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Nonlinear Dynamic Analysis of Reinforced Concrete Structures Using Proper Orthogonal Decomposition and Multi-Fiber Modeling

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Abstract

This research proposes the extension of the Proper Orthogonal Decomposition (POD) method to reinforced concrete (RC) structures with material nonlinearities subjected to earthquakes, aiming to reduce the numerical cost of dynamic time history analysis. The approach involves reducing the structural model by projecting it onto the dominant POD modes. Material nonlinearity in RC structures, caused by steel ductility and concrete damage, is modelled using the multi-fiber section technique. Two reduction models are presented. The first is for a single base excitation: a full nonlinear analysis is performed on an initial small-duration earthquake segment, and the dominant POD modes are extracted. These modes are then used to reduce the dynamic model for the remaining earthquake duration. The second technique addresses multiple earthquakes: a complete analysis for one selected event identifies the dominant POD modes, which are then applied to reduce the model for subsequent earthquake analyses. The reduced models deliver results comparable to full models while significantly decreasing computational time (up to 96% reduction). This study demonstrates that the POD method effectively reduces the numerical cost of dynamic time history analysis for RC structures with nonlinearities under seismic loading.

Keywords: reinforced concrete, nonlinear dynamic analysis, proper orthogonal decomposition, multi-fiber, finite element method, model reduction, material non-linearities

1 Introduction

An extension of the Proper Orthogonal Decomposition method (POD) to nonlinear dynamic analysis of reinforced concrete multistory frame structures is shown in this study. The multi-fiber section is used to simulate the material nonlinearity.

The cross section of the structural element is divided into a number of longitudinal fibers for the multi-fiber section model. According to the uniaxial stress-strain behavior of its corresponding material, each fiber, which is composed of a single material, has the ability to exhibit nonlinear inelastic longitudinal deformation [1]. It is recommended to use nonlinear solving methods when dealing with nonlinearities. The Newton-Raphson method and its derivatives, the displacement control approach, and the arc length technique are the traditional and most often used nonlinear solvers.

Dynamic excitations in structures are usually studied using direct integration time history analysis. In this approach, temporal discretization is considered and the direct time integration is conducted using implicit methods like Newmark- β [2], Wilson θ [3], HHT- α [4] or explicit methods like central difference and Runge-Kutta. The main concern in using the direct time integration analysis for linear and nonlinear models is its high computational cost especially when applied in structural seismic analysis. In fact for seismic analysis, the structure is subjected to dynamic excitations at its base. These excitations are generally based on the accelerograms of previously recorded quakes in the region. In order to cover all probable scenarios, the structure should be subjected to multiple accelerograms vibrating in all different directions which greatly increases the time cost of this analysis technique.

In order to reduce the time cost of the dynamic time history analysis, a number of model reduction strategies have been proposed as a result of this setback. For linear systems, modal truncation can be used to define the most influential mode shapes of the structure and then this truncated modal base is used to reduce the dynamic equation of the structural system [5, 6]. Research has been done to find an analogy between nonlinear and linear normal modes for nonlinear structures, based on the work of [7]. However, because of its limitations when non-smooth nonlinearities are present in the structure, this nonlinear modal analysis is not commonly applied [8].

The Proper Orthogonal Decomposition (POD) is a data driven method based on the statistical Principal Component Analysis (PCA) of observations dataset. In other words, the best subspace for reproducing the complete dataset with the fewest possible errors is identified by analyzing data gathered from observations at various time intervals (snapshots). This subspace is later used to reduce the model under consideration in calculation. Since the 1930s, the POD approach has been used in fluid mechanics to reduce fluid flow models, reduce structural dynamics models, identify damage, reduce dynamic models for microelectromechanical systems, and many other fields.

There are two conventional techniques to conduct seismic analysis of reinforced concrete structures that take into account nonlinear material behaviour. The first is the pushover analysis, a static nonlinear technique that tries mimicking the dynamic behavior of the structure by considering it to respond dynamically according to its fundamental mode shape only. As nonlinear material behavior is taken into consideration, horizontal loads are gradually increased and distributed on the structure according to this fundamental mode shape vector. This method is only applicable to normal low-rise buildings where the fundamental mode shape is the predominant mode of vibration and no response in function of time is needed (only maximum values are given). In other situations, the direct integration nonlinear time history analysis described above is applied.

The POD method has never been employed, as far as the authors are aware, to reduce the direct integration nonlinear time history analysis of a Reinforced Concrete (RC) structure, where the multi-fiber section approach is used to describe material nonlinearity. This study investigates a nonlinear multi-fiber RC multistory frame structure subjected to seismic excitations at its base. To reduce the computational cost of the direct integration time history analysis, the Proper Orthogonal Decomposition (POD) method is applied. In Section 2, the nonlinear material behavior of a RC beam element is modeled using the multi-fiber section approach. Section 3 focuses on the dynamic analysis of a RC element with material nonlinearities, comparing the classical approach with the reduced POD method. Section 4 demonstrates the application of the POD reduction technique to the multistory frame structure and compares the outcomes with the full model analysis. Finally, Section 5 concludes the study and discusses potential future developments.

