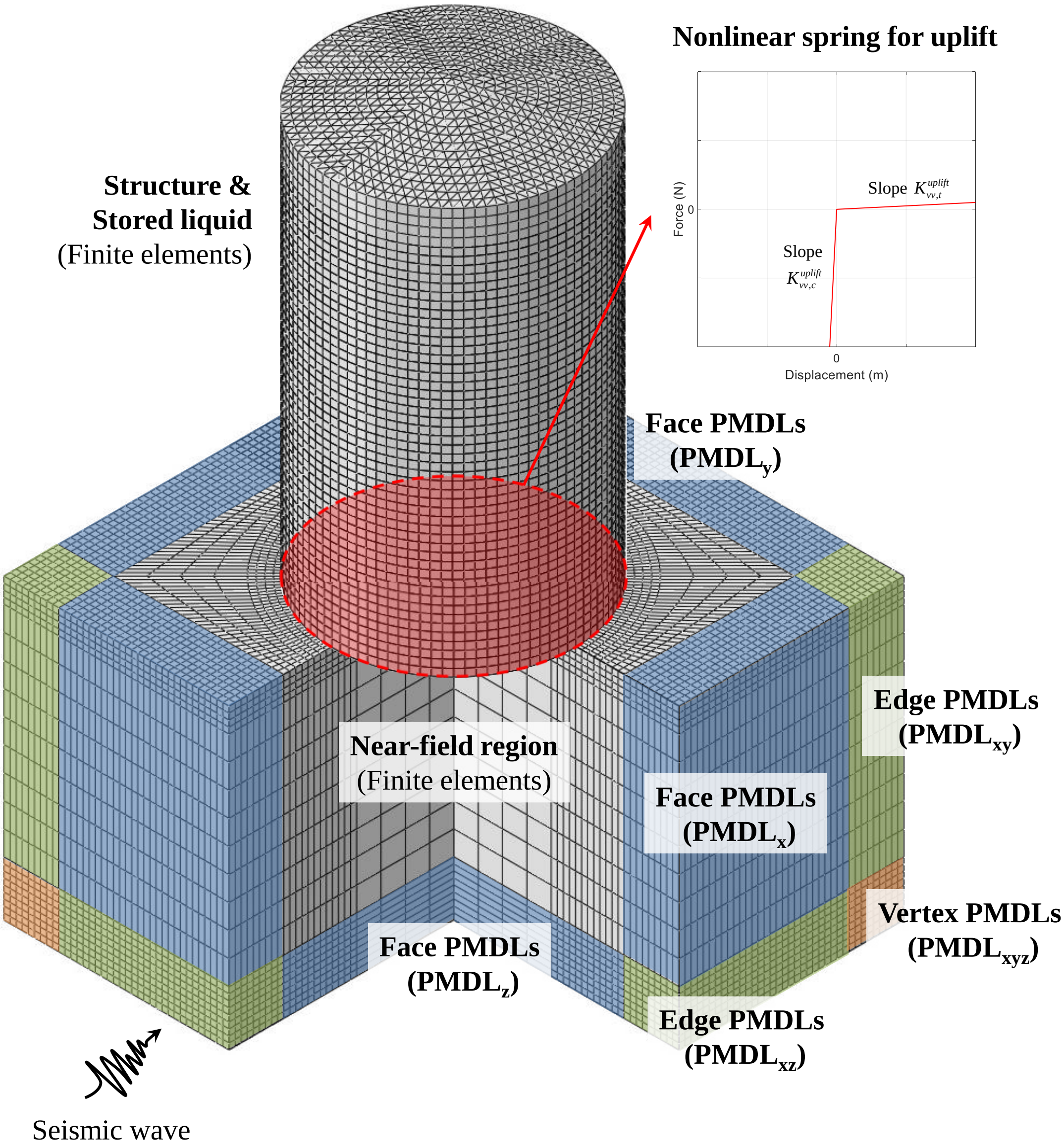

Figure 1: Cylindrical liquid storage tank on a half-space.

$$\mathbf{Q} = -\rho \mathbf{S}\ddot{\mathbf{U}} \tag{2e}$$

$$\mathbf{S} = \int \mathbf{N}^T \mathbf{v} \mathbf{N}_s dS \tag{2f}$$

where $\mathbf{P}(t)$ is the hydrodynamic pressure of the fluid, $\mathbf{N}(x, y, z)$ is the shape function of the fluid, $\mathbf{N}_s(x, y, z)$ is the shape function of the structure, $\mathbf{v}$ is the outward unit normal vector, $\ddot{\mathbf{U}}(t)$ is the total acceleration of the structure. $\mathbf{G}^{fs}$ from the free surface condition is calculated only for the elements located on the free surface. $\mathbf{S}$ in Equation (2f) denotes the interface element between the fluid and the tank structure.

