Proceedings of the Eighteenth International Conference on
Civil, Structural and Environmental Engineering Computing
Edited by: P. Iványi, J. Kruis and B.H.V. Topping
Civil-Comp Conferences, Volume 10, Paper 7.5
Civil-Comp Press, Edinburgh, United Kingdom, 2025
ISSN: 2753-3239, doi: 10.4203/ccc.10.7.5

Capsizing Risk Assessment of Nonlinear Ship Roll Motion Under Evolutionary Sea-Wave Excitation

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Abstract

An approximate analytical technique is developed for assessing the capsizing risk of ships rolling in beam seas subjected to non-white sea-wave excitations. The ship motion is modeled as a nonlinear roll system incorporating both softening and hardening restoring characteristics, nonlinear damping, and evolutionary stochastic excitation representative of ocean conditions. A stochastic averaging method is employed to derive time-dependent seakeeping probabilities in a computationally efficient manner. The method accounts for both bounded and unbounded ship roll motion associated with negative stiffness regions by introducing a tailored form of the non-stationary response amplitude probability density function (PDF), specifically designed to capture this critical ship dynamics behavior. A notable strength of the approach is its ability to handle stochastic excitations with time-varying intensity and frequency content, as commonly encountered in open-sea environments. Numerical examples involving nonlinear ship roll models are presented whereas comparisons with pertinent Monte Carlo simulation data demonstrate the efficiency and accuracy of the proposed technique.

Keywords: nonlinear ship rolling, statistical linearization, capsizing failure, stochastic averaging, seakeeping reliability, sea-wave evolutionary power spectrum

1 Introduction

Stability has always been a central concern in naval architecture, traditionally addressed through static criteria to ensure safe vessel operation. However, static analyses alone are insufficient to capture the complex dynamic behaviors encountered in real-world maritime environments, particularly under the influence of large-amplitude sea waves. In response to increased operational demands and safety standards, recent research has shifted toward dynamic stability assessments, with particular focus on the mechanisms of ship capsizing and excessive roll motions in beam seas. As seakeeping performance plays a critical role in determining operational limits, there is a growing need for tools that not only describe ship motion in adverse open sea conditions but also quantify the associated risks. Motivated by the need to quantify safety margins under such conditions, this work adopts a reliability-based perspective to assess the risk of dynamic instability in nonlinear ship rolling. The ship dynamics are modeled with respect to realistic nonlinear restoring and nonlinear damping characteristics whereas the influence of stochastic, non-white sea-wave excitations is explicitly considered. To ensure physical relevance and compatibility with current standards, the wave excitation is modeled using a JONSWAP-type sea spectrum, widely used in modern offshore and maritime design codes (e.g., [1, 2]).

Despite advances in design practices and safety regulations, incidents of ship capsizing continue to result in significant losses of vessels, cargo, and human life. These extreme events are often driven by rare but critical dynamic responses that exceed stability thresholds, making them well-suited to analysis through the lens of the first-passage problem (e.g., [3, 4]). In this regard, the likelihood of capsize is defined as the probability that the ship roll response exceeds a critical threshold for the first time under evolutionary stochastic excitation, assuming no prior exceedance has been occurred. This formulation provides a meaningful and quantifiable measure of the vessel seakeeping reliability. To assess the probability of such critical events, advanced Monte Carlo simulation (MCS) techniques have been widely applied in reliability analysis (e.g., [5]). However, in the context of complex nonlinear models (e.g., [6–8]) subjected to evolutionary stochastic excitation, MCS can become computationally burdensome, particularly when estimating higher-order statistical quantities such as response probability density functions (PDFs). This challenge motivates the development of efficient approximate analytical methods (e.g., [9]), frameworks related to modeling the response as an one-dimensional Markov process (e.g., [10]), probability density evolution schemes (e.g., [11]), and stochastic averaging/linearization techniques (e.g., [12–14]).

