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A Versatile Filter Model and Its Application in the Simulation of Track Irregularity and Fluctuating Wind Speed with Non-Rational PSD Functions

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Abstract

Environmental excitations acting on engineering structures, such as wind, ocean waves, and track irregularities, are often modelled as stationary Gaussian stochastic processes, with their statistical characteristics quantitatively described by the power spectral density functions. These excitations can be reproduced through stochastic simulation by filtering white noise using a constructed filter. A linear analog filter system is proposed in this study for simulating excitation power spectral density functions with non-rational characteristics. By introducing a fractional-order derivative operator, the proposed model can be applied to the simulation of turbulent wind speed with fractional-order asymptotic spectral properties. For track irregularity spectra with multiple segments, a corresponding piecewise form can be adopted in the model and efficient simulation can be achieved using a frequency-domain approach. The effectiveness and efficiency of the proposed method are validated through applications in the simulation of various commonly used fluctuating wind speed spectra and railway track irregularity spectra. To enhance adaptability, filter parameter conversion formulae are provided, allowing the benchmark model to be conveniently extended to scenarios with varied spectral parameters. The proposed method offers an efficient approach for the simulation of Gaussian stationary excitations with complex spectral properties. Further, it provides a foundation for stochastic response analysis and reliability analysis on the probability density level, say, by the method based on the dimension-reduced probability density evolution equation.

Keywords: stationary Gaussian processes, power spectral density functions, analog filters, fluctuating wind speed, track irregularity, fractional-order derivatives.

1 Introduction

Uncertainty is an inherent feature in engineering problems. Most environmental loads exhibit appreciable randomness [1], such as wind, ocean waves, railway track irregularity, and seismic ground motion, etc. Despite their different physical origins, these excitations can often be modelled as stationary Gaussian stochastic processes under certain conditions, with their statistical characteristics described by the second-order statistical moment, in most cases, the power spectral density (PSD) function [2].

The PSD functions of different excitations vary significantly due to their distinct physical backgrounds. While some PSD models, such as the spectra for earthquake ground acceleration [3], take the form of simple rational functions and are relatively easy to approximate and simulate, other excitations exhibit more complex spectral features. For instance, track irregularity and fluctuating wind speed spectra often demonstrate non-rational characteristics, including fractional-order asymptotic behaviour and multi-segment structures. In Chinese high-speed railway lines, track irregularity PSDs typically show both fractional-order and multi-segment properties [4], while turbulent wind speed spectrum, described by the Kaimal model [5], commonly exhibits fractional-order asymptotic decay with an exponent of -5/3.

To simulate such noise-like stochastic excitations, two main approaches are widely adopted: the spectral representation method and the filtering method. The spectral representation method, inspired by Rice's pioneering work in signal processing [6,7], was further developed by Borgman [8] and Shinozuka [9,10]. It has been applied in various fields and its performance has been significantly improved by some methods including the stochastic harmonic function method [11], and the Karhunen-Loeve expansion [12], etc. However, spectral representation primarily serves numerical analysis based on Monte Carlo simulation, which can be computationally demanding for the analysis of high-dimensional nonlinear structures [13]. On the other hand, filtering methods, including autoregressive (AR), moving-average (MA), autoregressive moving-average (ARMA), and analog filter models, provide a direct and efficient approach for simulating stochastic processes [14,15,16,17]. Unlike the spectral representation method, filter models can avoid the need for computing trigonometric functions and takes the advantage of making the problems more analytically tractable [18].

Recent advances in the probability density evolution method (PDEM) have renewed interest in the filtering methods. The dimension-reduced probability density evolution equation (DR-PDEE) [19,20], developed within the PDEM framework [1], offers an efficient and accurate approach for the stochastic response analysis and reliability analysis of high-dimensional nonlinear systems. In this context, constructing a linear filter system that combines analog and digital filters is essential for formulating an augmented system and establishing the associated Itô stochastic differential equation. Further, it has been illustrated that the DR-PDEE is also

applicable for nonlinear systems with fractional derivative elements and non-Markovian responses [21]. This theoretical framework has shown promising applications in ocean and earthquake engineering [16,17,22], motivating the development of a more versatile filter model capable of handling complex PSD characteristics.

