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Optimization of Combined Cable-Stayed Concrete Bridges

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Abstract

This paper presents an optimization-based approach for the automated design of combined cable-stayed concrete bridges. This approach combines an automated multi-start procedure with a gradient-based algorithm to solve the original non-convex optimization problem. The finite element method is used for the three-dimensional analysis considering dead load, road traffic live load and concrete time-dependent effects. The design is formulated as a cost minimization problem subject to constraints on the displacements and stresses considering service and strength criteria defined according to the provisions of the Eurocodes. A constraint aggregation approach is adopted to solve this problem by minimizing a convex scalar function obtained through an entropy-based approach. The discrete direct method of sensitivity analysis provides the structural response to changes in the design variables. The design variables are the deck and tower sizes, the cable-stays' and under-deck cables' prestressing forces and cross-sectional areas, the tower height, the maximum strut length and the distance from the tower to the backstays' anchor point. The optimization of a 90 m span bridge illustrates the features and applicability of the proposed approach. The optimum design features a deck slenderness of 1/72.1, maximum strut length-to-span ratio of 1/10.8 and height of the tower-to-span ratio of 1/4.4.

Keywords: cable-stayed bridges, under-deck cable-stayed bridges, combined cable-stayed bridges, optimization, concrete, cable prestressing forces, sizing design variables, shape design variables.

1 Introduction

Cable-stayed bridges are used worldwide ranging from small pedestrian bridges to medium-to-long-span road and railway bridges. Their popularity is owed to structural and construction efficiency, as well as their economic and aesthetic advantages. With their multiple inclined cable-stays, modern cable-stayed bridges offer continuous support and natural prestressing to the deck, allowing for long spans and slender designs. Besides the usual symmetrical three-span arrangement, these bridges may present a wide variety of arrangements (single-span, two-span, multi-span, symmetrical or asymmetrical). These are highly redundant structures with their static and dynamic response influenced by the stiffness and mass distribution of their load-bearing members (deck, tower and cable-stays) and the cable prestressing forces.

The design of cable-stayed bridges is a challenging task involving the definition of the structural system, determining the members' cross-sectional sizes, and calculating the cable forces distribution. The analysis should include geometrical nonlinearities, erection stages and the time-dependent behaviour of concrete. As usual in civil engineering practice, this design problem is still mostly guided by the designers' experience and relies on a trial-and-error procedure. This involves complex and time-consuming tasks and an optimized design may not be obtained. Optimization techniques, although not commonly adopted in practice, are particularly suited for assisting in this design problem, aiming at an efficient use of the materials and economic and sustainable solutions. Utilizing the increasing computational resources, it is relevant to promote the implementation of these techniques in practice, thus contributing to the automation of design procedures. A recent literature survey by Martins et al. [1] indicates that the optimization of cable-stayed bridges is a relevant research topic. Previous works on this topic can be categorized into two main research areas: cable forces optimization and optimal design. This survey also highlights some expected future developments, which can already be identified in recent works.

The optimum design considering wind [2, 3] and earthquake [4, 5] responses, the reliability-based optimum design, and the robust design including for example, cable loss scenarios [6] are relevant subjects for future research. Additionally, researchers are exploring the use of metaheuristic algorithms [7, 8, 9], artificial neural networks and surrogate models [4, 12]. They are also addressing the optimization of footbridges [7, 10], curved bridges [5, 10], long-span and multi-span bridges, with a focus on innovative cable arrangements like crossing-cables [11, 12]. Besides studying innovative cable arrangements in traditional cable-stayed bridges, it is also worth investigating the optimization of non-conventional cable-stayed bridges, such as under-deck cable-stayed bridges and combined cable-stayed bridges [13]. These represent rather innovative bridge typologies concerning the traditional use of prestressing in bridge design and the stay-cables layout. Specifically, the stay-cables in these typologies are positioned either below the deck, as seen in under-deck bridges, or both above and below the deck, in the case of combined bridges. These bridge types were developed by renowned structural engineers, including Leonhardt, Schlaich, Virlogeux, Manterola, Robertson, and Cremer. The main features, structural behaviour and design criteria of these novel bridge typologies, along with an overview of existing structures, are discussed in a series of works by Ana Ruiz-Teran and others [13, 14, 15, 16]. Nevertheless, the literature review reveals a knowledge gap in the optimization of these novel bridge typologies. Bearing this in mind, the optimization of under-deck cable-stayed concrete bridges was recently addressed by Martins et al. [17]. In this type of bridge, the cables are located below the deck, anchored at the deck's support sections, and deviated by inclined struts, forming a polygonal layout. The struts, acting in compression, introduce upward vertical forces that help support the vertical loads in the deck. Additionally, the compression forces from the inclined cables at their anchorages provide a natural prestressing effect, which enhances the structural behaviour of the concrete deck.

