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Quasi-Static Approximation in Topology Optimization of Beam Structure Subjected to Modal Inertial Forces

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Abstract

Topology optimization under dynamic constraints is challenging problem requiring special optimization procedures involving solution at each iteration step an eigenvalue problem. In this study we propose simple yet effective procedure of topology optimization under various dynamic loadings. The effectiveness of the proposed methodology is related with quasi-static approximation of the optimization process.

Effectiveness of the proposed approach has been demonstrated on a topology optimization of a beam supporting ramp structure. Based on a number of modes it was concluded that proposed method can increase selected natural frequency by a factor of 3.

Keywords: topology optimization, modal analysis, modular structures, structural dynamics, inertial forces, eigenvalue problem.

1 Introduction

Free-form pedestrian ramps are an element of building infrastructure that is susceptible to excessive vibrations, which must be taken into account at the design stage of such structures. Architectural form of an interesting example of this type of structure has been proposed recently by Zawidzki (2020) [1]. The advantage of such structures is their modular construction, which allows them to be freely shaped in even the most diverse terrain. However, one of the difficulties in their design is the correct prediction of the operating loads that cause the above-mentioned vibrations.

One of the first approaches to topological optimization of structures subjected to mechanical vibrations was presented in the work of Diaaz & Kikuchi (1992) [2]. As it was mentioned by in frequently cited book by Bendsoe and Sigmund (2004) [3] the main problem of these types of optimization problems is the non-smoothness of the eigenvalues. To tackle this problem special optimization algorithms are required.

Since the publication above mentioned paper by Diaaz and Kikuchi a great number of researchers have been working in the field of topology optimization subjected to dynamic loadings. Computational procedure for eigenfrequency optimization problem has been proposed by Olhoff and Du (2014) [4]. Their goal was to design the vibrating structures by maximizing their fundamental eigenfrequency, an eigenfrequency of higher order, or the gap between two consecutive eigenfrequencies of given order, subject to a given amount of structural material and prescribed boundary conditions. Level set-based topology optimization of systems vibrating under coupled acoustic–structural excitations was presented by Shu et al.(2014) [5]. Topology optimization framework for structures under stationary stochastic dynamic loads was investigated by Gomez and Spencer (2019) [6].

In this study we conduct research on a simple yet efficient method for topology optimization under dynamic constraints. The method is an extension of previous Authors' work on topology optimization of elastoplastic structures (Blachowski et al. 2020) [7].The effectiveness of the proposed method is demonstrated on an example of clamped beam supporting ramp deck.

2 Methods

The purpose of this study is to propose efficient approximate method allowing to design ramp-type structures subjected to dynamic loadings. The ramp under investigation is shown in Figure 1.

Traditional topology optimization problem for eigenfrequency maximization is replaced with the following approximate solution

$$\mathbf{Kq} = \omega_i^2 \mathbf{M} \boldsymbol{\varphi}^{(i)}, \quad i = 1,2,\dots \tag{1}$$

where $\mathbf{M}$ is mass matrix, $\mathbf{K}$ is stiffness matrix, $\mathbf{q}$ is displacement vector (quasi-static approximation of the mode shape), ω_i and $\boldsymbol{\varphi}^{(i)}$ are natural frequency and corresponding mode shape determined the whole design domain.

Such a formulation of the optimization problem allows us to use a simple variant of fully stressed design algorithm. During the topology optimization procedure formula (1) is used in such a way, that first we select index i, which provides information about inertia forces of i-th mode shape. Next, mass and stiffness matrices are modified at rows and columns corresponding to removed redundant material. It means that at each iteration of the optimization procedure we use the following update formula

$$\mathbf{K}(\boldsymbol{\rho}_l)\mathbf{q}(\boldsymbol{\rho}_l) = \omega_i^2\mathbf{M}(\boldsymbol{\rho}_l)\boldsymbol{\varphi}^{(i)}, \quad l = 1,2, \dots \text{no. of iterations} \tag{2}$$

where $\boldsymbol{\rho}_l$ denotes design variables vector (representing finite element density).