The nonlinear nature of ship rolling introduces substantial modeling challenges, particularly when attempting to capture the physics of large-amplitude motions and potential capsizing events. Although often phenomenological, nonlinear roll models employing odd-order polynomial representations of the restoring moment have proven effective in characterizing the essential dynamics of ship behavior under beam wave excitations (e.g., [15]). These formulations reflect key physical phenomena such as restoring asymmetry, softening and hardening effects, and loss of stiffness at high roll angles. In the present work, the nonlinear stiffness is represented using polynomial expressions up to fifth order, offering a balance between physical realism and analytical tractability while coupled with evolutionary stochastic wave excitation models dictated by modern maritime design codes. Additionally, nonlinear damping effects play a critical role in limiting large roll amplitudes and capturing energy dissipation mechanisms, and are incorporated to enhance the fidelity of the modeling. Lastly, the method rigorously captures both bounded and unbounded dynamic rolling by introducing a tailored form of the non-stationary response amplitude PDF, explicitly designated to reflect the critical features of nonlinear ship roll dynamics.

2 Mathematical Formulation

This section outlines the mathematical foundations underlying the proposed efficient stochastic dynamics vessel seakeeping reliability methodology. Emphasis is placed on the modelling assumptions and simplifications introduced to facilitate numerical efficiency, while maintaining consistency with the adopted complex nonlinear governing equation for ship rolling motion.

2.1 Sea-wave evolutionary excitation spectrum

The induced sea-wave excitation is modeled as a zero-mean Gaussian non-stationary stochastic process characterized by an evolutionary power spectrum (EPS). This modeling approach captures the time-varying distribution of energy in the roll-moment excitation spectrum and is defined as

$$S_w(\omega, t) = |g(t)|^2 |F_{\text{roll}}(\omega)|^2 S_{\text{JS}}(\omega) \tag{1}$$

where $S_{\text{JS}}(\omega)$ denotes the stationary JONSWAP wave-energy spectrum, $F_{\text{roll}}(\omega)$ is a frequency-dependent transfer function that maps wave energy into roll-moment excitation, and $g(t)$ is a time envelope function that introduces non-stationarity into the process. Particular attention is given to the JONSWAP model [16], which following the IEC 61400-3 guidelines [2], produces a narrow-banded spectrum with a pronounced peak at the dominant wave frequency. The inclusion of the function $F_{\text{roll}}(\omega)$, however, has been shown to broaden the band of the excitation spectrum. Subsequently, the adopted JONSWAP spectrum takes the form

$$S_{JS}(\omega) = 0.3125 T_p H_s^2 \left(\frac{\omega}{\omega_p}\right)^{-5} \exp\left[-1.25\left(\frac{\omega}{\omega_p}\right)^{-4}\right] M(\omega) \tag{2}$$

where $\omega_p = 2\pi/T_p$ is the peak frequency, and the term $M(\omega)$ defines the spectral peak enhancement

$$M(\omega) = (1 - 0.287 \log\gamma)\gamma^{\exp\left[-\frac{1}{2}\left(\frac{\omega/\omega_p - 1}{\sigma}\right)^2\right]} \tag{3}$$