Combined cable-stayed bridges provide another innovative structural solution for bridge design by integrating both above-deck and under-deck cable-supporting systems. The Obere Argen viaduct in Germany, designed by Jörg Schlaich and built in 1990, serves as a major reference of this bridge typology [18]. The design of this type of bridge aims for an appropriate balance between the stiffness of the load-bearing members (deck, tower, cable-stays and under-deck cables), with the cable prestressing forces playing a key role in controlling the structural response [15, 16]. From the literature review, and to the best of the authors' knowledge, the optimization of combined cable-stayed bridges has not yet been previously reported. Moreover, the specificities of the cable-supporting system of this bridge typology do not permit the direct application of existing algorithms developed for cable-stayed bridges.

The main objective of this work is to present an optimization-based approach for designing combined cable-stayed concrete bridges. To this end, a computer program was developed in the MATLAB environment to implement the proposed approach. This program integrates an automated procedure for generating multiple starting designs and a gradient-based optimization algorithm. Local optimum solutions are obtained from each starting design, and the least-cost solution is selected as the optimum design. This work represents a contribution to assist in the conceptual and preliminary design stages of this type of bridge. Furthermore, it also aims to provide more insight into the optimal design of this novel bridge typology.

Section 2 details the proposed optimization strategy. In Section 3, the optimization of a 90 m single-span combined cable-stayed concrete bridge illustrates the features of the proposed approach. The main conclusions and expected future developments are presented in Section 4.

2 Optimization Strategy

The design of combined cable-stayed bridges is posed herein as a cost minimization problem subject to constraints defined according to Eurocodes' provisions. This can be expressed by

$$
\begin{aligned}
\min \quad & C(\underline{x}) \\
s.t. \quad & g_j(\underline{x}) \leq 0 \qquad j = 1,2,\cdots,M \\
& \underline{x}_{\min} \leq \underline{x} \leq \underline{x}_{\max} \quad \underline{x} = \{x_1, x_2, \ldots, x_N\}^T
\end{aligned}
\tag{1}
$$

where $C(\underline{x})$ represent the total cost of the structure, $g_j(\underline{x})$ corresponds to the design constraints, $\underline{x}$ is the global design variable vector, M is the number of design constraints and N is the number of design variables, $\underline{x}_{min}$ and $\underline{x}_{max}$ are the vectors with the lower and upper bound values of the design variables, respectively.

As usual in structural optimization, this problem may be non-convex. This problem features a moderate number of design variables of different types (shape, sizing, and mechanical) and a somewhat large number of nonlinear and conflicting constraints due to the structural discretization and several load cases. Gradient-based and non-gradient-based strategies can be used to solve this problem. The latter do not require information on derivatives to minimize the objective function. These include techniques such as random search, branch-and-bound and metaheuristic algorithms (e.g., evolutionary algorithms, genetic algorithms, simulated annealing, and particle swarm optimization). Although versatile and easy to implement, these strategies may not be as efficient owing to their exponential convergence time with respect to the number of design variables. Additionally, the random nature of the search process in these techniques may pose some difficulty in finding appropriate values of the cables' prestressing forces in highly redundant cable-supported structures. Gradient-based strategies require the calculation of the gradients of the objective function and all design constraints concerning the design variables. The algorithm uses this information to determine the search direction and adjust the current design toward an optimum solution. These strategies feature polynomial convergence time to a local (not necessarily global) optimum solution. The information on the gradients makes these strategies efficient in searching for optimal designs comprising numerous design variables, such as a large number of cables' prestressing forces, in addition to sizing and shape design variables. Nevertheless, the algorithm requires defining an appropriate initial design, and a local minimum can be expected.

A gradient-based strategy is proposed herein to efficiently address the optimum design of an innovative concrete cable-supported bridge. This strategy combines a gradient-based algorithm with an automated multi-start procedure. This strategy facilitates the time-consuming task of defining appropriate starting designs. Moreover, by considering multiple starting designs, it contributes to avoiding the convergence to an inferior local optimum and simultaneously promotes the exploration of the design space. The analysis and optimization process is conducted for each starting design, and local optimum solutions are obtained. The least-cost design is selected as the optimum design.

