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Crowd-Induced Vertical Vibrations in Footbridges Evaluated Based on the Simplified Improved Multiplication Factor Method: A Parametric Study

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Abstract

This paper utilizes the improved multiplication factor method to predict vertical vibrations of pedestrian footbridges under crowd excitation, aiming to conduct a parametric study of crowd-induced effects on structures across varying crowd and structural parameters. The method amplifies the response of a virtual structure subjected to a representative single pedestrian, using an improved multiplication factor calibrated from extensive high-detail microsimulations that capture the complexities of pedestrian behaviour, including inter-subject variability, step-to-step fluctuations, and human-human interaction. Additionally, the method can account for human-structure interaction by incorporating the coupled crowd-structure system modal properties. Straightforward to apply and broadly applicable to both crowd and structural parameters, the method allows for extensive parametric analysis of crowd-induced structural accelerations. The results demonstrate that higher crowd densities do not necessarily represent the most critical condition for all structural configurations. This highlights the strength of the improved multiplication factor method, which is not restricted by specific crowd-structure combinations, providing a robust alternative to existing literature and conservative guidelines that typically exhibit a more limited range of applicability.

Keywords: multiplication factor approach, crowd excitation, pedestrian traffic, human-induced vertical vibrations, pedestrian bridges, serviceability assessment

1 Introduction

The demand for modern, aesthetically pleasing pedestrian footbridges has led to increasingly slender geometries, often resulting in structural natural frequencies that coincide with typical pedestrian walking frequencies. This near-resonance can cause excessive vibrations, leading to discomfort for pedestrians and potential serviceability failures. As a result, vibration serviceability has become a key consideration in footbridge design and evaluation [1, 2]. To address this, it is essential to accurately model the dynamic forces exerted by pedestrians, which serve as the source of vibrations. While modelling the forces of undisturbed pedestrians is straightforward, modelling the dynamics of a crowd is more complex and remains an active area of research [3].

This complexity arises from several factors, including intra-subject variability (unequal steps by the same person), inter-subject variability (differences in individuals gaits), human-human interaction (impact of people actions on one another), and human-structure interaction (mutual effects between walking pedestrians and vibrating structure). These elements affect pedestrian movement, including speed, step frequency, direction, and synchronization. As such, modelling crowd-induced loads requires detailed simulations, complicating footbridge design and assessment processes, and leading to the lack of widely accepted vibration serviceability standards.

The multiplication factor approach is a well-established method for simulating crowd effects by amplifying the structural response to a single pedestrian to estimate crowd-induced vibrations. Existing guidelines [4–7] rely on this method but are often criticized for conservative assumptions about crowd behaviour [8–10], typically overlooking intra-subject variability and inter-subject variability in dense traffic conditions, as well as interaction phenomena. While more recent literature models [11,13] are based on detailed simulations, they are limited in the range of crowd and structural parameters considered, raising concerns about their general applicability.

The improved multiplication factor method presented in [12] is adopted in this study to analyse structural vibrations induced by crowds across varying crowd and structure parameters. This is made possible by the broad applicability of the method, versatile to a wide range of traffic levels, footbridge geometries, and modal properties. The method is practical, while grounded on high-detail simulations to accurately capture crowd dynamics, including inter-subject variability, step-to-step fluctuations, and human-human interaction. Additionally, it can incorporate human-structure interaction by considering the modal properties of the coupled crowd-structure system [15].

The paper is structured as follows. Section 2 reviews the background simulations that inform the method, including the calculation of structural vibrations induced by crowds and virtual single pedestrians, which help generate the improved multiplication factor dataset. Section 3 details the calibration of the simulated improved multiplication factors. Section 4 outlines the application of the method to evaluate structural accelerations induced by crowds. Section 5 presents a parametric study to analyse the predictions of the method under varying crowd-structure scenarios. Section 6 discusses the key findings and their contribution to footbridge serviceability assessments.