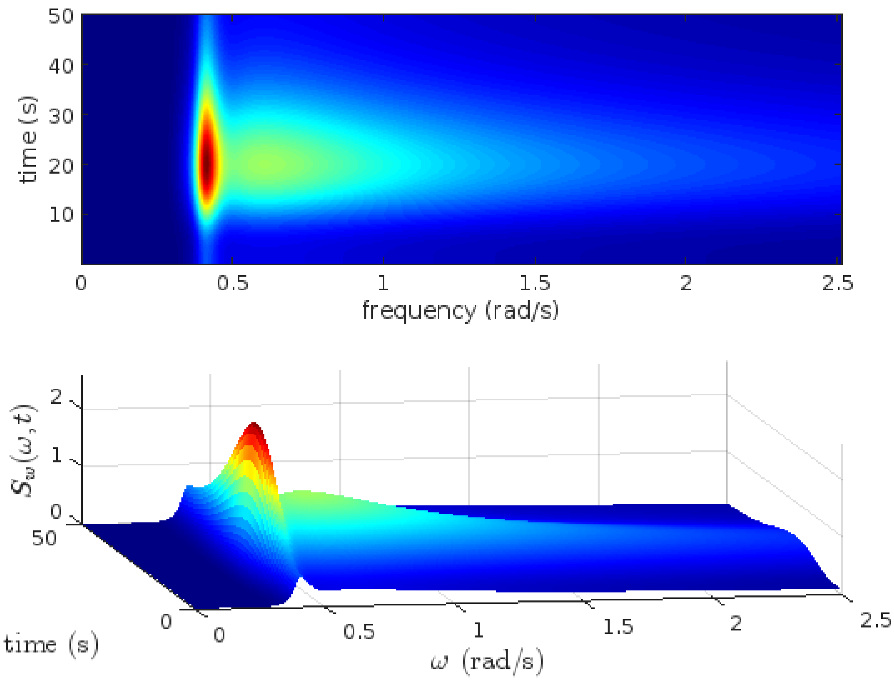

Figure 1: Evolutionary roll-moment excitation spectrum $S_w(\omega, t)$

In line with IEC 61400-3 (e.g., [2, 17]) and established practice, the peak-shape parameter is set to γ=3.3, with $\sigma = 0.07$ for frequencies below the peak frequency ω_p and $\sigma = 0.09$ otherwise. The peak period T_p and significant wave height H_s are determined from representative measurements for demonstration purposes in the numerical application part. The function F_{roll}, is defined as $|F_{\text{roll}}|^2 = C\omega^4$ with $C = 3$ reflecting beam-sea loading conditions and system properties. To model non-stationarity, the time envelope function $g(t)$ is adopted in the form

$$g(t) = \left\{ 0.2 + 0.8 \times \left[\frac{t}{a} \exp\left(1 - \frac{t}{a}\right) \right]^b \right\}^{0.5} \tag{4}$$

where the shaping parameters are chosen as $\alpha = 20$ and $b = 5$ governing the growth and decay characteristics of the envelope function. The resulting EPS is shown in Fig. 1, where the prominent narrow-band peak of the JONSWAP spectrum is evident. The broadening of the high-frequency tail arises due to the presence of the function F_{roll} which works towards injecting additional energy into the higher-frequency range of the excitation profile.

2.2 Governing nonlinear ship rolling motion equation

The ship rolling under stochastic sea-wave excitation is governed by a second-order differential equation featuring linear and nonlinear damping terms, along with a nonlinear restoring moment. While the restoring moment is commonly approximated using odd-order polynomials, damping formulations vary widely across the literature (e.g. [12, 15, 18]). In the herein study, we adopt a formulation proposed by Taylan, in which the roll motion is modeled using a quintic polynomial representation for the righting arm (GZ) curve, combined with a B1-type damping scheme as appear in [15]. The governing dynamics nonlinear ship rolling equation is expressed as

$$(I_{xx}+\delta I_{xx})\ddot{\phi}(t)+B_L\dot{\phi}(t)+B_N\dot{\phi}(t)|\dot{\phi}(t)|+\Delta(C_1\phi(t)+C_2\phi^3(t)+C_3\phi^5(t)) = S_w(\omega,t) \tag{5}$$

Here, $\phi(t)$ is the roll angle, I_{xx} is the ship roll moment of inertia, and B_L and B_N are the linear and nonlinear damping coefficients, respectively. Note that the squared velocity term is expressed as $\dot{\phi}(t)|\dot{\phi}(t)|$, ensuring that the damping force always opposes the motion. This formulation guarantees that, regardless of the sign of $\dot{\phi}(t)$, the damping moment remains directed opposite to the roll velocity, thereby accurately modeling energy dissipation due to nonlinear hydrodynamic effects. The coefficients C_1, C_3, C_5 correspond to the linear, cubic, and quintic terms of the restoring moment, derived from the ship GZ curve. These are defined as