This study develops an efficient analog filter system for simulating excitation PSD functions with complex characteristics, including asymptotically fractional-order and multi-segment properties. The proposed method has been applied to the simulation of fluctuating wind speed with the Kaimal spectrum and railway irregularities with the Chinese high-speed track irregularity spectra. The model is expected to enhance the accuracy and computational efficiency of stochastic simulation and provide a solid foundation for reliability analysis on the probability density level in engineering problems.

2 Basic Formulation of the Versatile Filter Model

A stochastic process X(t) could be expressed as the output of a filter system with its input being white noise, that is,

$$X(t) = G[\xi(\tau), 0 \le \tau \le t],$$
(1)

where G is a generalized linear filter operator, and $\xi(t)$ is Gaussian white noise.

By solving the optimization problem of minimizing the integral of the squared error between the approximated and target PSD functions, the filter system G can be designed with high accuracy. The basic form of the versatile filter model could be written as [17]

$$\begin{cases} \ddot{X}_{1} + 2\zeta_{1}\omega_{1}\dot{X}_{1} + \omega_{1}^{2}X_{1} = \xi(t), \\ \ddot{X}_{2} + 2\zeta_{2}\omega_{2}\dot{X}_{2} + \omega_{2}^{2}X_{2} = G_{1}[X_{1}], \\ \ddot{X}_{3} + 2\zeta_{3}\omega_{3}\dot{X}_{3} + \omega_{3}^{2}X_{3} = G_{2}[X_{2}], \\ \ddot{X}_{4} + 2\zeta_{4}\omega_{4}\dot{X}_{4} + \omega_{4}^{2}X_{4} = G_{3}[X_{3}], \\ \ddot{X}_{5} + 2\zeta_{5}\omega_{5}\dot{X}_{5} + \omega_{5}^{2}X_{5} = G_{4}[X_{4}], \end{cases}$$

$$(2)$$

where X_i , \dot{X}_i and \ddot{X}_i ($i=1,2,\cdots,5$) are the displacement, velocity and acceleration of the i-th DOF, respectively; G_i (i=1,2,3,4) are linear operators. There are 11 parameters to be determined in Equation (2) and they can be expressed as a vector η

$$\eta = \begin{pmatrix} S_0 & \omega_1 & \zeta_1 & \omega_2 & \zeta_2 & \omega_3 & \zeta_3 & \omega_4 & \zeta_4 & \omega_5 & \zeta_5 \end{pmatrix}^{\mathrm{T}},$$
(3)

where S_0 is the intensity of the white noise $\xi(t)$.

Given a target PSD, the linear operators in Equation (2) should be selected so that the output PSD conforms with the characteristics of the target one, and the unknown parameters in η can be identified using optimization algorithms, for instance, the genetic algorithm [17].

The output signal of the filter could be written as a unified form,

$$Y = G_5[X_i], \tag{4}$$

where the linear operator G_5 could be the identity operator, the integer- and evenfractional-order differential operator, etc. Clearly, Equation (4) plays a similar role as the observation equation in the optimal control theory. The filter model mentioned above has been successfully applied to the simulation of random ocean waves with arbitrary JONSWAP spectra [17].

2 Application in the Simulation of Track Irregularities

In the dynamics of vehicle-track coupled systems, the structural vibration majorly stems from track irregularities caused by the random distribution of track geometric state [4]. To quantify the randomness involved in this problem, the track irregularity is generally expressed as stochastic processes and their PSD function describing the second-order statistical information is utilized in the stochastic simulation.

In this section, the proposed versatile filter model is applied in the approximation of the Chinese high-speed railway track irregularities with piecewise non-rational properties.

2.1 Track Irregularity Spectrum of Chinese High-speed Railway Lines

Typically track irregularity spectra take the form of integer-order rational functions, for instance, the railway track irregularity spectra in the USA [23], in Germany [24], and the track irregularity spectra for main railway lines in China [4]. It is not difficult to approximate these types of spectra using the filter model and optimization method as described in [17].