The fluid force $\mathbf{F}^{liquid}(t)$ in Equation (1) can be obtained using the hydrodynamic pressure $\mathbf{P}(t)$.

$$\mathbf{F}^{liquid} = \int \mathbf{N}_S^T \mathbf{v}^T \mathbf{N} dS\, \mathbf{P} = \mathbf{S}^T \mathbf{P} \tag{3}$$

In this study, only the base uplift is considered for an unanchored structure. The base uplift must be considered in the vertical interaction force $\mathbf{F}_v^{soil}(t)$. When the tank base is lifted, the vertical displacements at the tank base are different from those on the soil surface. This introduces an additional degree-of-freedom $\mathbf{U}_v^g(t)$. The two displacements $\mathbf{U}_v(t)$ and $\mathbf{U}_v^g(t)$ are then related by the stiffness matrix $\mathbf{K}_{vv}^{uplift}$, which consists of nonlinear springs with a negligible coefficient in tension and a very large coefficient in compression for the relative motion between $\mathbf{U}_v(t)$ and $\mathbf{U}_v^g(t)$, as shown in Figure 1 [18, 19].

The near-field region can be simulated using finite elements, but the far-field region requires a numerical model that can account for energy radiation into the infinite domain as shown in Figure 1. Various methods have been developed for this purpose, but in this study we use a perfectly matched discrete layer (PMDL) to simulate the far-field region [15, 20, 21]. The PMDL is the preferred method for geotechnical simulation because the accuracy can be adjusted to the user's desired level and it can be easily combined with finite element methods. The PMDL model of the far-field region is used to estimate the interaction force $\mathbf{F}^{soil}(t)$. As shown in Figure 1, three types of PMDLs are commonly used to represent the semi-infinite domain. We use PMDL_i, PMDL_{ij}, and PMDL_{ijk} ($i, j, k = x, y, z$) to represent the regions that are infinite in only one direction, in two directions and in all three directions, respectively. We assume that the two vertical and one horizontal boundary surfaces of the near-field region are perpendicular to each other. Therefore, all three types of PMDLs are rectangular in shape. For example, the dynamic stiffness of PMDL_x, PMDL_{xy} and PMDL_{xyz} for a viscoelastic soil with a hysteretic damping ratio of η can be obtained as follows:

$$\begin{aligned}\mathbf{S}_x &= i\omega(1+i\eta)\mathbf{C}_x^s + i\omega\mathbf{C}_x^m + (1+i\eta)\mathbf{K}_x + \frac{1+i\eta}{i\omega}\mathbf{R}_x \\ &\approx i\omega\mathbf{C}_x^s + i\omega\mathbf{C}_x^m + \mathbf{K}_x + \frac{1}{i\omega}\mathbf{R}_x - \omega^2\frac{\eta}{\omega_0}\mathbf{C}_x^s + i\omega\frac{\eta}{\omega_0}\mathbf{K}_x + \frac{\eta}{\omega_0}\mathbf{R}_x\end{aligned} \tag{4a}$$

$$\begin{aligned}\mathbf{S}_{xy} &= (1+i\eta)\mathbf{K}_{xy}^s + \mathbf{K}_{xy}^m + \frac{1+i\eta}{i\omega}\mathbf{R}_{xy} - \frac{1+i\eta}{\omega^2}\mathbf{T}_{xy} \\ &\approx \mathbf{K}_{xy}^s + \mathbf{K}_{xy}^m + \frac{1}{i\omega}\mathbf{R}_{xy} - \frac{1}{\omega^2}\mathbf{T}_{xy} + i\omega\frac{\eta}{\omega_0}\mathbf{K}_{xy}^s + \frac{\eta}{\omega_0}\mathbf{R}_{xy} + \frac{1}{i\omega}\frac{\eta}{\omega_0}\mathbf{T}_{xy}\end{aligned} \tag{4b}$$

