Proceedings of the Eighteenth International Conference on
Civil, Structural and Environmental Engineering Computing
Edited by: P. Iványi, J. Kruis and B.H.V. Topping
Civil-Comp Conferences, Volume 10, Paper 9.1
Civil-Comp Press, Edinburgh, United Kingdom, 2025
ISSN: 2753-3239, doi: 10.4203/ccc.10.9.1

A Study on Parametric Latent Dynamics Identification for Aerodynamic Flow Modelling

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Abstract

Learning physical models from full-order data has become an effective approach to reducing computational costs in various engineering problem. This paper presents a data-driven framework for dynamical modeling of parameterized vortex strength. The framework employed model-order reduction method with parametric sparse identification of nonlinear dynamics (SINDy) to construct a low-dimensional dynamical model. The training data covering the parameter space are represented by both linear and nonlinear model-order reduction method. The application of vortex panel method in vortex strength shows the effectiveness and accuracy of proposed framework.

Keywords: potential flow, latent space dynamics identification, data-driven method, model order reduction, proper orthogonal decomposition, autoencoder.

1 Introduction

In the modern era, physical simulations play an essential role in significant developments in engineering, technology, and science faster than ever before. It provides solutions of full-order models (FOMs) or high-fidelity mathematical representations of real-world physical systems capturing detailed dynamics with minimal simplification. These models are typically derived from first principal equations, such as Navier-Stokes equations in fluid dynamics or finite element method in structural mechanics. While FOMs provide accurate and comprehensive simulations, they often require significant computational resources due to their large number of degrees of freedom. As a result, performing full-order simulations for real-time applications, optimization, uncertainty quantification, or digital twins may be impractical.

To overcome this challenge, several surrogate models or data-driven models have been developed to approximate FOMs behaviour while keeping high-fidelity and lowering computational cost. One of those models is projection-based reduced order models (pROMs), in which the linear or nonlinear compression techniques are employed to approximate the full state domains. While the former includes proper orthogonal decomposition (POD) [1], and reduced basis method [2], the latter is auto-encoder [3-5]. The compression-based pROMs have been successfully applied to various problems, such as nonlinear heat conduction problem, Boltzmann transport problem, Naver-Stokes equation, computational fluid dynamics simulations for aerospace, and aero-elastic wing design. pROMs are split in two classes [6]: intrusive methods, in which one deals with directly the governing equations, and nonintrusive methods that only requires simulation data to approximate the full state fields. Because the flexibility of nonintrusive methods by using pure data, in this paper, we will focus on developing those methods to approximate the full state field using data

In terms of nonintrusive methods, numerous approximation techniques are used to construct a nonlinear model that forecasts new outputs from new inputs. These include, for example, Gaussian processes, Kriging, radial basis functions, and deep neural networks. They have been applied to physical problems as mentioned earlier and neural networks currently has become the most well-known framework because of their wealthy demonstration capability assisted by the universal approximation theorem. Nevertheless, these methods somehow exist limitations in means of interpretability due to the black-box nature. Furthermore, the generalization of surrogate models also is considered.

To enhance the interpretation and generalization, we employ a data-driven framework developed by Fries [7] for dynamical modelling of parameterized vortex strength that can compress the entire state field data into a reduced space. There are two common different types of compression such as linear and nonlinear. The prevalent linear one can be accomplished by the proper orthogonal decomposition (POD). The nonlinear one can be realized by the autoencoder (AE) in which the encoder and decoder are constructed by neural networks. Indeed, once the compression is performed, the data size is reduced significantly. Furthermore, the complexity of dynamic systems within the reduced space becomes simpler than that of the FOMs. Besides, to achieve simulation data, the panel method is carried out in order to resolve the full state field of vortex strength.