Gradient-based nonlinear programming algorithms may encounter difficulties when addressing problems with many design variables and constraints. This can be tackled via constraint aggregation, using classical (p-norm and Kreisselmeier-Steinhauser (KS) function) or induced aggregation methods (induced exponential and induced power aggregation) [19]. Therefore, the problem in Equation 1 can be solved as a single objective optimization to minimize the cost subject to aggregated constraints. Alternatively, it can be addressed as a multi-objective optimization by aggregating the cost and all the design goals (defined by the constraints) in a single objective function [17]. The latter approach was adopted herein. The multi-objective problem is solved indirectly by the minimization of the convex scalar function given by Equation (2), obtained through an entropy-based approach [20]

$$\min F(\underline{x}) = \min \frac{1}{\rho} \ln\left[\sum_{j=1}^{M} e^{\rho\left(g_j(\underline{x})\right)}\right] \tag{2}$$

where ρ is a control parameter that should be tuned for each problem and must not be decreased through the analysis and optimization process. This function corresponds to the KS function, aggregates all the design goals and creates a convex approximation close to the boundaries of the original non-convex domain. This function includes all constraints with different probabilities of becoming active. As the iterations progress, uncertainty decreases about which constraints are most relevant for finding the optimum. This procedure reduces the cost design goal compared to previous iterations while keeping all constraints within limits.

The design goals, $g_j(\underline{x})$, do not have an explicit algebraic form. Therefore, the problem is solved using an explicit approximation given by the Taylor series expansion of all the goals, around the current design variable vector, truncated after the linear term, given by

$$\min F(\underline{x}) = \min \frac{1}{\rho} \ln\left[\sum_{j=1}^{M} e^{\rho\left(g_j(\underline{x}) + \sum_{i=1}^{N} \frac{dg_j(\underline{x})}{dx_i} \Delta x_i\right)}\right] \tag{3}$$

where $g_j(\underline{x})$ is the *j-th* design goal and $dg_j(\underline{x})/dx_i$ is the sensitivity of the *j-th* design goal with respect to *i-th* design variable. The value of the aggregation parameter ρ should be tuned for each problem (usual values range from 100 to 2000) and must not be decreased during the optimization process. To ensure the accuracy of the explicit approximation, bound constraints with move limits were used. The MATLAB function *fmincon*, which minimizes a scalar function of several variables subject to bound constraints using an interior-point algorithm, was selected to minimize the objective function.

This design problem comprises different types of design variables (shape, sizing, and mechanical). The shape design variables concern the maximum strut length, the height of the tower and the distance from the tower to the backstays' anchor point. These design variables characterize the cable-supporting system and affect the mass, stiffness, and cost of the structure. The second type refers to the mechanical design variables corresponding to the prestressing forces of the cable-stays and under-deck cables. Although these variables do not directly influence the cost, they are fundamental in controlling the structural response of combined cable-stayed bridges. The sizing variables correspond to the cross-sectional dimensions of the deck, tower and struts, as well as the cross-sectional areas of cable-stays and under-deck cables. Figure 1 depicts the 23 design variables considered.

Besides minimizing cost, the design should comply with a set of criteria concerning service and safety requirements. In the constraint aggregation approach adopted, the cost is considered the first design goal and can be expressed as

$$g_1(\underline{x}) = \frac{C}{C_0} - 1 \leq 0 \tag{4}$$

where C is the current cost of the structure, and C_0 is the initial cost of each analysis and optimization cycle. With this approach, cost is always a primary objective for optimization. The second set of objectives concerns limiting the vertical displacements of the deck and the tower's horizontal displacements under service conditions and considering the time-dependent effects

$$g_2(\underline{x}) = \frac{|\delta|}{\delta_0} - 1 \leq 0 \tag{5}$$

where δ and δ_0 are the displacement value and the limit value for the displacement under control, respectively. For the long-term analysis of the bridge, values of $L/1000$ and $H/1000$ were considered as limits for vertical and horizontal displacements, respectively [21]. L and H correspond to the main span length and the height of the tower, respectively.