$$\mathbf{S}_{xyz} = \frac{1+i\eta}{i\omega}\mathbf{R}_{xyz}^s + \frac{1}{i\omega}\mathbf{R}_{xyz}^m \approx \frac{1}{i\omega}\mathbf{R}_{xyz}^s + \frac{1}{i\omega}\mathbf{R}_{xyz}^m + \frac{\eta}{\omega_0}\mathbf{R}_{xyz}^s \tag{4c}$$

where the element matrices are defined in Lee et al. [21].

Following the descriptions in the above, the interaction force $\mathbf{F}^{soil}(t)$ between the soil and the structure can be expressed as follows:

$$\begin{Bmatrix} \mathbf{F}_h^{soil} \\ \mathbf{F}_v^{soil} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{Bmatrix} = -\begin{bmatrix} \mathbf{M}_{hh}^g & \mathbf{0} & \mathbf{M}_{hv}^g & \mathbf{M}_{hg}^g & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{M}_{vh}^g & \mathbf{0} & \mathbf{M}_{vv}^g & \mathbf{M}_{vg}^g & \mathbf{0} \\ \mathbf{M}_{gh}^g & \mathbf{0} & \mathbf{M}_{gv}^g & \mathbf{M}_{gg}^g + \mathbf{M}_{gg}^f & \mathbf{M}_{gf}^f \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{M}_{fg}^f & \mathbf{M}_{ff}^f \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{U}}_h \\ \ddot{\mathbf{U}}_v \\ \ddot{\mathbf{U}}_v^g \\ \ddot{\mathbf{U}}_g - \ddot{\mathbf{U}}_g^* \\ \ddot{\mathbf{U}}_f \end{Bmatrix} - \begin{bmatrix} \mathbf{C}_{hh}^g & \mathbf{0} & \mathbf{C}_{hv}^g & \mathbf{C}_{hg}^g & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{C}_{vh}^g & \mathbf{0} & \mathbf{C}_{vv}^g & \mathbf{C}_{vg}^g & \mathbf{0} \\ \mathbf{C}_{gh}^g & \mathbf{0} & \mathbf{C}_{gv}^g & \mathbf{C}_{gg}^g + \mathbf{C}_{gg}^f & \mathbf{C}_{gf}^f \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{C}_{fg}^f & \mathbf{C}_{ff}^f \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{U}}_h \\ \dot{\mathbf{U}}_v \\ \dot{\mathbf{U}}_v^g \\ \dot{\mathbf{U}}_g - \dot{\mathbf{U}}_g^* \\ \dot{\mathbf{U}}_f \end{Bmatrix}$$

$$-\begin{bmatrix} \mathbf{K}_{hh}^g & \mathbf{0} & \mathbf{K}_{hv}^g & \mathbf{K}_{hg}^g & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{vv}^{uplift} & -\mathbf{K}_{vv}^{uplift} & \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{vh}^g & -\mathbf{K}_{vv}^{uplift} & \mathbf{K}_{vv}^{uplift} + \mathbf{K}_{vv}^g & \mathbf{K}_{vg}^g & \mathbf{0} \\ \mathbf{K}_{gh}^g & \mathbf{0} & \mathbf{K}_{gv}^g & \mathbf{K}_{gg}^g + \mathbf{K}_{gg}^f & \mathbf{K}_{gf}^f \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{K}_{fg}^f & \mathbf{K}_{ff}^f \end{bmatrix} \begin{Bmatrix} \mathbf{U}_h \\ \mathbf{U}_v \\ \mathbf{U}_v^g \\ \mathbf{U}_g - \mathbf{U}_g^* \\ \mathbf{U}_f \end{Bmatrix} - \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{R}_{gg}^f & \mathbf{R}_{gf}^f \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{R}_{fg}^f & \mathbf{R}_{ff}^f \end{bmatrix} \begin{Bmatrix} \bar{\mathbf{U}}_h \\ \bar{\mathbf{U}}_v \\ \bar{\mathbf{U}}_v^g \\ \bar{\mathbf{U}}_g - \bar{\mathbf{U}}_g^* \\ \bar{\mathbf{U}}_f \end{Bmatrix}$$