2 Methodology

2.1 Panel method

The panel methods assume irrotational and incompressible flow around the airfoil configuration. Therefore, they can be used to solve Laplace's equations for potential flow in the vortex field. Considering the constant strength vortex (γ) distribution on the airfoil surface, the velocity potential can be expressed as:

$$\phi_i^* = -\frac{\gamma}{2\pi}\int \tan^{-1}\frac{z}{x-x_0}\,dx_0, \tag{1}$$

where γ is vortex strength, and x and z are collocation points on the surface panel. The integral equation can be solved by applying the Dirichlet boundary condition to each of the collocation points in Eq. 2, where the total inner potential can be set equal to the freestream potential. It is derived from the requirement that the tangential velocity on the boundary surface should be defined by freestream and panel geometry. For steady flow condition, it can be expressed as Eq. 3. Here, β_i is an angle between the freestream direction and normal direction to the panel as shown in Fig. 1.

$$\phi_i^* = (\phi + \phi_\infty) = \phi_\infty \tag{2}$$

$$\frac{\partial \phi_i^*}{\partial s_i} = V_\infty \sin\beta_i \tag{3}$$

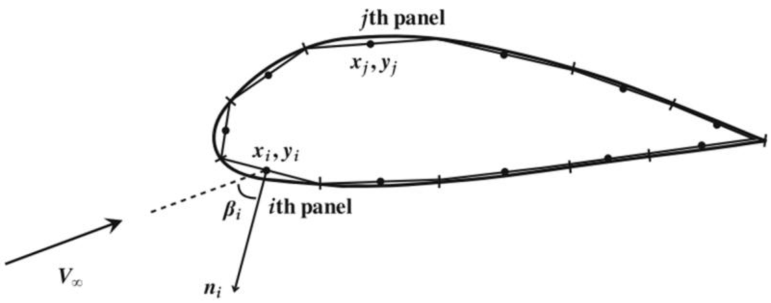


Fig. 1 Vortex panel distribution over the surface of body

Eq. 1 for the unknown vortex strength on the airfoil surface can be established as linear algebraic equation, as shown in Eq. 4 by enforcing the Kutta condition at the trailing edge as in Eq. 5. From the solution of linear equation, the pressure distribution on the airfoil surface and aerodynamic loads can be evaluated.

$$\sum_{i=1}^{N} A_i\gamma_i = V_\infty\, sin\,\beta \tag{4}$$

$$\gamma_1 + \gamma_N = 0 \tag{5}$$

2.2 Latent space dynamics identification

Latent dynamics identification is a powerful data-driven approach for discovering low-dimensional, interpretable dynamical systems from high-dimensional simulation data. In the context of aerodynamic flow modelling, especially in potential flow governed by inviscid and incompressible assumptions, this approach enables efficient yet accurate characterization of system dynamics across a range of parametric conditions. Our study adopts a reduced-order modelling (ROM) strategy that first projects full-order data onto a latent space and then identifies the dynamics in that reduced space using sparse regression techniques.

Let $u(t;\, \mu) \in \mathbb{R}^N$ denote the full-order state vector (e.g., surface potential values or flow-related observables) at time t under parameter vector $\mu \in \mathbb{R}^d$, where N is degrees of freedom due to spatial discretization. The system evolves according to unknown high-dimensional dynamics,

$$\frac{du}{dt} = F(u;\, \mu), \tag{6}$$

where F is typically derived from the governing PDEs (such as the Laplace equation in the case of potential flow) and may include non-trivial boundary conditions and parametric dependencies (e.g., angle of attack, geometry, panel positions).

To obtain a tractable model, we first reduce the dimensionality of u via a projection-based method. Assume $u \approx \bar{u} + \Phi z$, where $\bar{u}$ is the mean state, $\Phi \in \mathbb{R}^{N\times r}$ is a basis matrix (constructed via proper orthogonal decomposition (POD) or autoencoders (AE)), and $z \in \mathbb{R}^r$ are latent coordinates representing the reduced dynamics with $r \ll N$.

In the case of POD, Φ is obtained by using singular value decomposition of the snapshot matrix $\mathrm{U} = [u_1, u_2, \dots, u_d]$

$$U = \Phi\Sigma V^T, \tag{7}$$

where $\Phi \in \mathbb{R}^{N\times N}$ and $\mathrm{V} \in \mathbb{R}^{d\times d}$are left and right singular vector, respectively. The basis vector $\{\phi_i\}$ form the singular value matrix Φ by selecting the leading r modes corresponding to the largest eigenvalues. Alternatively, autoencoders learn a nonlinear encoder $E\colon \mathbb{R}^N \rightarrow \mathbb{R}^r$ and decoder $D\colon \mathbb{R}^r \rightarrow \mathbb{R}^N$ such that $x \approx D\big(E(x)\big)$, capturing nonlinear structures in the data. In general, encoder and decoder is trained by minimizing those mean square error.