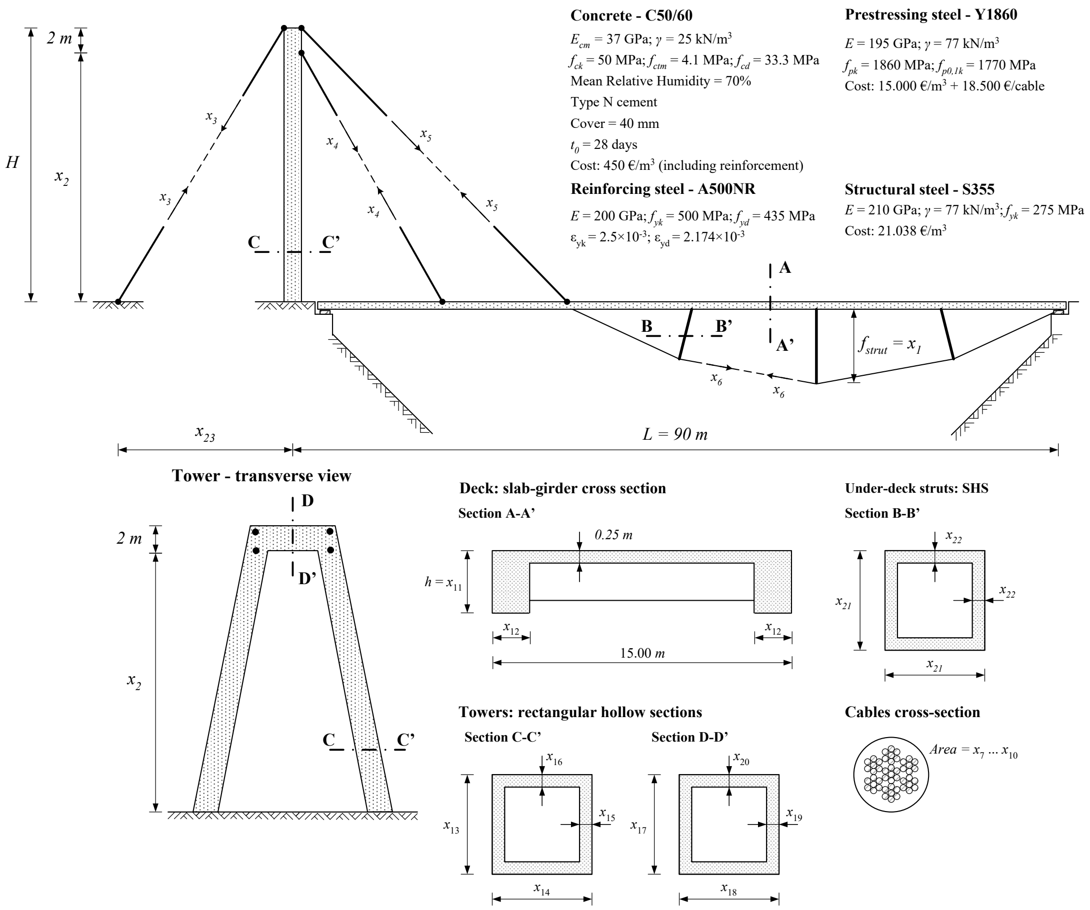

Figure 1: Design variables, material properties and unit costs.

The Eurocode 2 [22] design provisions were considered to define the stress constraints concerning the deck and tower members. Typically, these constraints can be expressed as follows

$$g_3(\underline{x}) = \frac{\sigma}{\sigma_{allow}} - 1 \leq 0 \tag{6}$$

where σ and σ_{allow} are the acting stress and the corresponding allowable stress, respectively. Different values for allowable stress were considered for both service conditions and strength verification of concrete members. For service conditions, the allowable stresses of 4.1 MPa for tension and 22.5 MPa for compression were used. For strength verification, the allowable value corresponds to the concrete member's structural resistance (including its reinforcement). This is assessed based on the acting internal forces, such as bending and axial force, biaxial bending and axial force, or shear force.

The steel struts that constitute the under-deck cable-staying system are subjected to high compression stresses. According to Eurocode 3 [23], the design constraint aiming to prevent the buckling of the struts can be expressed as

$$g_4(\underline{x}) = \frac{N_{Ed}}{N_{b,Rd}} - 1 \leq 0 \tag{7}$$

where N_{Ed} is the acting axial force and $N_{b,Rd}$ is the design buckling resistance of the member under compression. The latter is calculated considering pinned struts with a Class 1 cross-section.