Due to the mode switching phenomenon we keep constant term $\omega_i^2\boldsymbol{\varphi}^{(i)}$**.** It protects the convergence of the optimization procedure since the inertia forces remain constant during the optimization, so that we optimize our system for inertia forces related to i-th mode shape of the initial design domain.

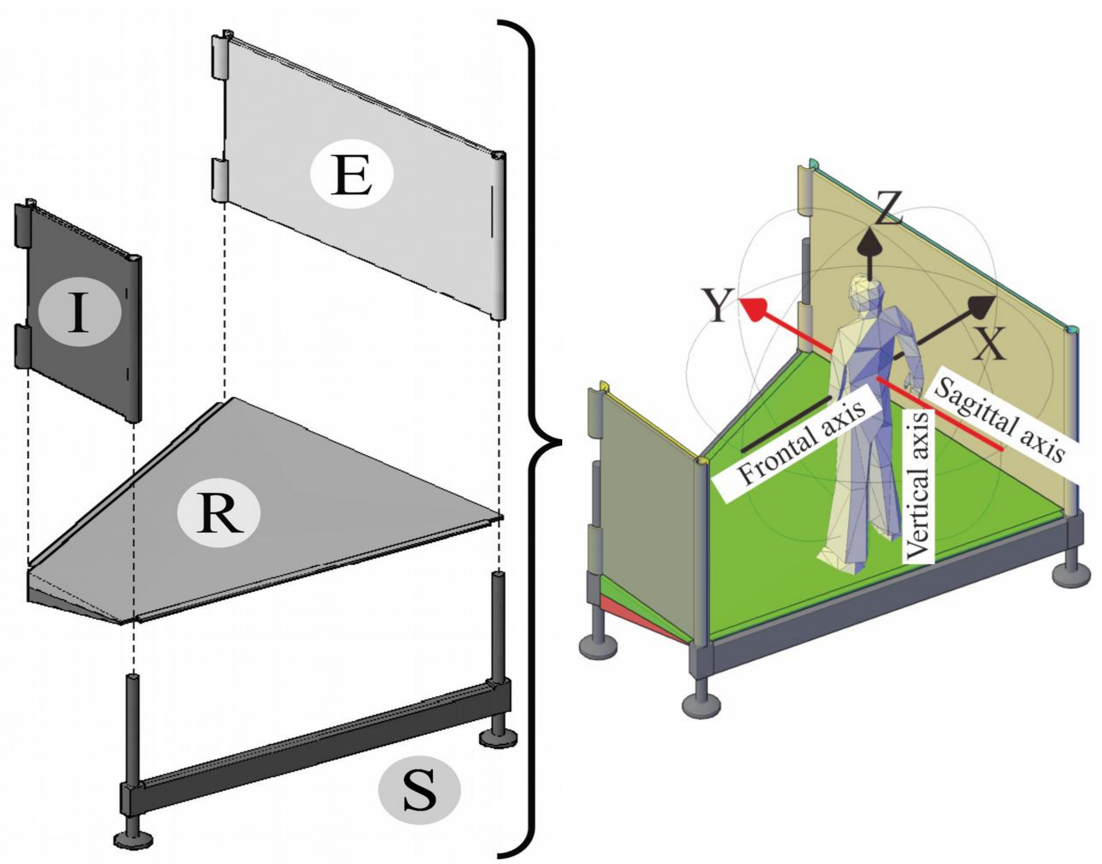


Figure 1: Ramp subjected to dynamic forces resulting to pedestrian traffic.

2.1. Supporting beam (S)

Beam supporting the ram is made of a material with the following properties: Young modulus Ex = 205 GPa, Poisson ration ν = 0.3 and material density ρ_0 = 7850 kg/m^3. Geometric dimensions are height, width and thickness L=9 m, H=1 m, B=1 m, respectively.