2 Simulation overview

This section covers the background simulations underlying the method, including crowd dynamics and loading (Section 2.1), single pedestrian forces (Section 2.2), structural response calculations (Section 2.3), and the improved multiplication factor definition (Section 2.4).

2.1 Crowd simulations

The study uses the social force model (SFM) [16] to simulate unidirectional flows of pedestrians on footbridges. This model describes pedestrian motion driven by "social forces" such as the desire for free movement, avoidance of obstacles, and attraction to groups or points of interest, enabling the simulation of human decision-making based on interactions with the surroundings. Specifically, the implemented version of the SFM [14] is updated to reflect the Weidmann's speed-density relation [17], which accounts for the decrease in mean crowd velocity with increasing traffic density.

Simulations are conducted on a footbridge with specific dimensions of 3 m in width and 40 m in length; however, the method is not limited to these dimensions, as it has been successfully tested on footbridges with varying geometries, including widths from 2.5 to 3.5 m and lengths from 20 to 60 m. The SFM model is employed to simulate the positions and velocities of pedestrians over time by solving the equation of motion at each time step, taking into account the current state of each pedestrian, including their location, speed, and the social forces acting upon them, which are influenced by both their personal motivations and the surrounding environment.

In this approach, pedestrian starting positions are randomly assigned, and their desired velocities are drawn from a well-established probabilistic distribution, with a mean of 1.34 m/s and a standard deviation of 0.26 m/s [18]. Simulations are conducted for crowd densities ranging from 0.2 to 1.5 ped/m^2, in increments of 0.1. To maintain a stable crowd density, the number of pedestrians on the footbridge is modelled to remain constant throughout the 400-second simulation. Additionally, each simulation for a given density is repeated 150 times with different random starting assignments, ensuring statistical reliability by capturing the inherent variability in pedestrian flow.

The outputs from the SFM are subsequently converted into crowd loading. Pedestrian body weights are modelled using a log-normal distribution, with average value and standard deviation of 725 N and 153.8 N [19]. For each individual, foot placements are determined along their 2-D trajectory, simulated by the SFM, based on their time-varying velocity, also modelled by the SFM, with the corresponding step frequency calculated accordingly, as described in [20]. Each step is associated with a step force, modelled using a Fourier series in compliance with the approach proposed by Li et al. [21]. This step-by-step reconstruction of the loading captures the intra-personal variability influenced by the surrounding environment; however, it does not account for inherent individual fluctuations in free walking, independent of external factors.

Ultimately, the total crowd load is obtained by summing the step forces of each individual, with each force linked to its corresponding location and timing based on the SFM-simulated pedestrian flow. This approach captures the crowd dynamic force accounting for inter-subject variability in desired walking, step-by-step fluctuations resulting from environmental influences, and human-human interaction, all modelled through the SFM.

2.2 Single pedestrian simulations

Each examined crowd density is paired with a representative single pedestrian, whose aim is to reflect the average behaviour of the crowd. This pedestrian generates a periodic walking force modelled as a continuous multi-harmonic load [22], which simplifies the computation compared to step-by-step force modelling [21] used for the crowd loading assembling procedure. The representative pedestrian weight is set to 725 N, matching the mean value of the crowd [19], and the steady walking speed is calculated based on the crowd density to be represented through [17]. The corresponding step frequency is derived from this steady speed based on [20]. The walking force is then computed as a time-dependent periodic load, represented by a sum of harmonic components in keeping with [22]. This load is treated as travelling with constant velocity in space, consistent with the calculated density-dependent walking speed.

The key formulations corresponding to the described procedure are detailed in Section 4.