Nevertheless, with the advancement of high-speed railway technology, ballastless tracks have been increasingly adopted in engineering practice. Consequently, their track irregularity spectra have attracted growing attention and research. For Chinese high-speed railway lines with design speeds in the range of 300 to 350 km/h, a piecewise non-integer-order PSD model was proposed in [25] based on statistical analysis of detection data. The spectrum has a unified form for each segment as [4,25]

$$S_{\text{high-speed}}(f) = \frac{A}{f^n},$$
 (5)

where f is the wavenumber in m^{-1} ; A and n are the characteristic parameters and they vary according to different segments and the irregularity type. The values of these parameters for the track cross-level irregularity of Chinese high-speed railway lines are listed in Table 1.

To verify the efficacy of the proposed versatile filter model for the approximation of track irregularity spectrum with piecewise non-rational properties, two approaches have been adopted based on the filter system in Equation (2), that is, a single filter model and a multi-segment model. The former has a smooth function form while the latter fits the target PSD with higher accuracy. One of these approaches can be selected for use based on the needs of the actual problem.

Segment 1		Segm	nent 2	Segment 3		
A	n	A	n	A	n	
3.6148×10 ⁻³	1.7278	4.3685×10 ⁻²	1.0461	4.5867×10 ⁻³	2.0939	
Wave number f_1 for separating point 1 (m ⁻¹)		Wave number f_2 for separating point 2 (m ⁻¹)				
0.0258		0.1163				

Table 1: Characteristic parameters of the track cross-level irregularity PSD of Chinese high-speed railway lines [4].

2.2 Single Filter for the PSD Approximation

In the track-vehicle coupled dynamics [4] and stochastic response analysis of structures [1], it is preferable under some circumstances to have a continuous and smooth filter model for the excitation simulation. Thus, a single function form of the proposed filter system is first introduced in this section.

Note that the wavenumber range of the high-speed railway track irregularity lies between $0.01~\text{m}^{-1}$ to $1~\text{m}^{-1}$, the filter model is designed as follows,

$$\begin{cases} \ddot{X}_{1} + 2\zeta_{1}\omega_{1}\dot{X}_{1} + \omega_{1}^{2}X_{1} = \xi(t), \\ \ddot{X}_{2} + 2\zeta_{2}\omega_{2}\dot{X}_{2} + \omega_{2}^{2}X_{2} = \dot{X}_{1}, \\ \ddot{X}_{3} + 2\zeta_{3}\omega_{3}\dot{X}_{3} + \omega_{3}^{2}X_{3} = \dot{X}_{2}, \\ \ddot{X}_{4} + 2\zeta_{4}\omega_{4}\dot{X}_{4} + \omega_{4}^{2}X_{4} = \dot{X}_{3}, \\ \ddot{X}_{5} + 2\zeta_{5}\omega_{5}\dot{X}_{5} + \omega_{5}^{2}X_{5} = \dot{X}_{4}, \end{cases}$$

$$(6)$$

and the filter parameters values are listed in Table 2.

Name	S_0	$\omega_{_{\! 1}}$	ζ_1	ω_2	ζ_2	ω_3
Value	47.8911	-1.6939	0.2074	0.1367	10.9183	0.0090
Name	ζ_3	ω_4	54	ω_5	ζ_5	
Value	-0.2241	0.0665	2.8762	-0.6752	-27.5352	

Table 2: Filter parameter values for the track cross-level irregularity PSD of Chinese high-speed railway lines.

The comparison between the target track irregularity spectrum and the filter approximation is displayed in Figure 1. From the figure, it can be seen that the overall approximation is achieved by the single filter model while minor deviation inevitably exists around the separating points.

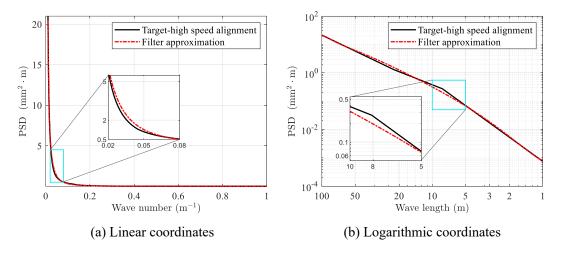


Figure 1: Comparison between the target track irregularity PSD and its approximation by the single filter model.