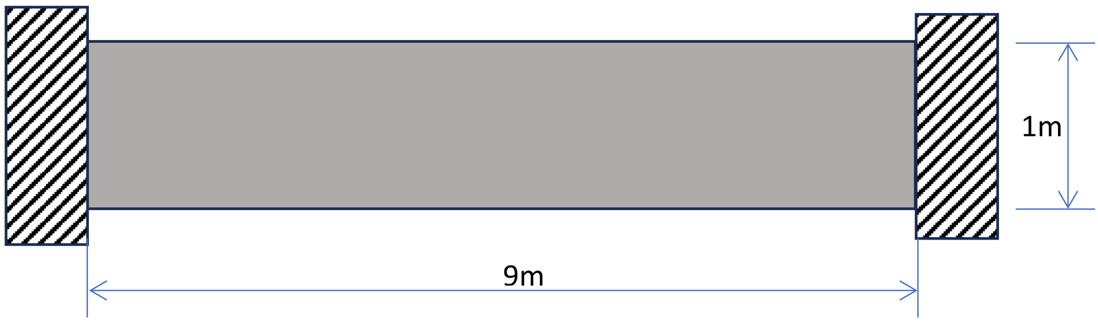


Figure 2: Design domain used in optimization process.

The first step in our method is to determine mode shape and corresponding natural frequencies of the whole design domain.