$$-\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_{gg}^f & \mathbf{T}_{gf}^f \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_{fg}^f & \mathbf{T}_{ff}^f \end{bmatrix} \begin{Bmatrix} \bar{\bar{\mathbf{U}}}_h \\ \bar{\bar{\mathbf{U}}}_v \\ \bar{\bar{\mathbf{U}}}_v^g \\ \bar{\bar{\mathbf{U}}}_g - \bar{\bar{\mathbf{U}}}_g^* \\ \bar{\bar{\mathbf{U}}}_f \end{Bmatrix} + \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ -\mathbf{P}_g^* \\ \mathbf{0} \end{Bmatrix} \tag{5a}$$

$$\bar{\mathbf{U}}(t) = \int_0^t \mathbf{U}(\tau)d\tau \tag{5b}$$

$$\bar{\bar{\mathbf{U}}}(t) = \int_0^t \bar{\mathbf{U}}(\tau)d\tau = \int_0^t\int_0^\tau \mathbf{U}(\sigma)d\sigma d\tau \tag{5c}$$

where $\mathbf{M}^g$, $\mathbf{C}^g$, and $\mathbf{K}^g$ are the mass, damping, and stiffness matrices for the near-field region of soil, respectively; $\mathbf{M}^f$, $\mathbf{C}^f$, $\mathbf{K}^f$, $\mathbf{R}^f$, and $\mathbf{T}^f$ denote the corresponding matrices given in Equation (4) for the far-field region of the ground represented by the PMDL. In Equation (5) the superscripts *g* and *f* represent the near- and far-field regions of soil, respectively, and the subscripts *g* and *f* denote the nodal points in the near-field region that is not contact with the tank base and those only in the PMDL of the far-field region, respectively. $\mathbf{U}_g^*(t)$ and $\mathbf{P}_g^*(t)$ denote the free-field motion caused by the incident seismic waves and the resulting nodal forces, respectively.

From Equations (1), (2), (3), and (5), the final equation of motion for the nonlinear fluid-structure-soil interaction system can be obtained as follows:

$$
\begin{bmatrix}
\mathbf{M}_{ss} & \mathbf{M}_{sh} & \mathbf{M}_{sv} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{M}_{hs} & \mathbf{M}_{hh}+\mathbf{M}_{hh}^{g} & \mathbf{M}_{hv} & \mathbf{M}_{hv}^{g} & \mathbf{M}_{hg}^{g} & \mathbf{0} & \mathbf{0} \\
\mathbf{M}_{vs} & \mathbf{M}_{vh} & \mathbf{M}_{vv} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{M}_{vh}^{g} & \mathbf{0} & \mathbf{M}_{vv}^{g} & \mathbf{M}_{vg}^{g} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{M}_{gh}^{g} & \mathbf{0} & \mathbf{M}_{gv}^{g} & \mathbf{M}_{gg}^{g}+\mathbf{M}_{gg}^{f} & \mathbf{M}_{gf}^{f} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{M}_{fg}^{f} & \mathbf{M}_{ff}^{f} & \mathbf{0} \\
\rho\mathbf{S}_{s} & \mathbf{0} & \rho\mathbf{S}_{v} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}^{fs}
\end{bmatrix}
\begin{Bmatrix}
\ddot{\mathbf{U}}_{s} \\ \ddot{\mathbf{U}}_{h} \\ \ddot{\mathbf{U}}_{v} \\ \ddot{\mathbf{U}}_{v}^{g} \\ \ddot{\mathbf{U}}_{g} \\ \ddot{\mathbf{U}}_{f} \\ \ddot{\mathbf{P}}
\end{Bmatrix}
+
\begin{bmatrix}
\mathbf{C}_{ss} & \mathbf{C}_{sh} & \mathbf{C}_{sv} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{C}_{hs} & \mathbf{C}_{hh}+\mathbf{C}_{hh}^{g} & \mathbf{C}_{hv} & \mathbf{C}_{hv}^{g} & \mathbf{C}_{hg}^{g} & \mathbf{0} & \mathbf{0} \\
\mathbf{C}_{vs} & \mathbf{C}_{vh} & \mathbf{C}_{vv} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{C}_{vh}^{g} & \mathbf{0} & \mathbf{C}_{vv}^{g} & \mathbf{C}_{vg}^{g} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{C}_{gh}^{g} & \mathbf{0} & \mathbf{C}_{gv}^{g} & \mathbf{C}_{gg}^{g}+\mathbf{C}_{gg}^{f} & \mathbf{C}_{gf}^{f} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{C}_{fg}^{f} & \mathbf{C}_{ff}^{f} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}
\end{bmatrix}
\begin{Bmatrix}
\dot{\mathbf{U}}_{s} \\ \dot{\mathbf{U}}_{h} \\ \dot{\mathbf{U}}_{v} \\ \dot{\mathbf{U}}_{v}^{g} \\ \dot{\mathbf{U}}_{g} \\ \dot{\mathbf{U}}_{f} \\ \dot{\mathbf{P}}
\end{Bmatrix}
+
\begin{Bmatrix}
\mathbf{F}_{s}^{int}\left(\mathbf{U},\dot{\mathbf{U}}\right) \\ \mathbf{F}_{h}^{int}\left(\mathbf{U},\dot{\mathbf{U}}\right) \\ \mathbf{F}_{v}^{int}\left(\mathbf{U},\dot{\mathbf{U}}\right) \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0}
\end{Bmatrix}
$$