2.3 Structure vibration to simulated pedestrian loadings

Crowd loading is applied to over a thousand structures, with natural frequencies ranging from 0.5 to 5.5 Hz (at 0.05 increments) and various damping ratios, including 0.1, 0.2, 0.5, 0.8, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0 and 10.0%. The inclusion of such high damping ratios is meant to allow for the consideration of the human-structure interaction phenomenon, whose main effect can be modelled by implementing the damping ratio of the crowd-structure coupled system [15], higher compared to that of the empty footbridge as pedestrians act as damping elements. Simulated structures are simply supported beams (40 m long) with a modal mass of 25000 kg. The structure dynamic response is calculated by considering the contribution of the fundamental

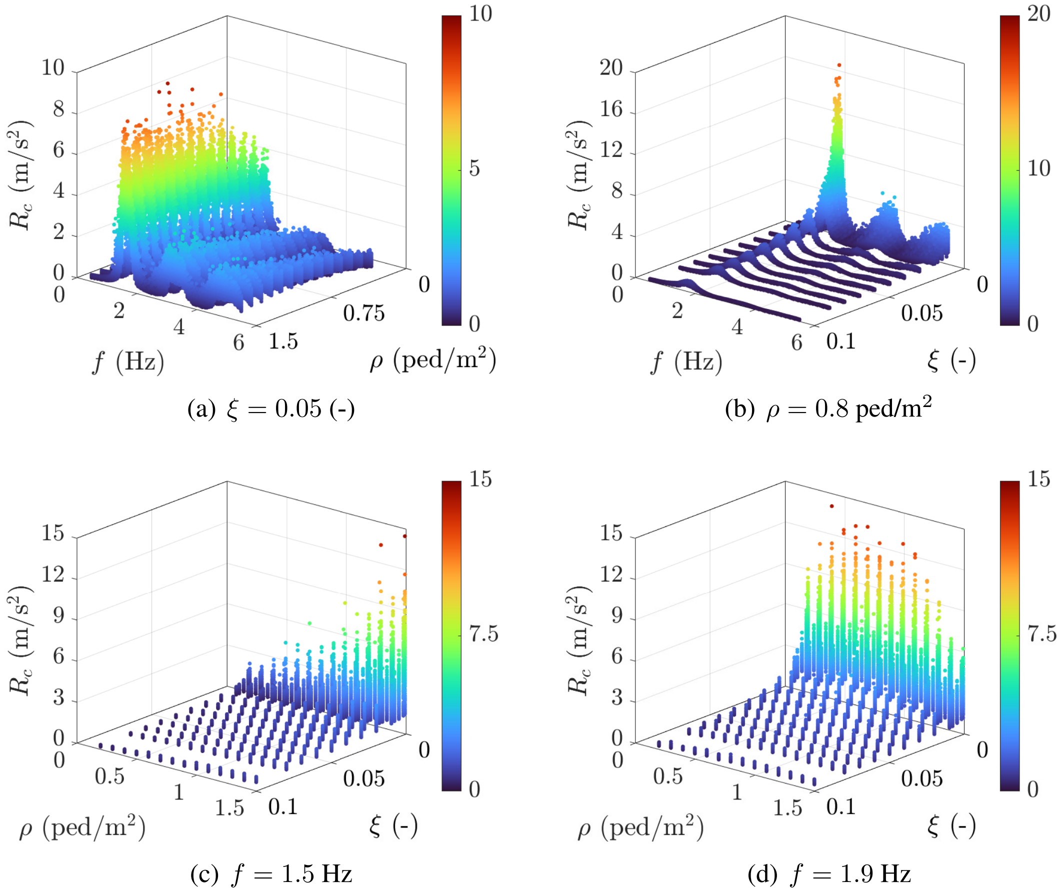


Figure 1: Simulations: maximum acceleration induced by the SFM simulated crowds on the examined footbridges, with fixed (a) structural damping set at 0.05, (b) crowd density of 0.8 ped/m^2, (c-d) natural frequency equal to 1.5 and 1.9 Hz.

bending mode with a half-sine shape, where each step force contributing to the crowd loading is weighted by the mode shape amplitude at its respective application point. In this, the footfall locations, simulated in 2-D based on the SFM flow, are projected onto the footbridge centreline, as only flexural vibrations are considered. The dynamic vibration of the structure due to crowd loading is determined by numerically solving the modal equation of motion, and the crowd-induced structural response is defined by the maximum acceleration across time at the mid-span. By combining simulated crowd flows with the examined footbridge modal parameters, a dataset of nearly three million crowd-induced maximum accelerations is obtained.