Another set of constraints concerns the cable-stays and under-deck cables' stresses and can be written as

$$g_4(\underline{x}) = \frac{\sigma}{k \cdot f_{pk}} - 1 \leq 0 \tag{8}$$

$$g_5(\underline{x}) = 1 - \frac{\sigma}{0.10 \cdot f_{pk}} \leq 0 \tag{9}$$

where σ and f_{pk} are the acting stress and the characteristic value of the prestressing steel tensile strength, respectively. The value of k in Equation (8) was considered equal to 0.50 for service conditions and 0.74 for strength verification. Equation (9) concerns a lower limit for tension in the cable-stays and under-deck cables to ensure their structural efficiency.

Combined cable-stayed bridges feature small geometrical nonlinearity and can be appropriately analysed through linear analysis [13, 15]. The finite element method was used for the three-dimensional analysis under static loading (dead load and road traffic live load) including concrete time-dependent effects. Euler-Bernoulli beam elements with 2-node and 12-degrees of freedom were used to model the deck and tower. Due to the short length and the stress level on the cable-stays and under-deck cables, the geometrical nonlinearity owing to the cables' sag can be neglected. Therefore, these members were modelled as 2-node bar elements. The struts were also modelled using 2-node bar elements so that the connections between the struts and the deck are pinned, thus avoiding the transmission of bending moments between the struts and the deck. The structural concrete of the deck and tower was modelled as a linear viscoelastic material, and the time-dependent effects of ageing, creep and shrinkage of concrete were computed according to the Eurocode 2 [22] formulations. Equivalent nodal forces were used to model the time-dependent effects of creep and shrinkage. These forces produce the same displacements field as the time-dependent effects. The stresses are then computed using only the elastic constitutive relationship between stresses and mechanical origin deformations. Detailed information about the time-dependent effects modelling can be found in previous work by Martins et al. [24]. A linear elastic behaviour of the materials (structural concrete, reinforcing steel, structural steel, prestressing steel) was considered. Material nonlinearity was considered when formulating the stress design constraints in the different structural members. Homogeneous concrete cross-sections were assumed, and the steel reinforcement was considered for design purposes only. The flowchart of the proposed strategy is depicted in Figure 2.