$$C_1 = \frac{d(GZ)}{d\phi} = GM \tag{6}$$

$$C_3 = \frac{4}{\phi_v^4}(3A_{\phi v} - GM\phi_v^2) \tag{7}$$

$$C_5 = -\frac{3}{\phi_v^6}(4A_{\phi v} - GM\phi_v^2) \tag{8}$$

In these expressions, GM denotes the metacentric height, ϕ_v is the vanishing stability angle, and $A_{\phi v}$ is the area under the GZ curve. By dividing through $I_{xx} + \delta I_{xx}$ and substituting the C_1, C_3, C_5, Eq.(5) is reformulated to emphasize the role of nonlinear contributions in the ensuing reliability analysis. The nonlinear components are each scaled by distinct weighting factors ε_1, ε_2, and ε_3, yielding the final form

$$\ddot{\phi}(t) + b_L\dot{\phi}(t) + \varepsilon_1 b_N\dot{\phi}(t)|\dot{\phi}(t)| + \omega_\phi^2\phi(t) + \varepsilon_2 m_3\phi^3(t) + \varepsilon_3 m_5\phi^5(t) = S_w(\omega,t) \tag{9}$$

where the normalized coefficients are given by

$$\omega_\phi^2 = \frac{\Delta GM}{I_{xx} + \delta I_{xx}} \tag{10}$$

$$m_3 = \frac{4\omega_\phi^2}{\phi_v^2}\left(\frac{3A_{\phi_v}}{GM\phi_v^2} - 1\right) \tag{11}$$

$$m_5 = -\frac{3\omega_\phi^2}{\phi_v^4}\left(\frac{4A_{\phi v}}{GM\phi_v^2} - 1\right) \tag{12}$$

$$b_L = \frac{B_L}{I_{xx} + \delta I_{xx}} \tag{13}$$

$$b_N = \frac{B_N}{I_{xx} + \delta I_{xx}} \tag{14}$$

The weighting factors introduce modeling flexibility by enabling the representation of physically meaningful damping and restoring behaviors. They enhance adaptability, allowing the model to capture a wide range of ship-specific dynamic responses and design requirements.

2.3 Capsizing seakeeping reliability assessment

Having defined both the excitation stochastic process and the nonlinear roll motion equation, this subsection focuses on evaluating the ship capsizing probability; see also [3, 19]. The approach relies on a stochastic averaging treatment applied to the system dynamics, reducing the original nonlinear second-order differential equation to a first-order stochastic differential equation (SDE) governing the evolution of the roll angle amplitude. Assuming the system of Eq. (9) is lightly damped and excited by Eq. (1), it is expected to exhibit a pseudo-harmonic behavior under non-capsizing conditions, characterized by a slowly varying with time roll amplitude $A(t)$ and a slowly varying with time phase $\phi(t)$. Therefore, the roll angle satisfies

$$\phi(t) = A(t)\cos(\psi), \quad \dot{\phi}(t) = -\omega(A)A(t)\sin(\psi) \tag{15}$$

with ψ defined as:

$$\psi = \omega(A)t + \theta(t) \tag{16}$$

Here, $A(t)$ and $\theta(t)$ denote slowly time-varying functions, treated as constants over a single oscillation cycle. This enables the use of equivalent linearization (e.g., [20]), transforming Eq. (9) into

$$\ddot{\phi}(t) + \beta(A)\dot{\phi}(t) + \omega^2(A)\phi(t) = S_w(\omega, t) \tag{17}$$