Figure 1 illustrates the maximum structural accelerations induced by crowds simulated using the SFM on the examined footbridges, with variations in modal parameters (natural frequency and structural damping) and traffic density. Specifically, each dot in Figure 1 represents a specific combination of crowd density and modal parameters. The simulation dataset is presented by varying all parameters except one: structural damping set at 0.05 in Figure 1(a), traffic density equal to 0.8 ped/m^2 in Figure 1(b), and natural frequencies of 1.5 Hz and 1.9 Hz in Figure 1(c) and Figure 1(d), respectively. These variations enable a detailed analysis of how each parameter influences the structural accelerations induced by the crowd. The results show peaks corresponding to resonance conditions, particularly when the crowd, depending on its density, results in a step frequency that aligns with the structure natural frequency (or multiples). This highlights the dependence of the structural response on the interaction between the footbridge natural frequency and crowd density. Furthermore, structural damping determines how the structure mitigates the vibrations, with its impact on structural accelerations being clearly evident.

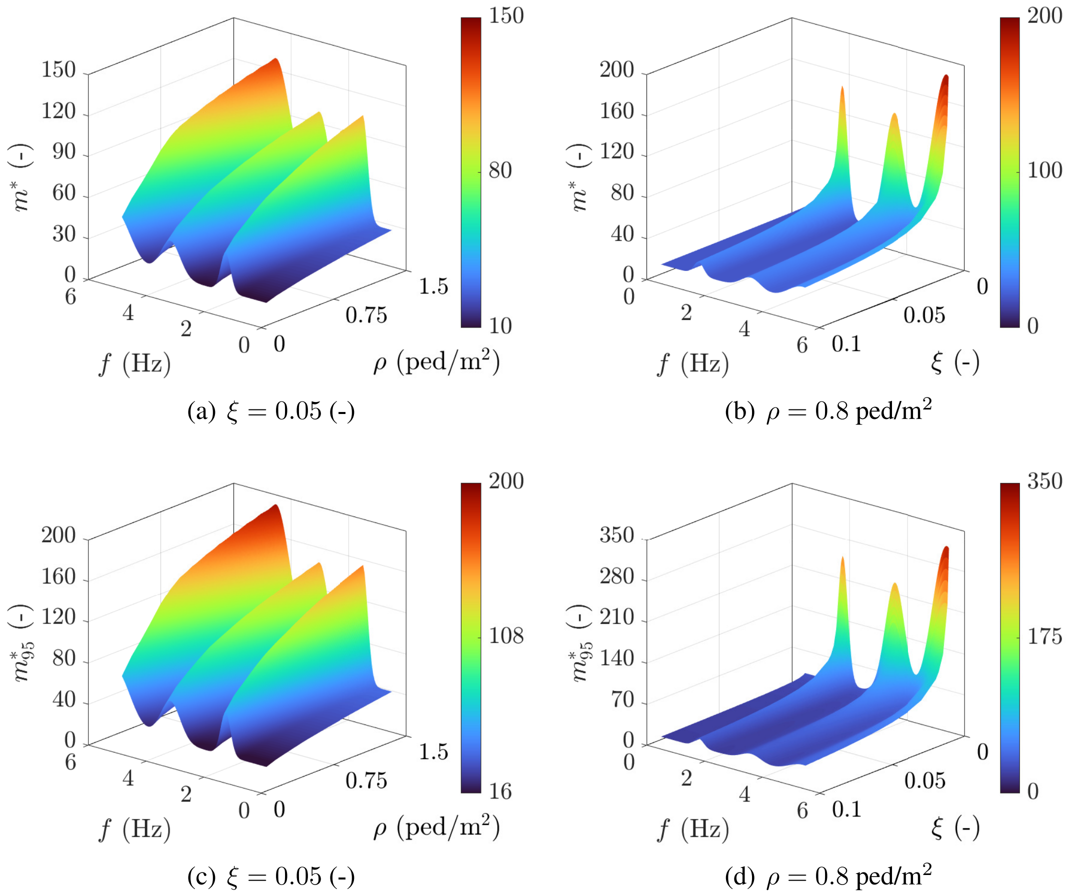
For each traffic density, a probabilistic approach based on 150 simulations is used to simulate crowd loadings (see Section 2.1), while a deterministic approach is applied to define the loading of a single pedestrian representative of the crowd density level (see Section 2.2). To simulate the variability of crowd dynamics in the associated single pedestrian, an additional damping term is introduced in footbridges crossed by single pedestrians, compared to structures crossed by crowds. This improves the model by reflecting the randomness in crowd dynamics, even when considering just a single pedestrian. The extra damping ξ^* varies with crowd density ρ as:

$$\xi^* = 0.005595\rho^{-1.013} + 0.07885 \tag{1}$$

The formulation is specifically calibrated based on the standard deviation of pedestrian average step frequencies, as modelled by the SFM. This results in a structure response to single pedestrians that better mirrors the crowd behaviour.

2.4 Improved multiplication factors

For each pedestrian group crossing a specific footbridge, an improved multiplication factor is calculated by dividing the maximum crowd-induced structural acceleration by the maximum acceleration produced by its associated single pedestrian, who walks



(a) $\xi = 0.05$ (-) (b) $\rho = 0.8$ ped/m^2

(c) $\xi = 0.05$ (-) (d) $\rho = 0.8$ ped/m^2

Figure 2: Calibration: analytically formulated (a-b) average and (c-d) design improved multiplication factors, with fixed (a-c) structural damping set at 0.05, (b-d) crowd density of 0.8 ped/m^2.

on a virtual over-damped footbridge. This factor is derived for each examined case, resulting in a dataset comprising nearly three million elements.

The modal mass and mode shape affect both the crowd and single pedestrian accelerations equally, so they do not influence the improved multiplication factor. As a result, the improved multiplication factor remains independent of these modal parameters and is applicable to any vertical mode, even though the simulations are based on a fundamental half-sine bending mode with a specific modal mass.

3 Calibration process

This section details the calibration of the simulated improved multiplication factors. The average and 95th percentile trends of the improved multiplication factor dataset, m^* and m^*_{95}, are analytically described to predict average and design (i.e., 95th percentile) structural maximum crowd-induced accelerations. The average trend is first

a_n	c_n	d
$a_1 = 0.4105\sqrt{\rho A}\,\xi^{-0.5021}$	$c_1 = 0.24$	$d = 1.868\sqrt{\rho A}\,\xi^{-0.01086}$
$a_2 = 0.9a_1$	$c_2 = 2c_1$	
$a_3 = 1.3a_1$	$c_3 = 3c_1$	

Table 1: Analytical definitions of model parameters.

addressed, calibrated based on a sum of three bell-shaped functions, representing resonance between the structure natural frequency f and the typical step frequency f_s (connected to the crowd density via [17]) and multiples:

$$m^*(f) = d + \sum_{n=1}^{3} a_n \exp\left[-\left(\frac{f - nf_s}{c_n}\right)^2\right] \tag{2}$$

Parameters featuring these bell-functions are fitted based on crowd density ρ, deck area A, and structural damping ξ, with calibrated definitions provided in Table 1. This results in an analytical representation of m^* for predicting average maximum structural accelerations due to crowd excitation.