$$
+
\begin{bmatrix}
\mathbf{K}_{ss} & \mathbf{K}_{sh} & \mathbf{K}_{sv} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{S}_{s}^{T} \\
\mathbf{K}_{hs} & \mathbf{K}_{hh}+\mathbf{K}_{hh}^{g} & \mathbf{K}_{hv} & \mathbf{K}_{hv}^{g} & \mathbf{K}_{hg}^{g} & \mathbf{0} & \mathbf{0} \\
\mathbf{K}_{vs} & \mathbf{K}_{vh} & \mathbf{K}_{vv}+\mathbf{K}_{vv}^{uplift} & -\mathbf{K}_{vv}^{uplift} & \mathbf{0} & \mathbf{0} & -\mathbf{S}_{v}^{T} \\
\mathbf{0} & \mathbf{K}_{vh}^{g} & -\mathbf{K}_{vv}^{uplift} & \mathbf{K}_{vv}^{uplift}+\mathbf{K}_{vv}^{g} & \mathbf{K}_{vg}^{g} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{K}_{gh}^{g} & \mathbf{0} & \mathbf{K}_{gv}^{g} & \mathbf{K}_{gg}^{g}+\mathbf{K}_{gg}^{f} & \mathbf{K}_{gf}^{f} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{K}_{fg}^{f} & \mathbf{K}_{ff}^{f} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H}
\end{bmatrix}
\begin{Bmatrix}
\mathbf{U}_{s} \\ \mathbf{U}_{h} \\ \mathbf{U}_{v} \\ \mathbf{U}_{v}^{g} \\ \mathbf{U}_{g} \\ \mathbf{U}_{f} \\ \mathbf{P}
\end{Bmatrix}
$$

$$
+
\begin{bmatrix}
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{R}_{gg}^{f} & \mathbf{R}_{gf}^{f} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{R}_{fg}^{f} & \mathbf{R}_{ff}^{f} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}
\end{bmatrix}
\begin{Bmatrix}
\bar{\mathbf{U}}_{s} \\ \bar{\mathbf{U}}_{h} \\ \bar{\mathbf{U}}_{v} \\ \bar{\mathbf{U}}_{v}^{g} \\ \bar{\mathbf{U}}_{g} \\ \bar{\mathbf{U}}_{f} \\ \bar{\mathbf{P}}
\end{Bmatrix}
+
\begin{bmatrix}
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_{gg}^{f} & \mathbf{T}_{gf}^{f} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_{fg}^{f} & \mathbf{T}_{ff}^{f} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}
\end{bmatrix}
\begin{Bmatrix}
\bar{\bar{\mathbf{U}}}_{s} \\ \bar{\bar{\mathbf{U}}}_{h} \\ \bar{\bar{\mathbf{U}}}_{v} \\ \bar{\bar{\mathbf{U}}}_{v}^{g} \\ \bar{\bar{\mathbf{U}}}_{g} \\ \bar{\bar{\mathbf{U}}}_{f} \\ \bar{\bar{\mathbf{P}}}
\end{Bmatrix}
$$