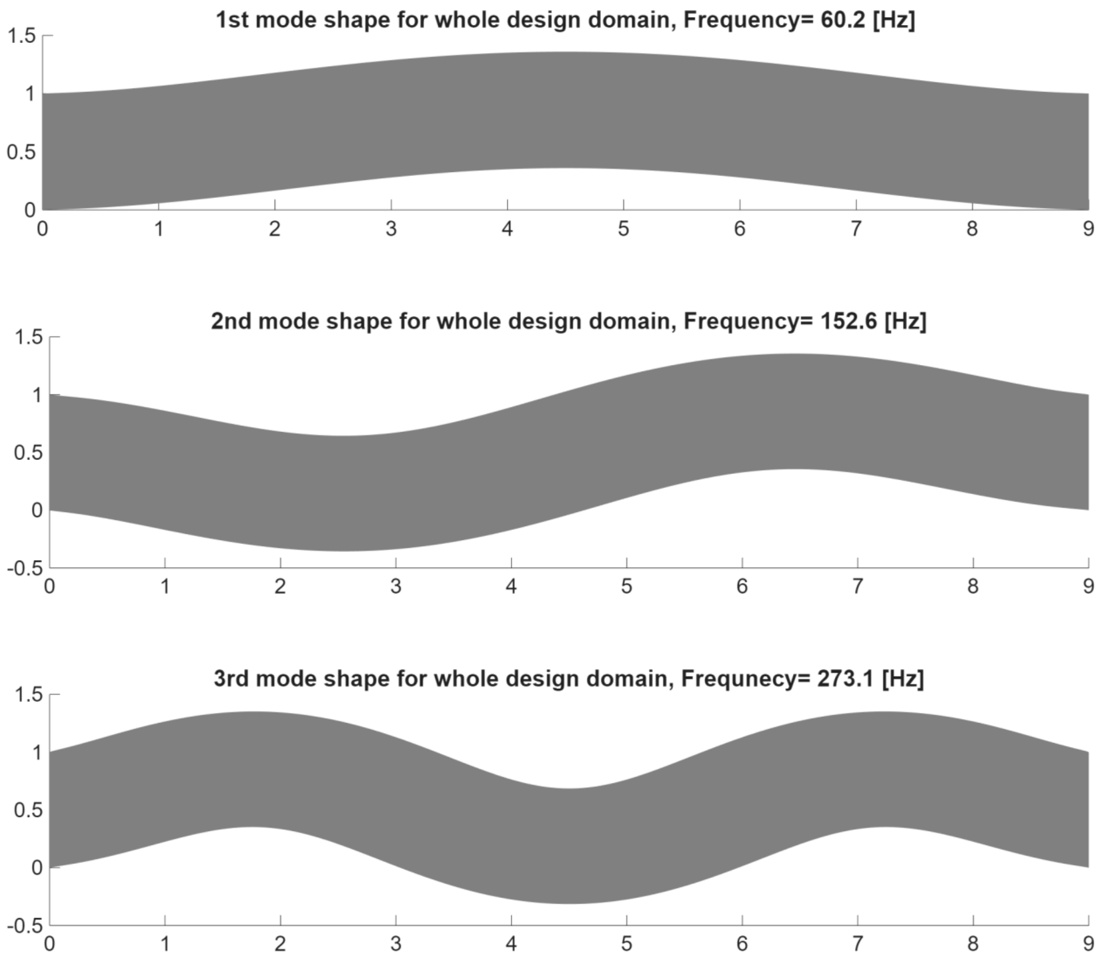


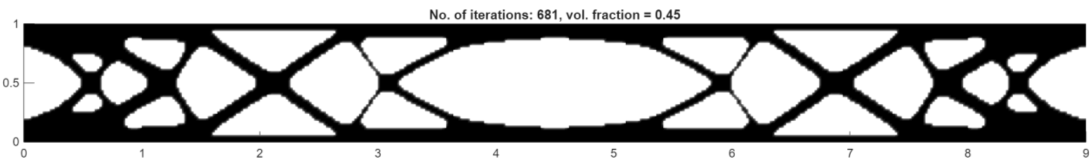
Figure 3: First three mode shape of the design domain.

3 Results

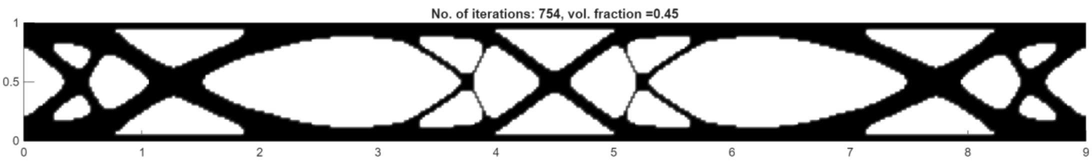
3.1 Optimal topologies under inertia forces of the first three modes

In the first section of the results we present optimal topologies obtained using our quasi-static approximation of the eigenvalue problem. Topologies optimized for the inertia forces generated using 1st, 2nd and 3rd mode of design domain are shown in Figure 3.

a) Optimal topology under 1st inertia force



b) Optimal topology under 2ndt inertia force



c) Optimal topology under 3rd inertia force

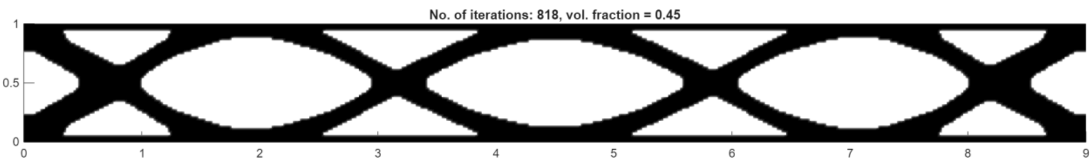


Figure 3: Optimal topologies obtained for the first three inertial forces

Having obtained optimal topologies for the inertia forces related to the first three modes of the design domain, it is interesting to compare their modes with initial modes of the whole design domain.