Therefore, the projection yields the latent space representation,

$$z(t;\, \mu) = \Phi^{\mathrm{T}}(u(t;\, \mu) - \bar{u}). \tag{8}$$

Our goal is to identify a governing system of the form

$$\frac{dz}{dt} = f(z;\, \mu), \tag{9}$$

where $f(\cdot)$ is an unknown, possibly nonlinear vector-valued function capturing the evolution of latent variables. Instead of assuming an a priori form for f, we construct a dictionary $\Theta(z)$ of candidate nonlinear functions (e.g., polynomials, trigonometric terms) and identify the active terms using Sparse Identification of Nonlinear Dynamics (SINDy) [8].

Given latent trajectories $Z = [z_1, \dots, z_T]^T$ and their time derivatives $\dot{Z} = [\dot{z}_1, \dots, \dot{z}_T]^T$, the SINDy regression problem is formulated as

$$\dot{Z} = \Theta(Z, \mu)\Xi + \varepsilon, \tag{10}$$

where $\Xi \in \mathbb{R}^{K\times r}$ is the sparse coefficient matrix that maps features to each latent variable's dynamics, and ε is the residual error. To enforce sparsity in Ξ, an optimization problem of the following form is solved

$$\min_{\Xi} \left\| \dot{Z} - \Theta(Z, \mu)\Xi \right\|_2^2 + \lambda \|\Xi\|_1, \tag{11}$$

where $Z \in \mathbb{R}^{M\times r}$ is the latent trajectory matrix collected at M time snapshots, $\dot{X}$ is the corresponding time derivative (computed via numerical differentiation or learned jointly), $\Xi \in \mathbb{R}^{K\times r}$ contains the sparse coefficients, and λ is a regularization parameter that promotes sparsity. Each column of Ξ represents the dynamics of one latent variable as a sparse linear combination of candidate terms.

Figure 2 illustrates the overall framework of latent space dynamics identification based on autoencoder- or POD-based dimensionality reduction followed by sparse regression via SINDy.