2 Multi-fiber beam model

For modeling material nonlinearity in structural elements, the most straightforward and widely used method is concentrated (lumped) plasticity. This method, however, is predicated on the premise that nonlinear material behavior only manifests at specific concentrated places of the structural member—a significant simplification. Furthermore, the interplay between bending moments and fluctuating axial forces at the plastic hinge is not considered. Furthermore, characteristic curves (Moment versus Rotation or Force vs Displacement) supplied by the seismic codes characterize the behavior of plastic hinges. The accuracy of these curves is diminished because they are predicated on imprecise predictions and assumptions. However, as we shall explain in this section, the multi-fiber beam model ensures that nonlinear material behavior is dispersed throughout the cross section and over the length of the structural element (distributed plasticity method). Furthermore, this method can be used on elements with non-typical cross sections and accounts for the interplay between bending moments and axial stresses. Even though the fiber model approach requires more computing power than the focused plastic hinge technique, it is still effective and highly useful, particularly for wall elements that are modeled using the comparable beam approach. The multi-fiber beam approach is used in this work to model the nonlinear material in 1D finite elements (beams, columns, equivalent beam model for walls) while taking into account the Euler-Bernoulli hypothesis (planar

sections before deformation remain planar and perpendicular to the element's center line after deformation) because 2D finite elements are not covered in this paper.
As previously stated, the multi-fiber beam technique entails splitting the cross section of the structural element into a number of longitudinal fibers. Consequently, a three-level analysis is necessary when employing this modeling technique.

The fundamental level of analysis is fibers. The fibers of reinforced concrete members are composed of a single substance, which may be either confined or unconfined concrete or steel reinforcements.The fiber axial stress σ_{fiber} and the longitudinal tangent Young modulus $E_{T\,fiber}$ are determined in function of the longitudinal fiber axial strain ε_{fiber}.

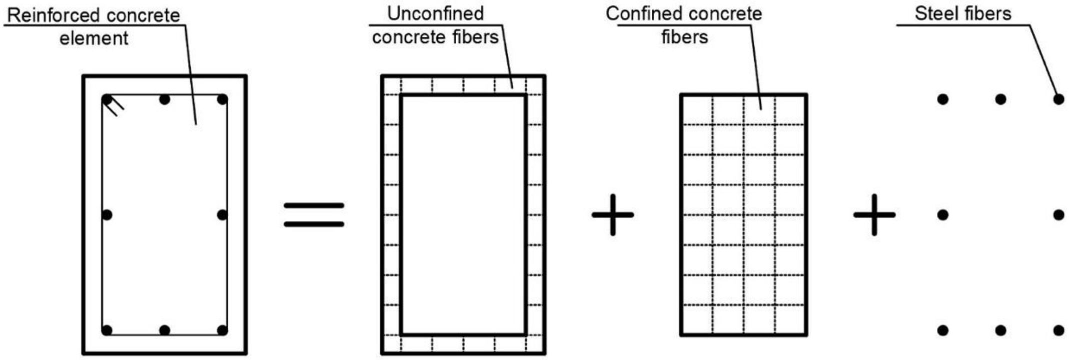

Figure 1. Multi-fiber reinforced concrete section

The element’s cross section is the second level of analysis. Applying the Euler-Bernoulli hypothesis will result in perfect bond conditions between fibers (no sliding of a fiber with respect to another is allowed). In 2D structural analysis, for a fiber having its centroid located at the ordinate y in the section reference, the axial longitudinal strain in the fiber $\varepsilon(y)$ can be determined in function of the section’s uniform axial strain along x axis ε_x and the section’s curvature along z axis ϕ_z

$$\varepsilon(y) = \varepsilon_x - y\phi_z = \{1 \quad -y\}\begin{Bmatrix}\varepsilon_x \\ \phi_z\end{Bmatrix} \tag{1}$$

since in nonlinear analysis the calculation is done by increments and we get

$$\Delta\varepsilon(y) = \Delta\varepsilon_x - y\Delta\phi_z = \{1 \quad -y\}\begin{Bmatrix}\Delta\varepsilon_x \\ \Delta\phi_z\end{Bmatrix} \tag{2}$$

this axial strain increment of the fiber $\Delta\varepsilon(y)$ causes an increment in the section’s internal axial force ΔN and bending moment along z axis ΔM_z.