3.2 Vibration modes of the beam optimized for inertial force of the 1st mode

Mode shapes of the 1st optimal topology have been shown is Figure 4,

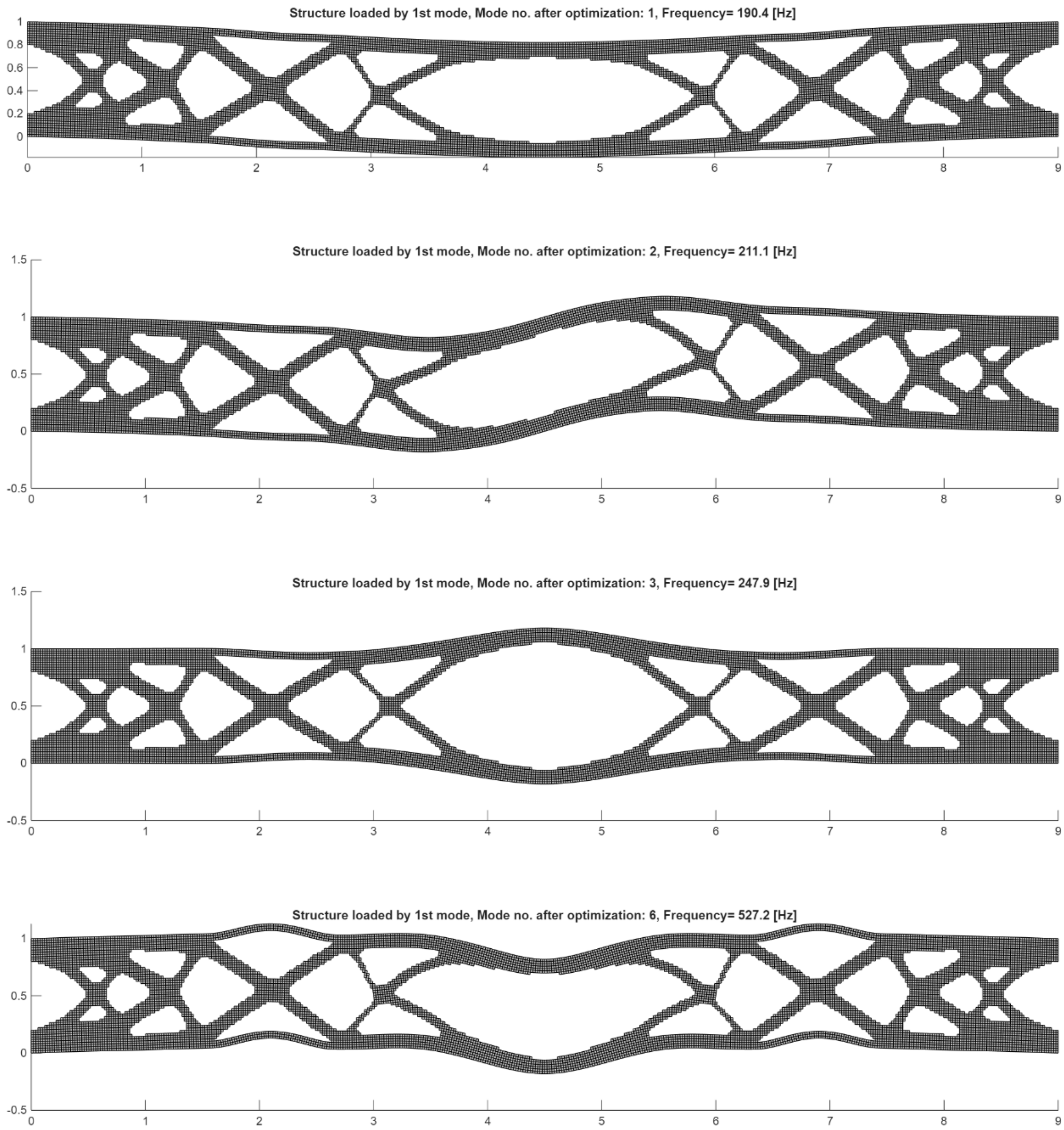


Figure 4: Mode shapes of the 1st optimal topology

Since this topology has been obtain for quasi-statically applied inertial forces of the 1st mode of the design domain (Figure 3) it could be expected that the distribution of the mass is connected with inertial forces of the first mode of the design domain. However, it can be noticed that local modes appear in a set of new modes such as 6th mode shown in Figure 4.

3.3. Vibration modes of the beam optimized for inertial force of the 2nd mode

Similarly as in the case of the topology optimized for inertia force of the 1st mode of the design domain, the topology optimized for inertia forces of the 2nd mode of the design domain are reflecting intensity of the inertial forces of that mode. However, in this case also local modes appear. It can be noticed in Figure 5, where 3 mode after optimization do not require bending of the beam, only deformation symmetric to the neural axis is visible.