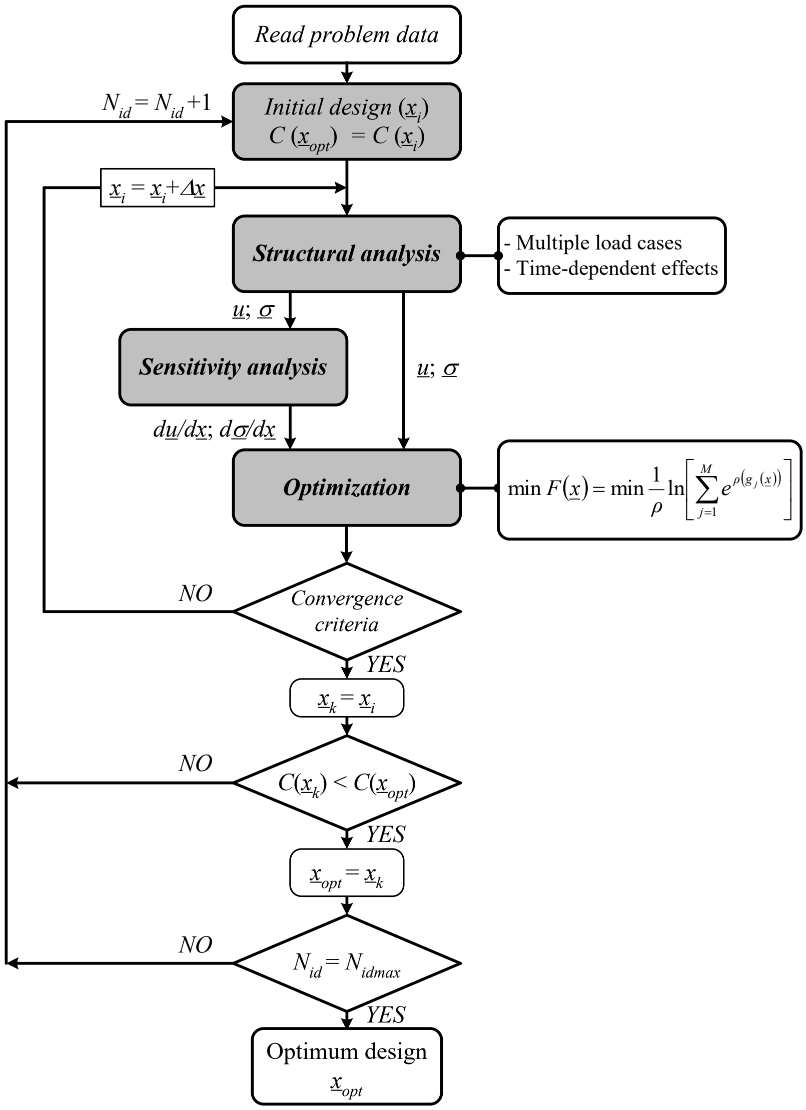

Figure 2: Flowchart of the optimization strategy.

The sensitivity analysis provides the optimization algorithm with the gradients of the objective function and all the design constraints with respect to the design variables. To this aim, the discrete direct method was selected, utilizing both analytical and semi-analytical derivatives. The former was used for sizing and mechanical design variables, and the latter was used for shape design variables.

The computer program begins by reading the problem data and generates a finite element model based on the design variables that describe the bridge's geometry. An iterative procedure is used to determine the prestressing forces and cross-sectional areas of the cable-stays and under-deck cables for an initial design. This automated procedure involves structural analysis, an influence matrix approach, and preliminary design calculations. The gradient-based algorithm utilizes the sensitivities information to minimize the objective function and improve the current design. The design variables are updated, and the analysis and optimization process is repeated until the changes in the design variables and the structure's cost become small. The definition of an initial design and its optimization is repeated for each of the N_{idmax} solutions considered. The optimal design is selected as the least-cost solution from the local optimum solutions (in the Pareto sense) obtained for each N_{id} starting design.

3 Numerical Example and Results

The numerical example concerns the optimization of a single-span combined concrete cable-stayed bridge with a total length of 90 m and a deck width of 15 m (Figure 3). The deck is simply supported at the abutments, featuring a slab-girder cross-section. An "A"-shaped tower with a height given by x_2+1 m was considered. The above-deck suspension features two planes of two cables providing the deck with lateral suspension. Two cables deviated by three struts constitute the under-deck suspension system. The anchoring points of the cable-stays and the struts divide the span into 15-m segments. Longitudinally, the struts are oriented along the bisector of the angle formed by the under-deck cables. In the transverse direction, "v-shaped" struts were adopted. The eccentricities of the under-deck cable-staying system are expressed in terms of the design variable, x_1, corresponding to the maximum eccentricity of the cable-staying system at the central strut. The lateral struts present an eccentricity of $3x_1/4$, corresponding to the eccentricity of a parabola with ends at the under-deck cables anchoring points in the deck, and a maximum eccentricity of x_1 at the central strut. Detailed information regarding the calculation of the nodal coordinates of the struts can be found in a recent article by Martins et al. [17].

Beam elements were used to model the towers and the deck. The main girders are modelled with longitudinal beam elements. The transverse beam elements represent the deck slab and the deck slab plus cross-beams, at the abutments and in the sections where cable-stays and struts are placed. The bridge finite element model is shown in Figure 3 and has a total of 96 nodes and 145 finite elements. The properties and unit costs of the materials considered are presented in Figure 1.

Eight load cases were considered to check the relevant service and strength design constraints. The first case concerns the bridge subjected to dead load at the end of construction. This includes its self-weight and an additional dead load of 2.5 kN/m^2 (flooring, walkways, safety barriers and guardrails). The second case refers to the long-term analysis (18,250 days) of the bridge under the quasi-permanent load combination (dead load plus 20% of road traffic live load). Six additional load cases were considered to account for the most unfavourable effects of the road traffic live load (5 kN/m^2). These correspond to the live load placed on the entire deck length, or only on adjacent, or alternate spans. The construction stages may be relevant in the design of these bridges. Nevertheless, the current paper focuses on the static response of the complete bridge and thus, the erection stages were not directly considered.