$$\Delta N = E_{T\,fiber}A_{fiber}\Delta\varepsilon(y) \tag{3.a}$$

$$\Delta M_z = -y\Delta N = -yE_{T\,fiber}A_{fiber}\Delta\varepsilon(y) \tag{3.b}$$

for a single fiber, the resulting increment of internal forces in the section is

$$\{\Delta F_{Section}\} = \begin{Bmatrix}\Delta N \\ \Delta M_z\end{Bmatrix} = E_{T\,fiber}A_{fiber}\begin{bmatrix}1 & -y \\ -y & y^2\end{bmatrix}\begin{Bmatrix}\Delta\varepsilon_x \\ \Delta\phi_z\end{Bmatrix} \tag{4}$$

for all the fibers, the entire resulting increment of internal forces in the section is

$$\{\Delta F_{Section}\} = \begin{Bmatrix}\Delta N \\ \Delta M_z\end{Bmatrix} = \underbrace{\sum_{i=1}^{n_{fiber}} E_{T\,fiber\,i}\,A_{fiber\,i}\begin{bmatrix}1 & -y_i \\ -y_i & {y_i}^2\end{bmatrix}}_{[K_T]}\begin{Bmatrix}\Delta\varepsilon_x \\ \Delta\phi_z\end{Bmatrix} \tag{5}$$

where $n_{fiber}$is the total number of fibers in the section and $[K_T]$ is the section's tangent stiffness matrix.
The entire element is the third level of analysis. Linear shape functions are considered for longitudinal translation and Hermite cubic shape functions are used for bending. Applying the principle of virtual work we get

$$\{F_{int}\} = \int [B(x)]^T \begin{Bmatrix} 1 \\ -y \end{Bmatrix} \sigma_{fiber}(x,y)dV \quad (5)$$

$$[K_T] = \int [B(x)]^T \begin{Bmatrix} 1 \\ -y \end{Bmatrix} E_{T\,fiber}(x,y)\{1 \quad -y\}[B(x)]dV \quad (6)$$

where $\{F_{int}\}$ is the internal nodal force vector of the element, $[K_T]$ is the element's tangent stiffness matrix and $[B(x)]$ is the gradient operator containing the derivatives of shape functions.
The volume integral required for the calculation of $\{F_{int}\}$ and $[K_T]$ is split into a surface integral on the cross section and a 1D integral along the longitudinal axis of the element. Since the element's cross section is already divided into fibers, we substitute the surface integration by the summation of fiber areas. Next, the longitudinal 1D integration is done by Gauss points.

3 Full and reduced dynamic models

As already mentioned, the classical time costly approach for capturing the nonlinear seismic response of a structure in function of time is the full model implicit direct integration nonlinear time history analysis. The Newmark-β method is one of the famous implicit direct integration techniques used for linear and nonlinear time history analysis. For this method, knowing the structural system state at instant t_i (displacement, velocity and acceleration vectors) and assuming a variation pattern for acceleration between instants t_i and t_{i+1} (i.e. constant average acceleration) makes it possible to express the dynamic equation of the structural system at instant t_{i+1} with only one unknown (the displacement vector at instant t_{i+1}) and thus solving easily the system.
The proper orthogonal decomposition POD also known as the Principal Component Analysis PCA and the Karhunen-Loève Decomposition KLD is a statistical analysis of observation data. Let's consider a data matrix $[X]$ containing n observation vectors $[X] = [\{X_1\} \quad \cdots \quad \{X_n\}]$ and each observation vector is made of m dimension

$$[X] = [\{X_1\} \quad \cdots \quad \{X_n\}] = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \quad (7)$$

$\{S_i\} = \{x_{i1} \quad \cdots \quad x_{in}\}$ is row i in matrix $[X]$ and represents all the data collected on dimension i. If data set $\{S_i\}\ \forall\ i$ has a zero mean, the variance of $\{S_i\}$ becomes

$$\sigma^2(\{S_i\}) = \frac{1}{n-1} \times \sum_{k=1}^{n} \left(x_{ik} - mean(\{S_i\})\right)^2 = \frac{1}{n-1} \times \sum_{k=1}^{n} (x_{ik})^2 = \frac{1}{n-1}\{S_i\}\{S_i\}^T \quad (8)$$

and the covariance of $\{S_i\}$ and $\{S_j\}$ becomes

$$COV(\{S_i\},\{S_j\}) = \frac{1}{n-1} \times \sum_{k=1}^{n} \left(x_{ik} - mean(\{S_i\})\right)\left(x_{jk} - mean(\{S_j\})\right) \quad (9.a)$$

$$COV(\{S_i\},\{S_j\}) = \frac{1}{n-1} \times \sum_{k=1}^{n} (x_{ik})(x_{jk}) = \frac{1}{n-1}\{S_i\}\{S_j\}^T \quad (9.b)$$

High value of $\sigma^2(\{S_i\})$ indicates high action on dimension i and vice versa. High value of $COV(\{S_i\},\{S_j\})$ indicates high similarity between the actions on dimension i and dimension j. On the other hand, $COV(\{S_i\},\{S_j\}) = 0$ indicates zero resemblance (total independence) between the actions on dimension i and dimension j.