2.3 Multi-segment Filter for the PSD Approximation

To improve the accuracy of the filtering approximation, a multi-segment filter model is proposed based on the characteristic of target PSD function. Since the function form for a single segment is relatively simple compared with the whole curve, the degrees of freedom of the corresponding filter for that segment could be reduced. The equation of motion for each segment of the filter model could be expressed as

$$\begin{cases} \ddot{X}_{1} + 2\zeta_{1}\omega_{1}\dot{X}_{1} + \omega_{1}^{2}X_{1} = \xi(t). \\ \ddot{X}_{2} + 2\zeta_{2}\omega_{2}\dot{X}_{2} + \omega_{2}^{2}X_{2} = \dot{X}_{1}. \end{cases}$$
(7)

Note that the signals of the three sub-systems influence each other, it is thus necessary to introduce a post-processing step. Denote $L[Y; f_L, f_H]$ as an operator that limits the wavenumber of the signal Y between f_L and f_H , the output of the multisegment filter system can be written as

$$Y = L[Y_1; 0, f_1] + L[Y_2; f_1, f_2] + L[Y_3; f_2, 1],$$
(8)

where Y_1 , Y_2 and Y_3 are the output signals of the three sub-filters, respectively. The operator L can be efficiently realized using Fourier transform and its inverse.

The parameters of the multi-segment filter system are listed in Table 3, and the comparison between the approximated PSD function and the target one is shown in Figure 2. It should be pointed out that the comparison is both shown in linear

coordinates and logarithmic coordinates, and no significant deviation can be detected. Thus, it can be seen that the approximation provided by the proposed multi-segment filter model matches the target spectrum with high accuracy, and the parameters involved does not significantly increase compared with the single filter model.

Parameter	Segment 1	Segment 2	Segment 3
S_0	599.0526	77.7836	38.7397
$\omega_{\rm l}$	-0.3315	-2.7432	0.4807
ζ_1	129.9852	19.2610	40.3037
ω_2	0.1104	-0.0116	-0.0474
ζ_2	37.2786	0.0026	-23.3827

Table 2: Parameter values for the multi-segment filter model.

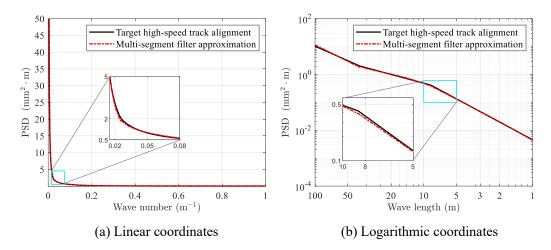


Figure 2: Comparison between the target track irregularity PSD and its approximation by the multi-segment filter model.

3 Application in the Simulation of Fluctuating Wind Speed

In engineering, wind speed is commonly decomposed into the mean wind speed component and the fluctuating wind speed component, and the latter is quantitatively described by the PSD function [26]. The widely used wind speed spectra include the Davenport spectrum [27] and the Kaimal spectrum [28]. Due to the asymptotic fractional-order characteristic of the PSD functions, in this section the versatile filter model is combined with a fractional-order operator to improve the accuracy of the filtering approximation.

3.1 Kaimal Spectrum of Fluctuating Wind Speed

Based on the similarity theory and collected wind field data, the Kaimal spectrum that describes the nonhomogeneous wind speed distribution in space was proposed by [28]. The PSD model can be expressed as [26]

$$S_{\text{Kaimal}}(\omega) = \frac{400\pi u_*^2 f}{\omega (1+50f)^{5/3}},$$
 (9)

where u_* is the shear velocity of the flow in the atmospheric boundary layer, z is the reference height above surface, v_z is the mean wind speed at the reference height, and

$$f = \frac{\omega}{2\pi} \frac{z}{v_z} \tag{10}$$

is referred to as the Monin or similarity coordinate [26].

3.2 Fractional-order Filter for the PSD Approximation

The Kaimal spectrum has an asymptotic -5/3 order decay trend, which is challenging for an integer-order filter model to accurately capture. To solve this problem, the fractional derivative was introduced and incorporated in the analog filter in [29]. In this study, a different approach with fractional derivatives is proposed based on the versatile filter model to improve the consistency.

The Caputo fractional-order derivative is one of the widely used definitions in the theory of fractional calculus [30]. The Caputo derivative of a fractional order α for a function g(t) is defined as [30]

$$D^{\alpha}g(t) = \frac{1}{\Gamma(1-\alpha)} \int_{a}^{t} \frac{g'(t)}{(t-\tau)^{\alpha}} d\tau, \ 0 < \alpha < 1,$$
(11)

where D^{α} is the Caputo derivative operator of order α , Γ denotes the Gamma function and g'(t) is the first-order derivative of g(t).