For forecasting the 95th percentile maximum crowd-induced accelerations (design case), the ratio Δ of the 95th to the 50th percentile improved multiplication factors is modelled as a power function of structural damping:

$$\Delta = \xi^{-0.08098} - 0.05682 \tag{3}$$

This allows for an efficient calculation of the 95th percentile m^*_{95} by multiplying the average m^* (computed as expressed by Equation (2)) with Δ.

This approach offers a streamlined method for predicting both average and design (95th percentile) crowd-induced maximum accelerations by computing the footbridge response to a representative single pedestrian and amplifying it using the calibrated analytical formulations of the average and design improved multiplication factors, m^* and m^*_{95}, illustrated in Figure 2 for varying crowd and structural parameters.

4 Method application

This section describes the application of the improved multiplication factor method [12], outlining its implementation procedure and validity scope.

The improved multiplication factors m^* and m^*_{95} offer a simple way to estimate the average and 95th percentile maximum crowd-induced structural accelerations, by calculating only the maximum response from a single virtual pedestrian. When considering specific natural frequency f, damping ratio ξ, and crowd density ρ in real serviceability assessments, the following procedure should be used. The method might include the effect of human-structure interaction on structural dynamic properties, implementing the equivalent modal parameters of the crowd-structure coupled system [15]. Therefore, f and ξ may reflect either the empty footbridge properties or the equivalent values, depending on human-structure interaction consideration. In this regard, it is highlighted that below, structural damping (as well as virtual over-damping ξ^*) is considered dimensionless and not expressed as a percentage.

To predict the structure average maximum acceleration due to crowd:

1. Identify v_s: Evaluate pedestrian velocity v_s consistent with crowd density ρ, using the Weidmann's speed-density relation [17]:

$$v_s(\rho) = 1.34 \left\{1 - \exp\left[-1.913\left(\rho^{-1} - 0.185\right)\right]\right\} \tag{4}$$

2. Evaluate f_s: Calculate step frequency f_s for pedestrian velocity v_s, via the experimentally calibrated formulation by Bruno and Venuti [20]:

$$f_s(v_s) = 0.35\, v_s^3 - 1.59\, v_s^2 + 2.93\, v_s \tag{5}$$

3. Define $P(t)$: Calculate the single pedestrian multi-harmonic loading $P(t)$:

$$P(t) = G + G \sum_{k=1}^{4} \mathit{DLF}_k \, \sin\left(2\pi k\, f_s\, t\right) \tag{6}$$

   where $G = 725$ N is the subject static weight [19], and DLF_k (-) is the k-th harmonic dynamic load factor, dependent on the step frequency as in [22].

4. Determine ξ_{TOT}: Evaluate extra damping ξ^* depending on crowd density via Equation (1), and compute total damping ratio ξ_{TOT} as $\xi + \xi^*$.

5. Calculate R_s^*: Evaluate the maximum acceleration R_s^* experienced by the virtual footbridge with total damping ξ_{TOT} and natural frequency f, based on the modal equation of motion:

$$\ddot{q}(t) + 4\pi\, f\, \xi_{\text{TOT}}\, \dot{q}(t) + (2\pi\, f)^2\, q(t) = P(t)\, \phi(t)\, /\, M \tag{7}$$

   where $q(t)$ is the modal coordinate, M the modal mass, and ϕ the mode shape.

6. Compute m^*: Determine the average multiplication factor m^* (Equation (2)) corresponding to the natural frequency f and the density-dependent step frequency f_s (defined as formulated in Equation (5)), based on parameters depending on ρ, ξ, and deck area A, as described in Table 1.

7. Estimate R_c: Compute the average crowd-induced structure acceleration R_c by multiplying m^* by R_s^*.

To predict the structure design maximum acceleration due to crowd (95th percentile), steps 1 to 6 should be followed, with the following additional step:

8. Calculate Δ: Compute factor Δ (Equation (3)) for the scaling from 50th to 95th percentile improved multiplication factors, depending on ξ.