If data set $\{S_i\}$ $\forall$ i has a zero mean, the covariance of matrix $[X]$ becomes

$$COV([X]) = \frac{1}{n-1}[X][X]^T \qquad (10.a)$$

$$COV([X]) = \begin{bmatrix} \sigma^2(\{S_1\}) & COV(\{S_1\},\{S_2\}) & \cdots & COV(\{S_1\},\{S_n\}) \\ COV(\{S_2\},\{S_1\}) & \sigma^2(\{S_2\}) & \cdots & COV(\{S_2\},\{S_n\}) \\ \vdots & \vdots & \ddots & \vdots \\ COV(\{S_n\},\{S_1\}) & COV(\{S_n\},\{S_2\}) & \cdots & \sigma^2(\{S_n\}) \end{bmatrix}$$

(10.b)

Determining the principal components of data matrix $[X]$ starts by finding a new orthonormal reference $[N]$. The initial data matrix $[X]$ is expressed in this new reference as $[X'] = [N]^T[X]$. For $[N]$ to be containing the principal components of the data observation, $COV([X'])$ should be a diagonal matrix. In other words, we have zero similarity between actions on different new dimensions in reference $[N]$ ($COV(\{S'_i\},\{S'_j\}) = 0 \; for \; i \neq j$).

Since $COV([X])$ is made up of $[X][X]^T$ so it is a symmetrical matrix and thus has real eigenvalues.

$$[X][X]^T[\emptyset] = [\emptyset][\lambda] \qquad (11)$$

where $[\emptyset]$ is the eigenvectors matrix and $[\lambda]$ is the diagonal matrix containing the eigenvalues. Eigenvectors are orthonormal vectors and we can demonstrate that the new reference $[N]$ we were talking about in the previous paragraph is in fact the eigenvectors matrix ($[N] = [\emptyset]$) of $[X][X]^T$. In fact for $[X'] = [\emptyset]^T[X]$ we get

$$COV([X']) = \frac{1}{n-1}[X'][X']^T = \frac{1}{n-1}[\emptyset]^T \underbrace{[X][X]^T[\emptyset]}_{[\emptyset][\lambda]} = [\lambda] \qquad (12)$$

$COV([X'])$ is a diagonal matrix and $\sigma^2(\{S'_i\}) = \lambda_i$. We notice that the higher λ_i is the more we have actions on dimension i in the eigenvectors reference. As a conclusion, principal components of the data set $[X]$ are the eigenvectors of $[X][X]^T$ and modes with high eigenvalues are the most influential in representing $[X]$.

The orthogonal eigenvectors obtained are called POD modes and the corresponding eigenvalues are called proper orthogonal values. The POD modes can be used to reconstruct the initial data matrix $[X]$. The higher the eigenvalue of a POD mode is, the more essential this mode is in recreating $[X]$.

By considering the most important s POD modes ($s < m$) and placing them in $[T] \in \mathbb{R}^{m \times s}$, the $\{X_t\}$ snapshot vector previously expressed in m dimensions can now be approximated in the lower s dimensions

$$\underbrace{\{X_t\}}_{\in \mathbb{R}^{m\times 1}} \cong \underbrace{[T]}_{\in \mathbb{R}^{m\times s}} \underbrace{\{Q_t\}}_{\in \mathbb{R}^{s\times 1}} \qquad (13)$$

where $\{Q_t\}$ contains the coordinates of the snapshot vector in the new reference $[T]$. The choice of the number s of POD modes to consider in the reduced new reference should satisfy 2 conditions:

The representation in the new reference should be accurate so the error should be minimal. The higher s is, the more accurate the approximation is.

$$error = \sum_{i=1}^{n} \left\| \{X_{t_i}\} - [T]\{Q_{t_i}\} \right\| \tag{14}$$

For the dimensions reduction to be efficient, the number of chosen POD modes s should be relatively small.