$$
=
\begin{Bmatrix}
\mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}_{gg}^{f}\ddot{\mathbf{U}}_{g}^{*} \\ \mathbf{M}_{fg}^{f}\ddot{\mathbf{U}}_{g}^{*} \\ \mathbf{0}
\end{Bmatrix}
+
\begin{Bmatrix}
\mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{C}_{gg}^{f}\dot{\mathbf{U}}_{g}^{*} \\ \mathbf{C}_{fg}^{f}\dot{\mathbf{U}}_{g}^{*} \\ \mathbf{0}
\end{Bmatrix}
+
\begin{Bmatrix}
\mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{K}_{gg}^{f}\mathbf{U}_{g}^{*} \\ \mathbf{K}_{fg}^{f}\mathbf{U}_{g}^{*} \\ \mathbf{0}
\end{Bmatrix}
+
\begin{Bmatrix}
\mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{R}_{gg}^{f}\bar{\mathbf{U}}_{g}^{*} \\ \mathbf{R}_{fg}^{f}\bar{\mathbf{U}}_{g}^{*} \\ \mathbf{0}
\end{Bmatrix}
+
\begin{Bmatrix}
\mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{T}_{gg}^{f}\bar{\bar{\mathbf{U}}}_{g}^{*} \\ \mathbf{T}_{fg}^{f}\bar{\bar{\mathbf{U}}}_{g}^{*} \\ \mathbf{0}
\end{Bmatrix}
+
\begin{Bmatrix}
\mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ -\mathbf{P}_{g}^{*} \\ \mathbf{0} \\ \mathbf{0}
\end{Bmatrix}
\tag{6a}
$$

$$
\bar{\mathbf{P}}(t) = \int_{0}^{t}\mathbf{P}(\tau)\,d\tau \tag{6b}
$$

$$
\bar{\bar{\mathbf{P}}}(t) = \int_{0}^{t}\bar{\mathbf{P}}(\tau)\,d\tau = \int_{0}^{t}\int_{0}^{\tau}\mathbf{P}(\sigma)\,d\sigma\, d\tau \tag{6c}
$$

By solving Equation (6), the time history of the dynamic response of an unanchored cylindrical liquid storage tank on flexible ground can be obtained, taking into account the material and geometric nonlinearities of the system. Prior to the nonlinear dynamic analysis of the system, the static response of the structure and the liquid due to its own weight must be obtained. The static solution is used as the initial state of the system. The subsequent nonlinear dynamic analysis provides the solution to Equation (6).

3 Application

Previous studies have shown that the dynamic behaviour of liquid storage tanks is strongly influenced by the fluid-structure-soil interaction, and this phenomenon has been observed in real earthquakes. Therefore, earthquake response analysis considering the fluid-structure-soil interaction is essential to ensure the seismic safety of liquid storage tanks. In particular, when a heavy structure such as a liquid storage tank rests on a flexible ground, nonlinear behaviour of the ground can occur and significantly affect the response of the whole system. One of the factors to be considered in the analysis of the soil-structure interaction is the energy radiation into the far-field region of the ground. Therefore, this study follows the procedure described in Section 2 to analyse the earthquake response of an unanchored liquid storage tank considering the nonlinear fluid-structure-soil interaction.

The dimensions and material properties of the structure, fluid, and soil of an example liquid storage tank capable of storing 200,000 *kl* of liquid are given in Table 1. The finite-element analysis code ABAQUS is used for the earthquake response analysis. The geometric and material nonlinear behaviour of the structure is considered. The material nonlinearity is assumed bi-linear. To consider the fluid-structure interaction in Equations (2) and (3), elements capable of representing the acoustic behaviour of the fluid has been implemented as user elements. To calculate the soil-structure interaction force expressed in Equation (5), the near- and far-field regions of the soil are modelled by solid elements and PMDLs, which can consider energy radiation to the infinite domain. The base uplift is considered by the nonlinear spring as shown in Figure 1.