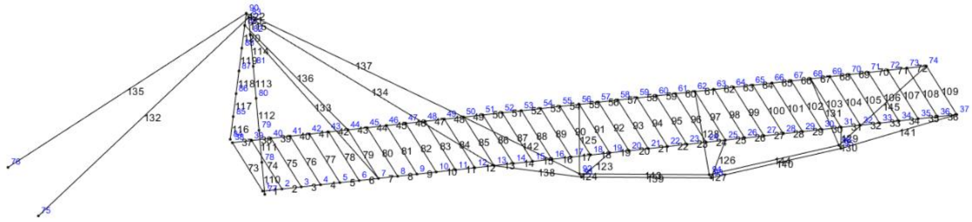

Figure 3: Finite element mesh of the bridge example.

The problem features 23 design variables and 1047 design constraints for the eight load cases. To explore the design space, twelve initial designs were considered and optimized. These were defined by varying the deck sizes, the maximum eccentricity of the under-deck cable-staying system and the height of the towers. This aimed to achieve a deck slenderness of 1/50 and 1/70, a maximum strut length-to-span ratio of 1/8, 1/10 and 1/12, and a height of the towers-to-main span ratio between 1/4 and 1/5. Figure 4 depicts the evolution of the bridge cost throughout the optimization process. The optimized solutions are obtained after a relatively small number of iterations, around 60 to 70 iterations. The example was run on a desktop personal computer (3.20 GHz processor, 64.0 GB RAM and operating system Windows 10 Pro). The average time of optimizing each starting design is approximately 14.3 minutes, corresponding to 80 iterations. Similar optimized solutions are obtained with the different initial designs. The least-cost solution is obtained with the starting design f10_h50_H4 characterized by x_1 = 9 m (f_{strut}/L = 1/10), x_{11} = 1.80 m (h/L = 1/50) and x_2 = 21.5 m (H/L = 1/4) (red curve in Figure 4).

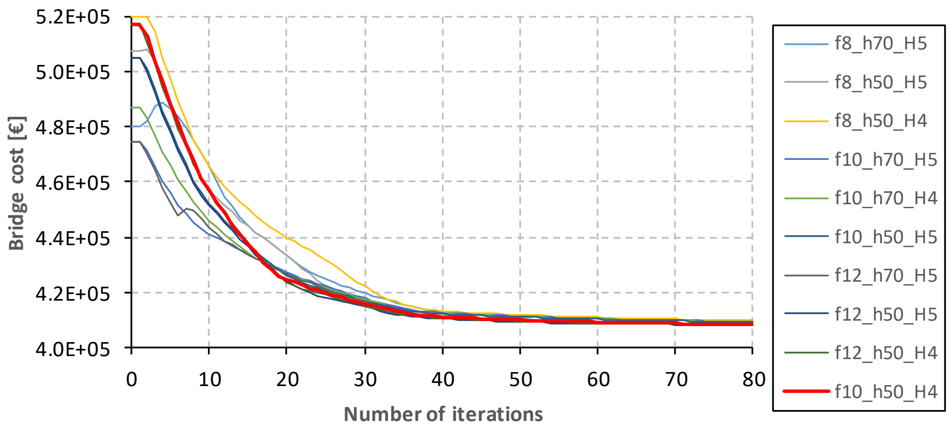

Figure 4: Bridge cost *vs.* number of iterations – multiple starting points.

The least-cost solution presents a cost reduction of 21.0% concerning the initial solution. This is due to a reduction in the sizing design variables of the deck and tower, as well as the tower height (Table 1). In the least-cost solution, the deck, cables, struts and tower represent 56.4%, 35.2%, 4.9% and 3.5% of the total cost, respectively. The value obtained for the cables is mainly due to a fixed cost (18.500 €/cable). Table 1 presents the initial and final values of the cost and the design variables corresponding to the least-cost solution. The least-cost solution presents a maximum value of 9.0 cm for the deck vertical displacements and 1.8 cm for the tower horizontal displacement, considering the time-dependent effects (Figure 5).