In order to balance between accuracy and efficiency, an energy criterion is considered to determine the optimal value of s. The Proper Orthogonal Value of a mode gives an indication on the energy carried by this mode. Generally, the first s POD modes carrying at least 99% of the total system energy are considered for the new reduced reference.

$$\frac{\sum_{i=1}^{s} \lambda_i}{\sum_{j=1}^{m} \lambda_j} \geq 99\% \tag{15}$$

In structural dynamics, the POD reduction can be applied on the direct integration time history analysis for linear or nonlinear structures. In order to get the observation data required for the POD, we initially do a classical implicit direct integration time history analysis of the full structural finite element model subjected to a specific base excitation. Let's consider a nonlinear structural system with m degrees of freedom and n snapshots were taken. We calculate the POD modes and proper orthogonal values of the data matrix $[X]$ and then choose the subspace $[T] \in \mathbb{R}^{m \times s}$ containing the first s POD modes satisfying the 99% energy criterion. The dynamic equation of the system is

$$[M]\{\ddot{X}(t)\} + [C]\{\dot{X}(t)\} + R(\{X(t)\}) = \{F(t)\} \tag{16}$$

By replacing $\{X(t)\}$ and its derivatives by $[T]\{Q(t)\}$ and multiplying both sides of the dynamic equation by $[T]^T$ we get

$$\underbrace{[T]^T[M][T]}_{[M_r] \in \mathbb{R}^{s \times s}}\{\ddot{Q}(t)\} + \underbrace{[T]^T[C][T]}_{[C_r] \in \mathbb{R}^{s \times s}}\{\dot{Q}(t)\} + \underbrace{[T]^T R([T]\{Q(t)\})}_{R_r([T]\{Q(t)\}) \in \mathbb{R}^{s \times 1}} = \underbrace{[T]^T\{F(t)\}}_{\{F_r(t)\} \in \mathbb{R}^{s \times 1}} \tag{17}$$

The previously m degrees of freedom dynamic system is reduced to s degrees of freedom. However, the nonlinear restoring force $R([T]\{Q(t)\})$ cannot be reduced and always needs to be calculated in the full coordinate model which makes this step the most time consuming part of the entire process. In this case, the most effective direct time integration technique to adopt will be the one with the least recurrence for the expensive nonlinear restoring force calculation.

Implicit direct time integration techniques are usually used in conjunction with the Newton-Raphson approach for solving nonlinear systems. In order to reach convergence with this approach, multiple iterations are required at each time step and for every iteration we need to calculate the tangent stiffness matrix, its inverse and the nonlinear restoring force which are all time costly. Using the constant stiffness Newton-Raphson approach will save us the need for the tangent stiffness calculation and its inverse but will increase the number of iterations required for convergence.

On the other hand and for explicit direct time integration techniques, the popular central difference method requires only one iteration per time step and no expensive calculation of the tangent stiffness matrix and its inverse are needed (only the nonlinear restoring force is required). However, the central difference approach is conditionally stable and needs to satisfy the following stability condition

$$\Delta t < \frac{2}{\omega_{max}} \tag{18}$$

where Δt is the time step and ω_{max} is the largest natural pulsation of the system. Generally, the full model of the structure has a relatively large number of degrees of freedom and will result in high natural pulsations (for high modes) hence requiring small time steps to maintain calculation stability and consequently increasing the computational cost. Nevertheless, when working with a reduced structural model, significantly fewer number of degrees of freedom are considered and thus the reduced system will have smaller natural pulsations which makes it possible to use larger time steps while maintaining numerical stability. For this reason, in this work the central difference method is considered to be the most effective direct time integration technique for reduced models.

This simplification of the POD nonlinear dynamic model can be used in a variety of ways. The structure is examined for a number of potential earthquakes and is examined and verified for each excitation (shock record) independently, as was previously indicated for the dynamic seismic analysis. We begin with the traditional full model implicit direct integration nonlinear time history analysis for the first excitation because we must perform an analysis for every excitation. In order to simplify the dynamic model in the examination of the remaining excitations, we can identify the key POD modes by gathering snapshots from this preliminary analysis. [9] suggested this method and used it to analyze seismic base isolators as well as on a small-scale steel frame. In this article, we will use this method and expand it to a multistory frame reinforced concrete structure, using the multi-fiber section technique to describe the material nonlinearity.

4. Application

At first we need to consider the base vibrations to use. The following 4 earthquake recordings obtained from the Center of Engineering for Strong Motion Data CESMD (www.strongmotioncenter.org) were considered (refer to Table 1).

Table 1: Considered earthquakes.