The earthquake ground motion shown in Figure 2 is used as rock outcrop motions. The peak ground acceleration of the input ground motion is assumed to be 0.16 g.

Figure 3 shows the time histories of the displacements at the top of wall and Figure 3 shows the yielding of the liquid storage tank wall. It can be concluded that the nonlinear earthquake response of an unanchored liquid storage tank on flexible soil can be successfully obtained using the nonlinear finite-element model for the fluid-structure-soil interaction system.

Parameters		Value
Radius		45 m
Liquid height		33.864 m
Free board		1.136 m
Wall thckness		2.47 cm
Roof thickness		2.47 cm
Structure	Density	7850 kg/m^3
	Young's modulus	208.9 GPa
	Poisson's ration	0.2
	Yield stress	515 MPa
	Plastic modulus	3.372 GPa
	Damping ratio	2 %
	Rayleigh damping parameter	$\alpha = 1.550\ s^{-1}$, $\beta = 0.155 \times 10^{-2}\ s$
Fluid	Density	480 kg/m^3
Soil	Depth to a bedrock	15 m
	S-wave velocity	300 m/s
	Poisson's ratio	0.333
	Density	2400 kg/m^3
	Damping ratio	5 %

Table 1: Properties of the liquid storage tank system.

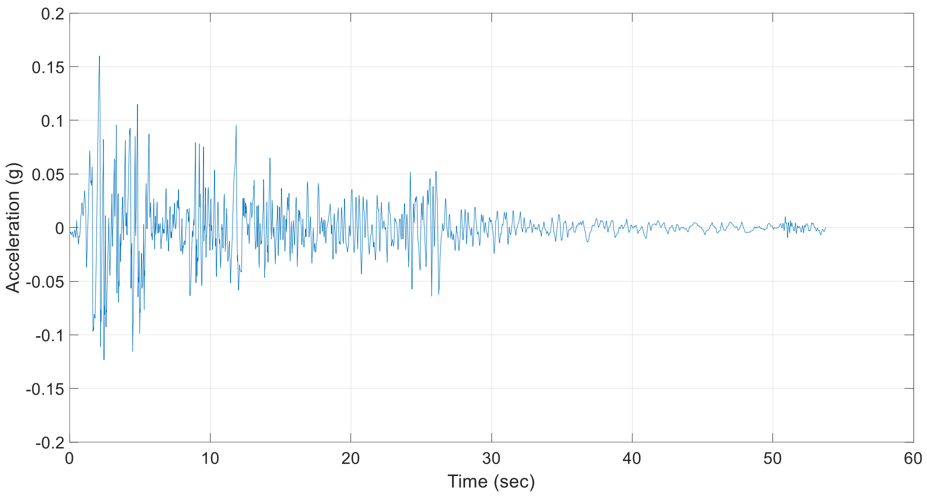

Figure 2: El Centro earthquake ground motion.

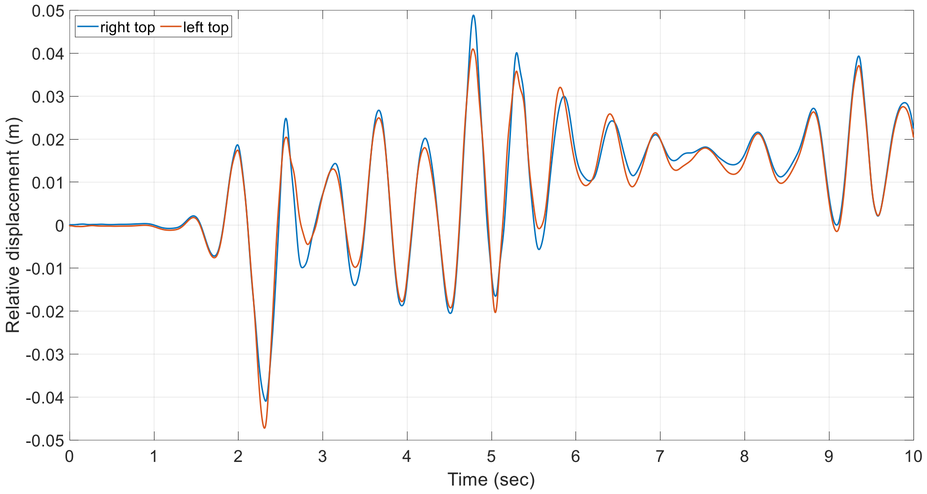

Figure 3: Time histories of the displacements at the top of wall.