9. Evaluate m^*_{95}: Compute the 95th percentile improved multiplication factor m^*_{95} by multiplying m^* by Δ.

10. Estimate $R_{c,95}$: Determine the 95th percentile structure maximum crowd-induced acceleration $R_{c,95}$ by amplifying R^*_s by m^*_{95}.

5 Parametric analysis

In this section, a parametric analysis of the results is presented and discussed. Specifically, the maximum structural acceleration induced by a crowd (whether average or design) is calculated by amplifying the maximum structural vibrations caused by a virtual representative single pedestrian, using the proposed analytical improved multiplication factor (average or design). The resulting forecasts from the method are then compared to background simulations and analysed parametrically.

Figure 3 represents the method prediction of the crowd-induced maximum average structural accelerations, calibrated to the mean trend of the simulated data presented in Figure 1. Similar considerations could also be made by analysing the design values (i.e., the 95th percentile). It is important to highlight the good match between the numerically simulated values and those analytically predicted by the method, as can be visually observed by comparing Figure 3 with Figure 1. The comparison shows a strong match in all the examined scenarios, with determination coefficients around 0.90 for all structure-crowd combinations (comprising both average and design prediction-to-simulation comparisons).

The worst-case scenario for maximum structural acceleration does not always occur at the highest crowd density. As density increases, the modal force generally rises, increasing the structural response, although this effect is not proportional due to human-human interactions. Additionally, as density grows, the average step frequency of pedestrians decreases, which can either amplify or reduce the structural response depending on how close the crowd mean step frequency is to the footbridge natural frequency. When walking is undisturbed, typical pedestrian speed and step frequency are around 1.34 m/s (Equation (4)) and 1.91 Hz (Equation (5)). Footbridges with a natural frequency around 1.91 Hz (or multiples), for example, may experience larger accelerations at lower densities, where pedestrians are less constrained and their step frequencies come closer to the footbridge natural frequency, leading to near-resonance.

Thus, the worst-case scenario is determined by the interplay between crowd density, pedestrian step frequency, and the natural frequency of the footbridge, rather than