The equivalent amplitude-dependent damping $\beta(A)$ and stiffness elements $\omega(A)$ are defined as

$$\beta(A) = b_L + \frac{S(A)}{A\omega(A)}, \quad \omega^2(A) = \frac{C(A)}{A} \tag{18}$$

with

$$C(A) = \frac{1}{\pi}\int_0^{2\pi} \cos\psi\Big(- \varepsilon_1 b_N \omega(A) A \sin\psi \cdot |-\omega(A)A\sin\psi| \\ + \omega_\phi^2 A\cos\psi + \varepsilon_2 m_3 (A\cos\psi)^3 + \varepsilon_3 m_5 (A\cos\psi)^5\Big)\,\mathrm{d}\psi \tag{19}$$

and

$$S(A) = -\frac{1}{\pi}\int_0^{2\pi} \sin\psi\Big(- \varepsilon_1 b_N \omega(A) A \sin\psi \cdot |-\omega(A)A\sin\psi| \\ + \omega_\phi^2 A\cos\psi + \varepsilon_2 m_3 (A\cos\psi)^3 + \varepsilon_3 m_5 (A\cos\psi)^5\Big)\,\mathrm{d}\psi \tag{20}$$

Assuming the roll angle amplitude $A(t)$ follows a truncated Rayleigh distribution of the form

$$p(A,t) = \frac{A}{c(t)}\exp\left(-\frac{A^2}{2c(t)}\right)\text{rect}(A) + \exp\left(-\frac{A_{cr}^2}{2c(t)}\right)\delta(A - A_\infty) \tag{21}$$

where $\text{rect}(A) = u(A) - u(A - A_{cr})$, with $u(\cdot)$ denoting the unit step function, $c(t)$ is a coefficient to be determined, and $\delta(\cdot)$ is the Dirac delta function. Considering the amplitude-dependent elements in Eq. (17) are approximated by corresponding time-varying elements forms

$$\ddot{\phi}(t) + \beta_{\text{eq}}(t)\dot{\phi}(t) + \omega_{\text{eq}}^2(t)\phi(t) = S_w(\omega, t) \tag{22}$$

where the time-varying equivalent elements are

$$\beta_{\text{eq}}(t) = b_L + \int_0^\infty \frac{S(A)}{A\omega(A)}\, p(A,t)\,\mathrm{d}A \tag{23}$$

and

$$\omega_{\text{eq}}^2(t) = \int_0^\infty \omega^2(A)p(A,t)\,\mathrm{d}A \tag{24}$$

Based on the nature of the roll angle amplitude PDF $p(A,t)$, the time-dependent elements comprise two parts; the bounded part, for $A \in [0, A_{cr}]$, and the unbounded part for $A \in (a_{cr}, \infty)$, which may lead to capsizing. In this context, the bounded equivalent stiffness $\omega_{eq,B}^2(t)$ element is given by

$$\omega_{eq,B}^2(t) = \int_0^{A_{cr}} \omega^2(A)p(A,t)\mathrm{d}t, \tag{25}$$

while the corresponding bounded equivalent damping $\beta_{eq,B}(t)$ element reads

$$\beta_{eq,B}(t) = \int_0^{A_{cr}} \beta(A)p(A,t)\mathrm{d}t. \tag{26}$$

To evaluate ship capsizing reliability, the critical roll angle amplitude A_{cr} is needed; this is defined as $\omega_{\text{eq}}^2(A_{\text{cr}}) = 0$. Notably, capsizing is noted when the roll amplitude A exceeds this critical threshold A_{cr}, leading to negative values of stiffness assisting the capsizing. In this setting, Eq.(23) and Eq.(24) yield:

$$\beta_{\text{eq,B}}(t) = b_L + \int_0^{A_{\text{cr}}} \frac{8}{3\pi}\,\varepsilon_1 b_N A\left(\omega_\phi^2 + \frac{3}{4}\varepsilon_2 A^2 m_3 + \frac{5}{8}\varepsilon_3 A^4 m_5\right)^{1/2} p(A,t)\,\mathrm{d}A \tag{27}$$

and

$$\begin{aligned}\omega_{\text{eq,B}}^2(t) = \omega_\phi^2 &+ \frac{3}{2}\varepsilon_2 c(t)m_3 + 5\varepsilon_3 c^2(t)m_5 \\ &- \frac{A_{\text{cr}}^2\left(5\varepsilon_3 m_5 A_{\text{cr}}^2 + 6\varepsilon_2 m_3 + 20\varepsilon_3 c(t)m_5\right)}{8\left(S(t) - 1\right)}\end{aligned} \tag{28}$$