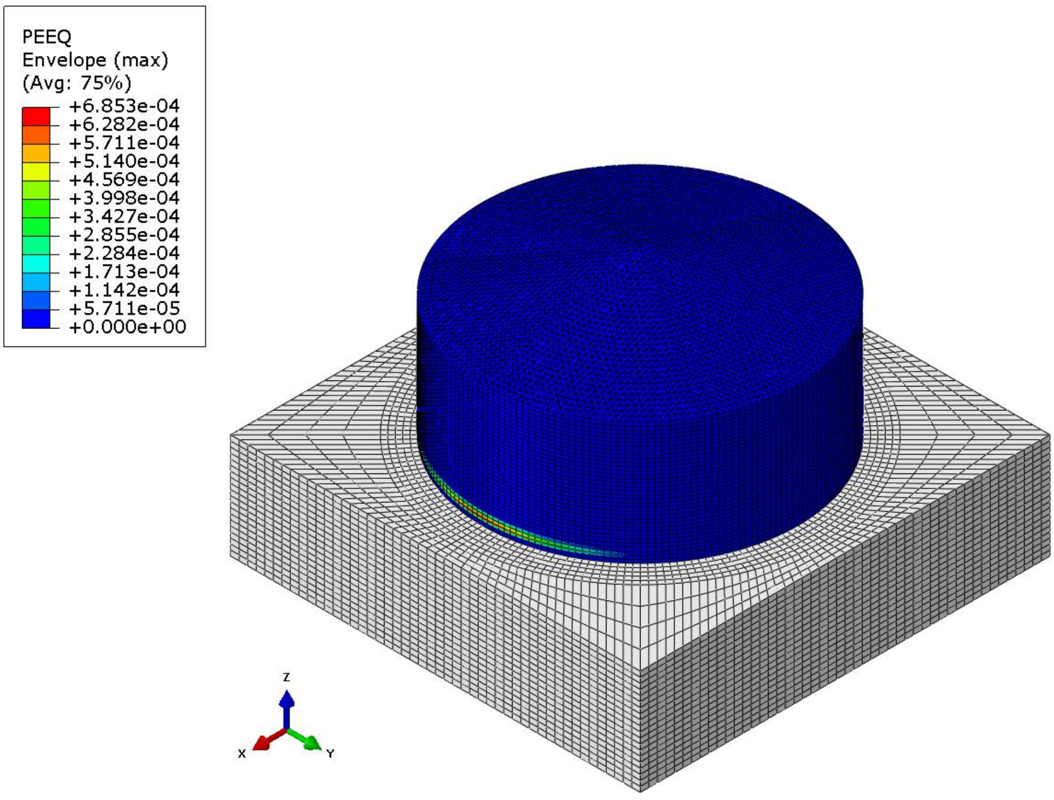

Figure 4: Yielding of the tank wall.

4 Conclusions

In this study, a nonlinear finite-element model was presented to reflect the dynamic characteristics of an unanchored cylindrical liquid storage tank resting on flexible ground. The tank structure was modelled using shell elements, which allowed geometric and material nonlinear behaviour to be taken into account. To consider the fluid-structure interaction, an acoustic element was implemented to represent the behaviour of the fluid, which was coupled to the structure using an interface element. To account for the soil-structure interaction, the near-field region of soil was modelled with solid elements and the far-field region was modelled with PMDL, which could account for energy radiation to infinity. The effective earthquake forces acting on the soil-structure interaction system were calculated by performing a free-field analysis of the ground.

Using the finite-element model, the nonlinear earthquake response analysis of the example unanchored liquid storage tank was performed to strictly consider the fluid-structure-soil interaction and base uplift. It can be concluded that the nonlinear earthquake response of an unanchored liquid storage tank on flexible soil can be successfully obtained using the proposed nonlinear finite-element model for the fluid-structure-soil interaction system. The earthquake response analysis method considering fluid-structure-soil interaction and base uplift developed in this study is expected to be useful for accurate seismic risk assessment of various liquid storage tanks and industrial facilities.

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