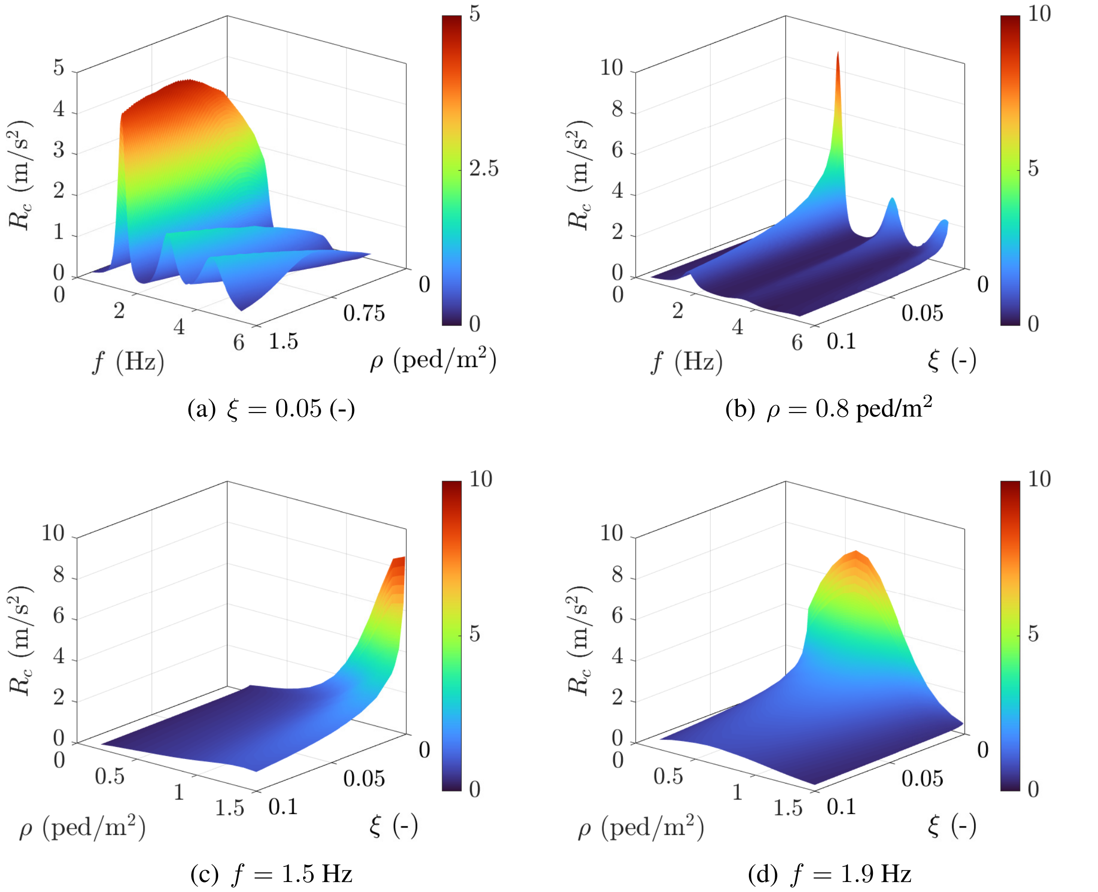


Figure 3: Predictions: average maximum structural crowd-induced accelerations calculated by the designed method, with fixed (a) structural damping set at 0.05, (b) crowd density of 0.8 ped/m^2, (c-d) natural frequency equal to 1.5 and 1.9 Hz.

density alone. This is illustrated in Figure 3(c) and Figure 3(d), where two example footbridges with different natural frequencies (1.5 Hz and 1.9 Hz) are analysed, respectively. These example cases show peak values corresponding to different crowd densities, demonstrating what stated above. This further emphasizes the advantages of the proposed method, which incorporates a broad range of structural and traffic parameters, enabling it to capture and simulate scenarios that may significantly differ from one another. In contrast, existing methods in the literature address a limited range of scenarios, and current guidelines typically assume high-density conditions to be the most critical, often neglecting the variability in pedestrian behaviour in medium-to-high density crowds where pedestrians are assumed to walk in unison. This highlights that the proposed method provides a substantial improvement over the existing body of literature in terms of serviceability analysis.

6 Concluding remarks

This study presents a comprehensive parametric analysis of crowd-induced vibrations in pedestrian footbridges. By exploiting the improved multiplication factor method, the analysis investigates the interplay between crowd density, footbridge geometries, and modal properties in determining structural accelerations. The results emphasize that higher crowd densities do not universally represent the most critical condition, suggesting that structural responses are more complex than commonly assumed.

The parametric study demonstrates that the relationship between crowd-induced excitation and structural characteristics plays a crucial role in determining footbridge serviceability, as these factors collectively govern the magnitude of potential structural accelerations, particularly in relation to the likelihood of near-resonance occurrences. Such conditions arise when the average crowd step frequency, influenced by crowd density that restricts undisturbed pedestrian motion, coincides with the natural frequency (or multiples) of the structure, with vibrations further amplified by typical low structural damping values.

This study also underscores the versatility of the improved multiplication factor method, which can be applied to a wide range of crowd and structural parameters, providing a comprehensive yet practical tool for evaluating footbridge serviceability under vertical vibrations. The parametric analysis - which reveals that each real-case scenario should be treated individually - coupled with the adaptability of the method, underscores its potential as a robust alternative to existing literature and conservative codes, which often have a more limited scope.

In conclusion, the results from this parametric study provide valuable insights into the structural response of footbridges under crowd excitation. By considering a broad range of traffic conditions and structural properties, this approach enhances the understanding of footbridge performance and offers a practical framework for ensuring their safety and serviceability in real-world scenarios.

Acknowledgements

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