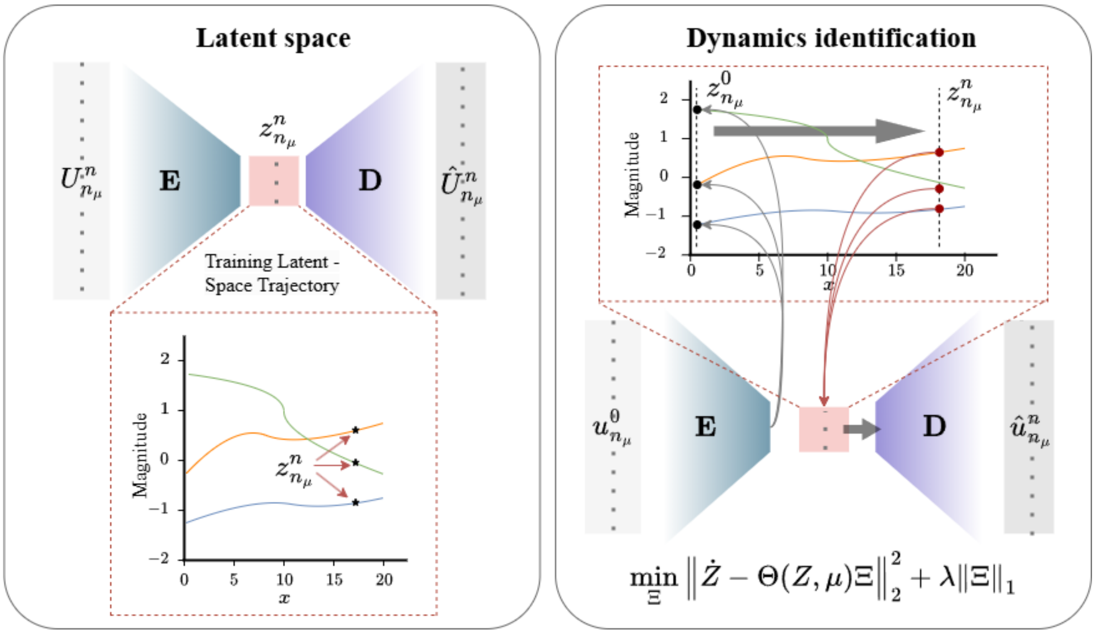


Fig. 2 Overview of latent space dynamic identification

One of the main advantages of operating in the latent space is the ability to model long-term flow evolution efficiently, especially for problems where vortical or unsteady effects dominate but are still low-dimensional. For example, in our vortex panel method-based simulations, the evolution of surface potential coefficients under varying angles of attack and time-varying inflow can be captured by a latent system with just a few degrees of freedom, without sacrificing accuracy.

Importantly, this framework is parametrically aware. Training data is generated by sampling across the parametric space μ, and the resulting library Θ may be augmented to include parametric terms explicitly, such as linear interactions with μ or cross-terms between z and μ. This allows the learned dynamical system to generalize across parameter values, enabling predictive modelling in unseen scenarios.

To ensure robustness, we apply a two-level cross-validation scheme: one across time (to avoid overfitting local transients), and another across parameter space (to ensure generalizability). Reconstruction of the full-order field is achieved through the inverse projection $u \approx \bar{u} + Wz$, enabling comparison with high-fidelity results and error quantification via standard norms (e.g., relative L^2 error).

Overall, latent dynamics identification using sparse regression in reduced-order coordinates offers a scalable and interpretable alternative to black-box sequence models such as recurrent neural networks. In the context of aerodynamic flow modelling, it provides a principled pathway for real-time surrogate modelling, control-oriented representations, and design optimization.

3 Numerical Results

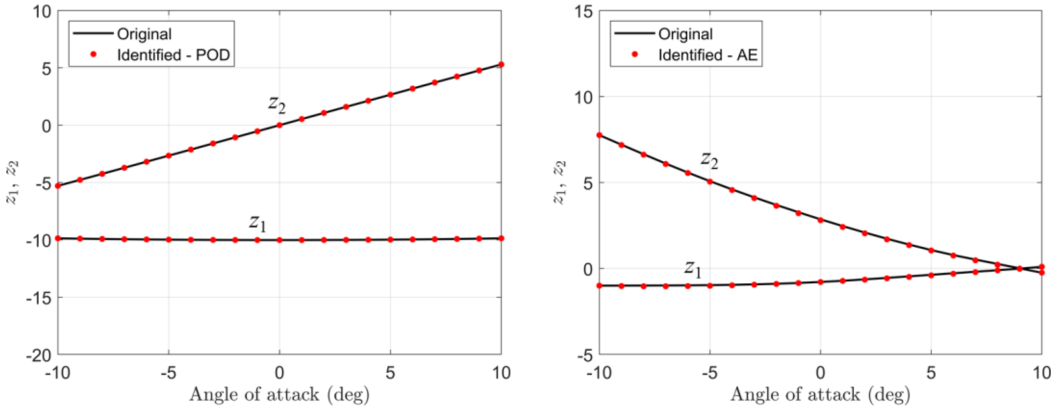
To evaluate the performance of latent space dynamics identification for aerodynamic flow modelling, the symmetric airfoil, van de Vooren airfoil with 15% thickness, is considered. The chord is unit length and the freestream velocity is 1 m/s. The airfoils are uniformly divided into 90 panels to obtain the full order model solution.

The FOMs are solved according to angle of attack $\alpha \in \{-10, -9, \ldots, 10\}°$, and total dimension of training data is $\mathrm{U} \in \mathbb{R}^{90\times 21}$. The accuracies of frameworks are verified for reproduction of training data and it is measured by the normalized root mean squared error (NRMSE), which is defined as

$$e(\boldsymbol{\alpha}) = \frac{1}{u_{max} - u_{min}} \sqrt{\frac{\sum_{i=1}^{N}(u_i - \tilde{u}_i)^2}{N}}. \tag{12}$$

Figure 3 shows the latent space of potential flow and dynamic identification according to the angle of attack. In this paper, the vortex strength is compressed into latent space $u \in \mathbb{R}^{2\times 21}$ using 2 POD basis and the encoder of AE. Using the dynamics identification [8], both latent spaces are represented by first order polynomial equation. This shows that dynamic identification is able to capture the latent spaces of potential flow.