where the time dependent factor $S(t)$ is determined by applying the normalisation condition $\int_0^\infty p(A,t) = 1$, and yields $S(t) = \exp[-A_{cr}^2/(2c(t))]$. A combination of deterministic and stochastic averaging yields a first-order SDE governing the evolution of the roll angle amplitude

$$\dot{A}(t) = -\frac{1}{2}\beta_{\text{eq,B}}(t)A(t) + \frac{\pi S_w(\omega_{\text{eq,B}}(t),t)}{2A(t)\omega_{\text{eq,B}}^2(t)} + \frac{[\pi S_w(\omega_{\text{eq,B}}(t),t)]^{1/2}}{\omega_{\text{eq.B}}(t)}\eta(t) \tag{29}$$

where $\eta(t)$ is a zero-mean and delta-correlated process of unit intensity, with $E(\eta(t)) = 0$; and $E(\eta(t)\eta(t+\tau)) = \delta(t)$ (e.g., [19, 21, 22]). Eq.(29) signifies that the amplitude process $A(t)$ is decoupled from the phase $\phi(t)$ and, thus, can be modeled as a one-dimensional Markov process, enabling the formulation of a Fokker-Planck equation that governs the associated response amplitude PDF

$$\begin{aligned}\frac{\partial p(A,t)}{\partial t} = &-\frac{\partial}{\partial A}\left\{\left(-\frac{1}{2}\beta_{\text{eq,B}}(t)A + \frac{\pi S_w(\omega_{\text{eq,B}}(t),t)}{2A\omega_{\text{eq,B}}^2(t)}\right)p(A,t)\right\} \\ &+ \frac{\pi S_w(\omega_{\text{eq,B}}(t),t)}{2\omega_{\text{eq,B}}^2(t)}\frac{\partial^2 p(A,t)}{\partial A^2}.\end{aligned} \tag{30}$$

Substituting the truncated Rayleigh PDF of Eq. (21) into Eq. (30), the following non-linear differential equation can be obtained for the computation of the time-varying $c(t)$

$$\dot{c}(t) = -\beta_{\text{eq,B}}(t)\,c(t) + \frac{\pi\,S_w(\omega_{\text{eq,B}}(t),t)}{\omega_{\text{eq,B}}^2(t)} \tag{31}$$

An adaptive discretization scheme is employed by dividing time into intervals of the form (e.g., [3, 12])

$$\begin{aligned}&[t_{i-1},\, t_i], \quad i = 1,2,\ldots,M, \quad t_0 = 0, \quad t_M = T, \quad \text{and} \\ &t_i = t_{i-1} + d_T T_{\text{eq}}(t_{i-1})\end{aligned} \tag{32}$$

where T_{eq} is the equivalent natural period of the ship rolling system $T_{\text{eq}}(t) = \frac{2\pi}{\omega_{\text{eq}}(t)}$ The survival probability $P_B(t)$ is computed via

$$P_B(T = t_M) = \prod_{i=1}^{M}(1 - F_i) \tag{33}$$

with F_i defined as the capsizing failure probability of the first-passage kind, meaning that the roll angle amplitude will cross the critical amplitude A_{cr} in the time interval $[t_{i-1}, t_i]$ given that no crossing has occurred prior to time t_{i-1}

$$F_i = \frac{\text{Prob}[a(t_i) \geq a_{cr} \cap a(t_{i-1}) < a_{cr}]}{\text{Prob}[a(t_{i-1}) < a_{cr}]} = \frac{Q_{i-1,i}}{H_{i-1}}, \tag{34}$$

where

$$H_{i-1} = \int_0^{A_{cr}} p(A_{i-1}, t_{i-1})dA_{i-1} \tag{35}$$

and

$$Q_{i-1,i} = H_{i-1} - \int_0^{A_{cr}} \int_0^{A_{cr}} p_{tr}(A_i, t_i | A_{i-1}, t_{i-1}) p(A_{i-1}, t_{i-1}) dA_i dA_{i-1} \quad (36)$$