Earthquake	Location	Date	Magnitude	Measurement station	Vibration direction	Total duration	Time step
Northridge	Los Angeles, USA	01/17/1994	6.4 ML	Newhall LA county fire station	0°	60s	20ms
Elcentro	California, USA	05/18/1940	6.9 Mw	Elcentro	0°	53.74s	20ms
L'Aquila	L'Aquila, Italy	04/06/2009	6.3 Mw	L'Aquila V.Aterno Centro Valle	90°	60s	20ms
Chile	Off the coast of central Chile	02/27/2010	8.8 Mw	Constitucion city	90°	120s	20ms

Referring to Figure 3, the structure is a 2D Reinforced Concrete (RC) multistory frame consisting of 10 stories and 5 spans, with a 3 m story height and a 5 m span length. The beams are subjected to a linear load of 1T/m, and the structural self-weight is disregarded. Every concrete column and beam is separated into finite parts of 1 m length and is thought to have a 40x40 cm square cross section with four 20 mm High Bond HB reinforcing bars on the top and bottom (see Figure 5). Two steel fibers and four concrete fibers make up the cross section. A 5% damping ratio for the first two eigenmodes was obtained using Rayleigh damping (the first two eigenmodes involve almost 90% of the total mass). Because of the intricacy of the reinforced concrete components, the energy criterion for choosing POD modes is set at 99.99%.
The components close to the beam-column connections at the first five stores are thought to exhibit material nonlinearity (see Figure 4). Initially linearly elastic and later plastic with strain hardening, the steel rebar is said to have a bilinear backbone curve. Its yielding stress is 400 MPa, its yielding strain is 2‰, its elastic Young modulus is 200 GPa, its ultimate stress is 420 MPa, and its ultimate strain is 2.5%.
The steel material will exhibit kinematic hysteresis behavior under cyclic stress if nonlinearity is achieved (see Figure 6). A simplified version of the Mander model [12], which accounts for the destructive effects, is used to simulate concrete, which is thought to be unconfined. The maximum tensile strength is 2.5 MPa (10% of the compressive strength) at a corresponding strain of 0.1 MPa, the ultimate compressive strain is 4 MPa, the elastic Young modulus is 25 GPa, and the maximum concrete compressive strength is 25 MPa at a corresponding strain of 2 MPa (see Figure 7).