In Fig. 4, the pressure coefficient distributions of airfoil surface at $\alpha = 5°$ obtained by both linear and non-linear model order reduction are compared with the corresponding FOM solution and analytical solution [9], showing that they are almost identical. Figure 5 shows the NRMSE of vortex strength reproduced by each MOR methods. The maximum NRMSE of POD is 1.11×10^{-9}, while the maximum NRMSE for AE is 7.4×10^{-3}. Therefore, this shows that the frameworks with both MOR methods can model the aerodynamic flow based on potential flow.



(a) POD (b) AE

Figure. 3 True and identified latent spaces using MOR methods and SINDy

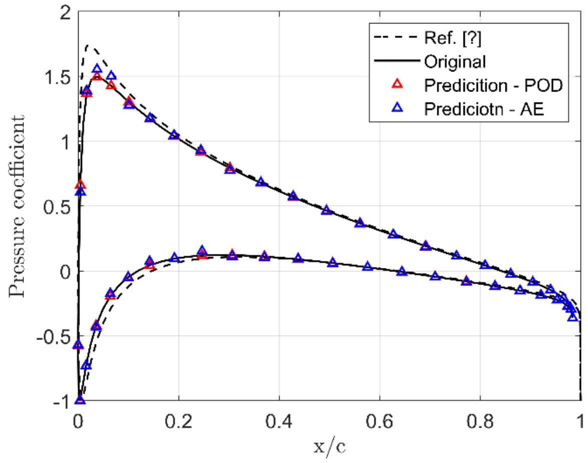
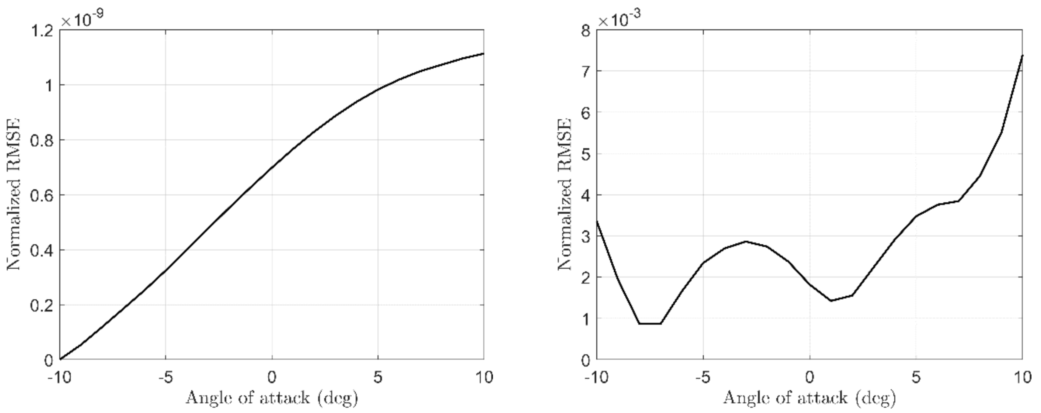


Figure. 4 Reconstruction of pressure coefficients at $\alpha = 5°$



(a) POD (b) AE

Figure. 5 NRMSE of reproduction of vortex strength according to MOR methods

4 Conclusions and Contributions

The application of latent space dynamics identification for aerodynamic flow modelling is performed using the penal method of potential flow. The aerodynamic flow is modelled by constant strength vortex element of panel method. To represent the latent spaces of aerodynamic flow, the linear and non-linear MOR methods such as POD and autoencoder are employed, and the latent spaces are identified by first order polynomial equation using SINDy.

The aim is to reproduce the training data in order to evaluate the frameworks using both linear and nonlinear MOR methods. For the symmetric airfoil, the training data is collected according to the angle of attack, and the present data-driven frameworks show that it is able to reproduce the training data with high accuracy, showing that the maximum NRMSE is less than 7.4×10^{-3}.

In future work, parameterization of both frameworks will be considered to include the unsteady potential flow induced by variations in freestream velocity.

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