In Eq. (36), p_{tr} stands for the transition amplitude PDF, provided in the form

$$p_{tr}(A_{i-1}, t_{i-1} \mid A_i, t_i) = \frac{A_i}{c(t_{i-1}, t_i)} \exp\left[-\frac{A_i^2 + h^2(t_{i-1}, t_i)}{2c(t_{i-1}, t_i)}\right] I_0\left(\frac{A_i h(t_{i-1}, t_i)}{c(t_{i-1}, t_i)}\right) \mathrm{rect}(A) \quad (37)$$

where $I_0(.)$ denotes the modified Bessel function of the first kind and zero order. The time-varying coefficients $h(t_{i-1}, t_i)$ and $c(t_{i-1}, t_i)$ [23] are given as

$$h(t_{i-1}, t_i) = A_{i-1}\sqrt{1 - \beta_{eq,B}(t_{i-1})\tau_i} \quad (38)$$

and

$$c(t_{i-1}, t_i) = \frac{\pi S_W\left[\omega_{\mathrm{eq,B}}(t_{i-1}), t_{i-1}\right]}{\omega^2_{\mathrm{eq,B}}(t_{i-1})}\tau_i \quad (39)$$

where $\tau_i = t_i - t_{i-1}$. This framework forms the basis for evaluating the time-dependent seakeeping reliability of a ship rolling under evolutionary sea-wave excitation processes, characterized by time-varying intensity and frequency content, as commonly encountered in open-sea environments.

3 Numerical results

This section presents the numerical results of the vessel seakeeping reliability assessment method based on the mathematical framework established in Section 2. Variations in the input significant wave height H_s as well as in the system weighting factors ε_1, ε_2, and ε_3, are considered. The results are compared against pertinent MCS data to validate the efficiency and reliability of the proposed approach. For all wave height scenarios, the excitation is characterized by a peak period of $T_p = 15.5$ s. The ship rolling parameters are $\omega_\phi = 1.5$ rad$/s$, $\phi_v = 0.96$ rad, $m_3 = -2.4$ s^{-2}, $m_5 = 0.2$ s^{-2}, $b_L = 0.042$ s^{-1}, and $b_N = 0.042$ (e.g., [24, 25]). The primary nonlinear damping weighting factor is held constant at $\varepsilon_1 = 0.1$, while variations in ε_2 and ε_3 are introduced with respect to the restoring force characteristics. To enhance the accuracy of the analysis under stronger excitations that induce pronounced nonlinear behavior—particularly when the equivalent natural period $T_{eq}(t)$ marches towards higher values—an adaptive time discretization step of $d_T = 0.1$ is prioritized. This preserves the validity of the assumption that the survival probability remains approximately constant over each time-interval (e.g., [3, 12]). Fig. 2 presents the time-dependent capsizing survival probability for a ship subjected to significant wave heights of $H_s = 6$ m and $H_s = 8$ m. The accuracy of the proposed methodology is assessed by juxtaposing its results with those obtained from Monte Carlo simulations involving 5,000 realizations (e.g., [26, 27]).