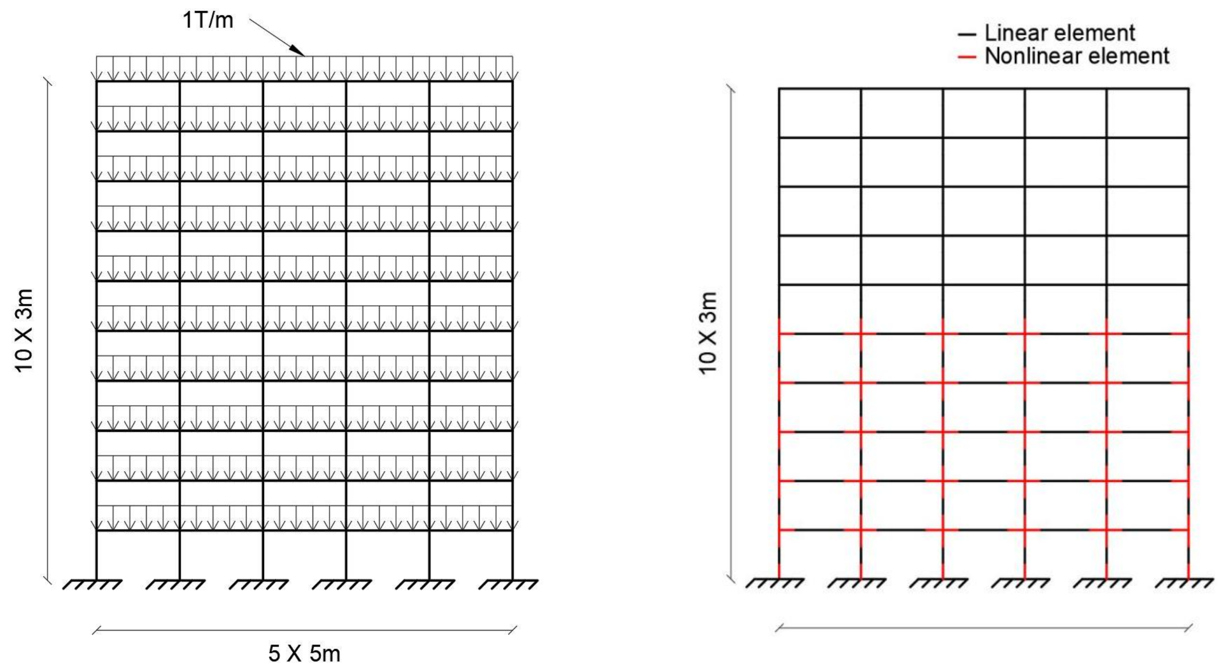

Figure 3. RC frame geometry and loading

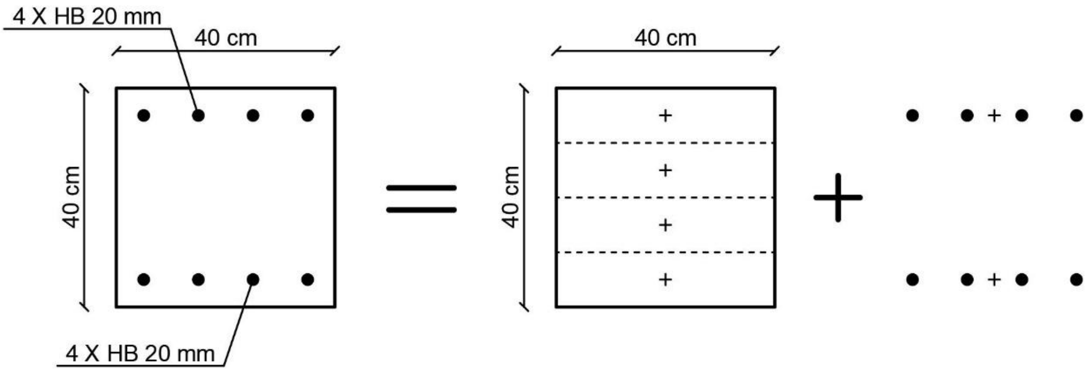

Figure 5. RC element multi-fiber section

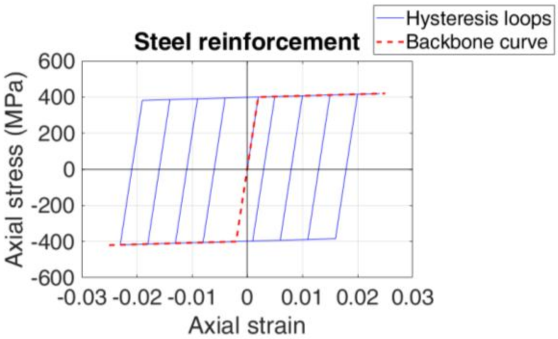

Figure 6. Steel reinforcement axial stress-strain curve

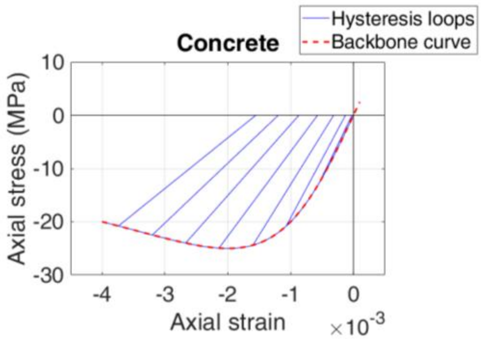

Figure 7. concrete axial stress-strain curve

The first vibrations is regarded as the Northridge earthquake. For this earthquake, a Full Model (FM) implicit nonlinear time history analysis was conducted using a time step of 20 ms. During the first 15 seconds of the vibration, when the majority of the violent exitation takes place, and at evenly spaced time intervals, fifty snapshots of the ensuing displacement vector were acquired. Based on the findings of the Northridge earthquake, the dynamic system was decreased and POD modes were taken out of the snapshot matrix. Next, for the remaining earthquakes (Chile, L'Aquila, and Elcentro), a 20 ms time step explicit nonlinear time history analysis using the Reduced Model (RM) was conducted. It should be mentioned that in order to have a baseline reference, FM implicit nonlinear time history analysis was carried out independently for the Chile, L'Aquila, and Elcentro earthquakes. Additionally, RM analysis was done for the Northridge earthquake in order to compare it with the entire dynamic model that was first constructed.

Since the first four POD modes account for over 99.99% of the system's energy, they make up the reduced base for this structure. This structure's original 1140 degrees of freedom are therefore reduced to just 4.