Design variable	Initial value	Final value	Design variable	Initial value	Final value
x_1 [m]	9.000	8.335	x_{13} [m]	3.000	1.580
x_2 [m]	21.500	18.250	x_{14} [m]	3.000	1.574
x_3 [kN]	12499	8004	x_{15} [m]	0.300	0.139
x_4 [kN]	2459	2449	x_{16} [m]	0.300	0.100
x_5 [kN]	6320	6132	x_{17} [m]	3.000	1.788
x_6 [kN]	5847	6254	x_{18} [m]	2.000	1.312
x_7 [m^2]	1.32×10^{-2}	8.83×10^{-3}	x_{19} [m]	0.300	0.194
x_8 [m^2]	3.45×10^{-3}	2.78×10^{-3}	x_{20} [m]	0.300	0.217
x_9 [m^2]	7.05×10^{-3}	7.15×10^{-3}	x_{21} [m]	0.300	0.337
x_{10} [m^2]	6.60×10^{-3}	6.70×10^{-3}	x_{22} [m]	0.016	0.012
x_{11} [m]	1.800	1.249	x_{23} [m]	15.000	28.324
x_{12} [m]	0.700	0.600			
Cost	Initial value	Final value	Cost	Initial value	Final value
Deck	271,519 €	230,178 €	Struts	25,076 €	20,165 €
Tower	73,817 €	14,258 €	Cables	146,573 €	143,688 €
Total cost	516,984 €	408,289 €			

Table 1: Initial and final values of the cost and design variables – least-cost solution.

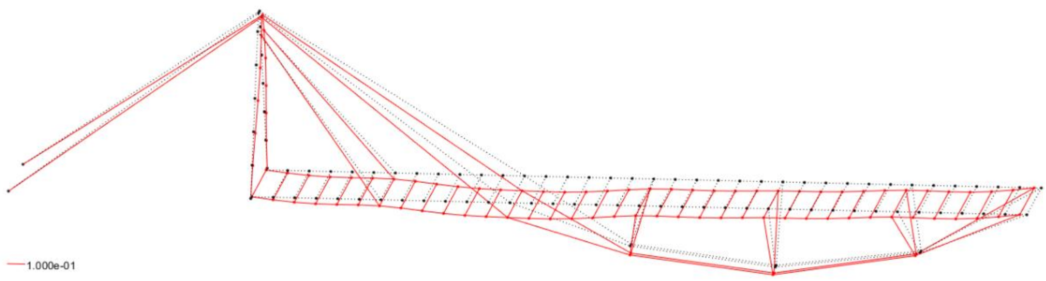

Figure 5: Deformed configuration of the bridge for load case 2 – least-cost solution.

The constraints concerning the deck vertical displacements for load case 2 and the deck normal stresses under service conditions are active at the optimum. Furthermore, the constraints regarding the cables' stresses and the buckling of the struts in load case 3 (bridge under dead load and live load applied on the entire deck) are also active at the optimum.

4 Conclusions and Contributions

The following conclusions can be drawn:

- An optimization-based approach was proposed to assist in the conceptual and preliminary design of combined cable-stayed concrete bridges. The design is posed as an optimization problem to minimize the cost subject to constraints on the displacements and stresses, considering service and strength criteria.
- An efficient convex optimization strategy is adopted to address the original non-convex optimization problem. A gradient-based algorithm is coupled with an automated procedure for defining multiple starting designs. Local optimum solutions are obtained from each starting design, and the least-cost solution is chosen as the optimum design. The original problem is solved indirectly through the minimization of an entropy-based convex scalar function that aggregates the cost and all design constraints in a single objective function.
- The proposed design approach automatically provides the design variables for this innovative bridge typology, balancing the stiffness of the deck and tower, and the suspension effect given by the above-deck and under-deck cable-staying system. This improves the structural behaviour and reduces the overall cost. The optimum solutions satisfy all the design constraints and feature cost reduction, primarily due to a decrease in the values of the sizing design variables of the deck and tower.
- In the least-cost solution, the deck, cables, struts and tower represent 56.4%, 35.2%, 4.9% and 3.5% of the total cost, respectively. The design is governed by the deck vertical displacements and normal stresses under service conditions, the buckling of the struts and the resistance of the cables. The optimum solution features a deck slenderness of 1/72.1, a maximum strut length-to-span ratio of 1/10.8 and a height of the tower-to-span ratio of 1/4.4.
- Future developments should consider topological design variables representing the number of struts and cable-stays, and towers with different typologies, such as “H”-shaped and “inverted Y”-shaped. The optimization considering different types of cross-sections (solid or voided slab, box girder) and solutions (prestressed concrete, steel, steel-concrete composite) for the bridge deck should also be addressed. Furthermore, optimum design for road bridges subject to seismic action and footbridges experiencing pedestrian-induced vibrations should be considered in future research.
- The development of metaheuristic algorithms for addressing this design problem should also be explored. Combining the gradient-based algorithm with a global search procedure will be addressed in upcoming research aiming at an improved exploration of the design space.

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