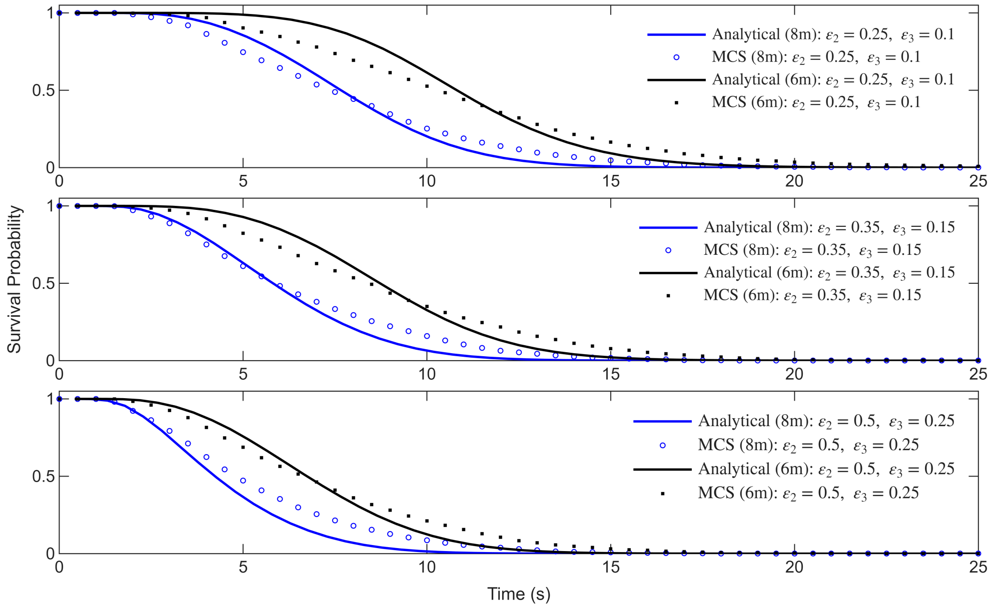

Figure 2: Capsizing survival probability of a nonlinear ship rolling system ($\varepsilon_1 = 0.1$) under evolutionary sea-wave excitation for noninal wave heights $H_s = 6$m and $H_s = 8$m; Comparisons with MCS data (5,000 realizations).

It can be noted that at any given time instant, the survival probability associated with the lower wave height condition ($H_s = 6\,$m) is consistently higher than that corresponding to the higher wave height ($H_s = 8\,$m). This observation aligns with engineering intuition, as lower wave heights impart reduced excitation energy to the system, thereby decreasing the likelihood of capsizing and improving the vessel seakeeping performance. The proposed methodology demonstrates a satisfactory degree of agreement with the MCS benchmark across a range of values for the considered weighting factors ε. Its consistent performance under varying wave heights further underscores the robustness of the approach in accommodating different excitation characteristics, rendering it well-suited for application across diverse and realistic sea states. In the presented results, specific values of ε_2 and ε_3 in the righting arm GZ curve were selected to ensure that the hardening effects associated with the fifth-order term remain of lower magnitude compared to the softening effects of the third-order term. This choice is further supported by the nature of the restoring coefficients m_3 and m_5, where m_3 is considerably larger in magnitude than m_5, reinforcing the dominance of softening behavior. The condition of $\varepsilon_2 < \varepsilon_3$ enables a balanced yet flexible representation of softening and hardening effects, fully consistent with ship roll modeling practices found in the literature (e.g., [15]). Mild to soft hardening behavior may also arise from small stabilizing fins mounted on the sides of large ships, which act like airplane wings and introduce limited restoring effects at larger roll angles. While this formulation facilitates a meaningful assessment of capsizing probability within the proposed seakeeping reliability framework, careful tuning of the damping and restoring weighting factors is essential to preserve physical realism and ensure the model captures relevant nonlinear roll dynamics. With these considerations in place, the proposed method offers a computationally efficient and practically applicable tool, particularly valuable in early-stage contexts aligned with performance-based analysis strategies.

4 Concluding remarks

This study proposes a novel framework to assess the capsizing survival probability of ships subjected to non-white stochastic sea-wave excitations in beam seas. The model incorporates nonlinear damping and both softening and hardening restoring moment characteristics, capturing essential dynamic behaviors of ship rolling under realistic ocean conditions. A key feature of the method lies in its ability to handle evolutionary stochastic excitation with varying intensity and frequency content, thus reflecting the nature of irregular sea states. The approach also addresses the challenge of unbounded roll responses by introducing a tailored non-stationary probability density function for the roll angle amplitude. The framework demonstrates significant computational efficiency and provides accurate estimates of seakeeping reliability when compared to benchmark Monte Carlo simulation results. By advancing the theoretical modeling of nonlinear ship rolling dynamics, this study contributes a practical and scalable tool for performance-based seakeeping analysis, especially in early-stage design or operational planning where full-scale simulations are often impractical.

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