According to the power order of the Kaimal spectrum, a fractional derivative of order 1/6 could be taken as G_5 and incorporated into the proposed filter model in Equation (2). The operators G_1 to G_3 are taken as the second-order derivative, and G_4 is taken as the first-order derivative, so that the output signal V(t) of the filter model is written as

$$V(t) = G_5[X_5(t)] = D^{1/6}\dot{X}_5(t).$$
 (12)

It should be pointed out that the fractional derivative in [29] is embedded in the equation of motion of the filter system, which could increase the computational cost if the equation is solved in the time domain. However, this issue can be avoided since the fractional derivative is relatively independent of the filter system.

The parameter values of the fractional filter system are shown in Table 3 for a benchmark case where the mean wind speed at the reference height is taken as 15 m/s. The output PSD of the fractional filter model together with the target Kaimal spectrum

are depicted in Figure 3. The comparison demonstrates that satisfactory approximation to the Kaimal spectrum is achieved by the proposed filter model for the benchmark case.

Name	S_0	$\omega_{_{\! 1}}$	ζ_1	ω_2	ζ_2	ω_3
Value	1.729	0.00528	0.00113	0.00908	0.198	0.00695
Name	ζ_3	ω_4	54	ω_5	ζ_5	
Value	0.0452	0.00267	0.457	0.0256	1.196	

Table 3: Parameter values for the fractional filter model.

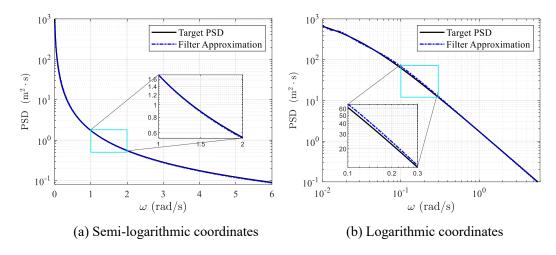


Figure 3: Comparison between the target Kaimal spectrum and its approximation by the fractional filter model.

The load condition of wind fields varies significantly, making the parameters in the Kaimal spectrum change accordingly. If the approximation of the proposed filter model is limited to the benchmark case, it will be difficult to find applications in general problems in wind engineering. To address this issue, the following conversion formulae are proposed to extend the filter model to target spectra with arbitrary parameters,

$$\omega_{1i} = \frac{v_{z1}}{v_z} \omega_{0i}, \ i = 1, 2, \dots, 5;$$

$$\zeta_{1i} = \zeta_{0i}, \ i = 1, 2, \dots, 5;$$
(13)

$$S_{K,01} = \left(\frac{v_{z1}}{v_z}\right)^{2/3} S_{K,00}.$$
 (15)

where v_{z1} is the altered reference speed in the new target PSD, $S_{K,00}$, ω_{0i} and ζ_{0i} ($i=1,2,\dots,5$) refer to the parameters in the benchmark filter model, while $S_{K,01}$, ω_{1i} and ζ_{1i} ($i=1,2,\dots,5$) refer to the filter parameters for the new target PSD function.

4 Conclusions and Contributions

A filter model capable of simulating various excitations with their PSD functions in non-rational forms is proposed. The model can achieve high accuracy for the simulation of excitations with integer-order spectra, including the JONSWAP ocean wave spectrum and the German railway track irregularity spectra. For the Chinese high-speed railway track irregularity with a piecewise PSD form, a multi-segment analog filter composed of three 2-DOF systems was designed. By applying the FFT algorithm, the model achieves high accuracy without compromising efficiency. Additionally, when incorporating fractional-order derivatives, the model can accurately simulate wind speed spectra with fractional-order asymptotic behaviour. The parameter conversion formula derived from the benchmark filter system extends the model to different spectral parameters, significantly expanding its applicability in engineering practices.

This method offers a reliable alternative for simulating various random processes and can be further enhanced by phase control techniques for simulating random fields. Its high accuracy and broad applicability make it suitable for facilitating stochastic response and reliability analysis on the probability density level.

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