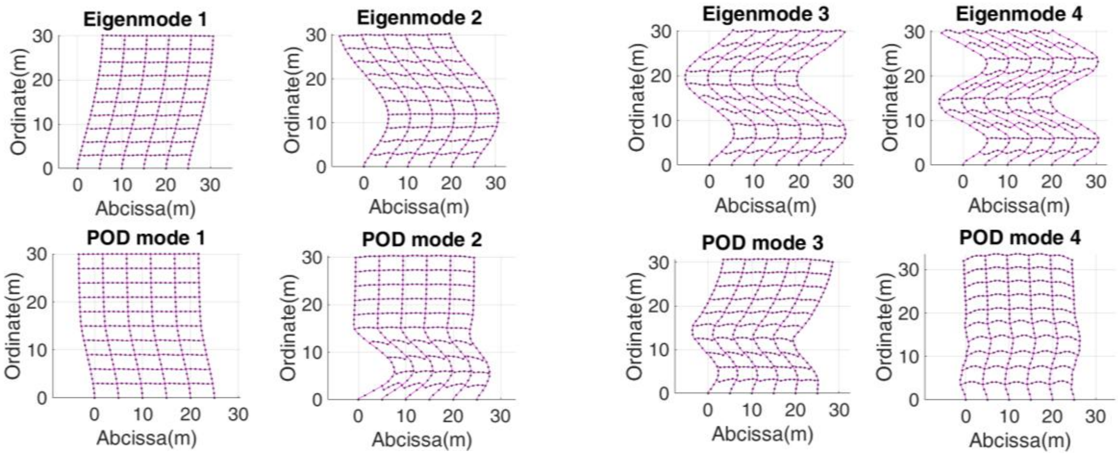

Figure 8. Classic structural eigenmodes Vs POD modes

By comparing the POD modes with the classical eigenmodes of the structure, we can clearly see the nonlinearity effect in the POD modes at the first 5 stories of the structure especially for modes 2 and 3.

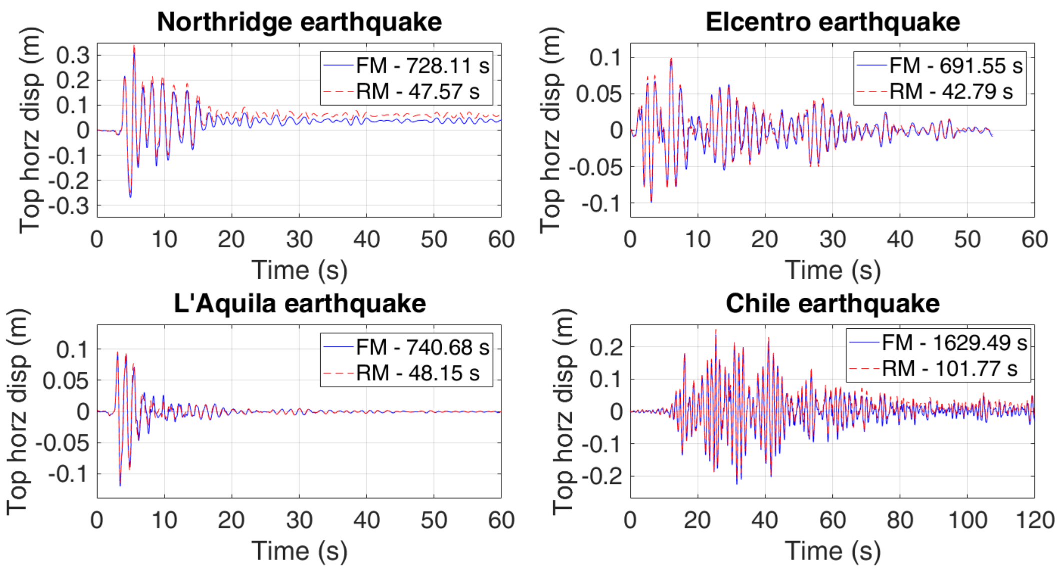

Figure 9. Structural top left corner horizontal displacement in function of time for Full Models (FM) and Reduced Models (RM)

As we can see in Figure 9 the reduced models results are very close to the full models and at a fraction of the time cost, for further details refer to the following table 2.

Table 2: The Reduced Model's (RM) accuracy and time savings in comparison to the Full Model (FM)

Earthquake	FM time	RM time	Time saving	Speedup	Average error	Max horiz displacement
Northridge	728.11 s	47.57 s	93.47%	15.3	2.07 cm	31.16 cm
Elcentro	691.55 s	42.79 s	93.81%	16.2	0.53 cm	9.96 cm
L'Aquila	740.68 s	48.15 s	93.50%	15.4	0.17 cm	12.05 cm
Chile	1629.49 s	101.77 s	93.75%	16.0	1.01 cm	23.81 cm

We can clearly see the time saving benefits of the POD modes in reducing the nonlinear structural system. Furthermore, the structural model subjected to additional excitations (Elcentro, L'Aquila, and Chile Earthquake) is reduced effectively by the POD modes that were taken from the Full Model (FM) study of the Northridge earthquake.

5. Conclusions

In this study, we expanded the use of Proper Orthogonal Decomposition to minimize nonlinear dynamic analysis numerical cost of reinforced concrete multistory frame structures, where the multifiber section technique was used to simulate material nonlinearity. We achieved to reduce a system with 1140 degrees of freedom to just 4 degrees while keeping an acceptable level of accuracy and obtaining a speedup of about 16 (the reduced model computation is 16 times quicker than the complete model). Additionally, it was demonstrated that POD modes derived from the examination of a complete structural model exposed to a particular base vibration could also be easily used to reduce the same model under various base excitation conditions. Having a snapshot matrix of the dynamic system that is accurately representational is crucial in this case. We are interested in extending this method to 2D reinforced concrete structural components (plates, shells, and membranes) utilizing the layered 2D element model in order to increase time savings and accuracy of findings.

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