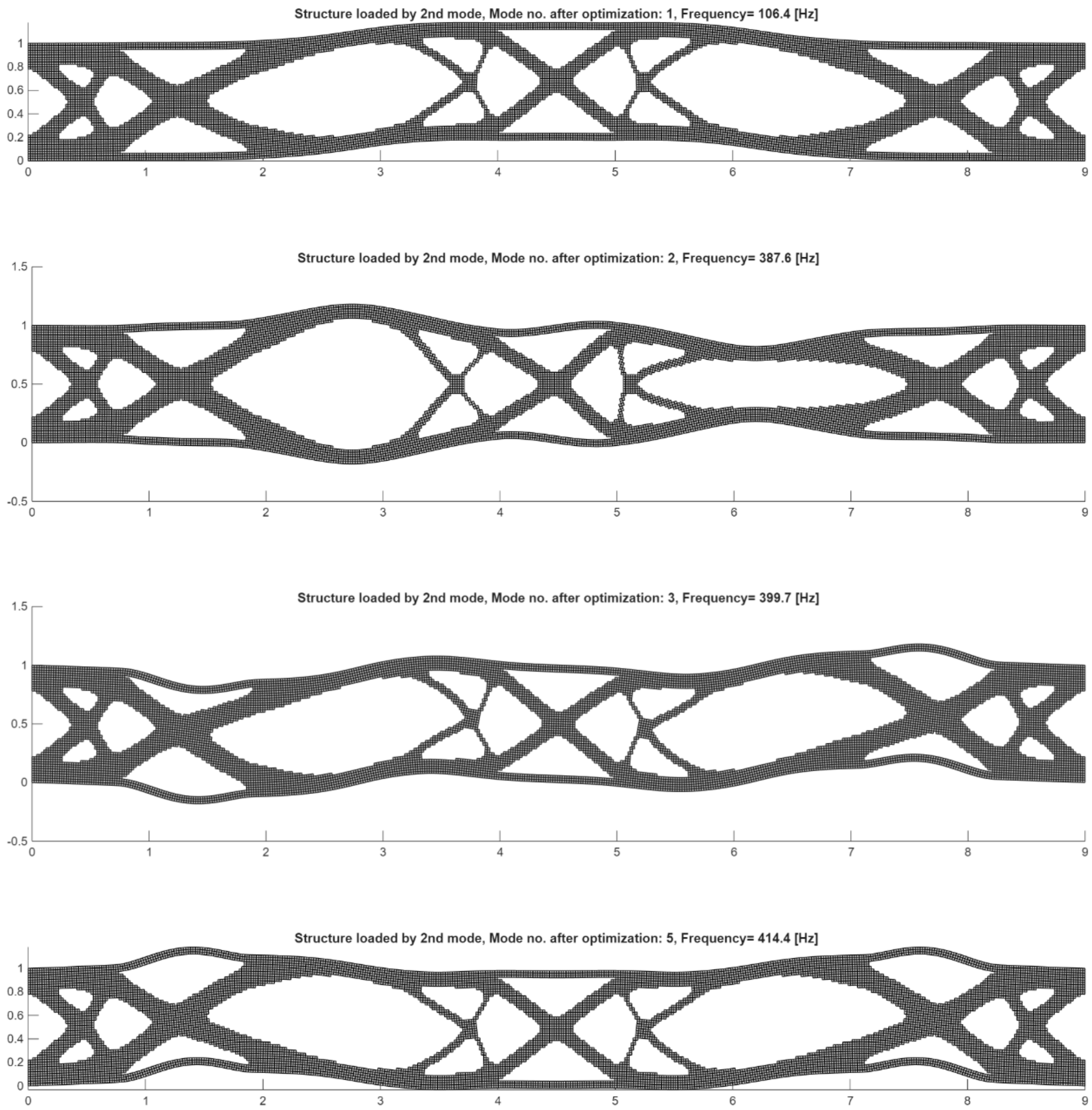


Figure 5: Mode shapes of the 2nd optimal topology

3.4. Vibration modes of the beam optimized for inertial force of the 3rd mode

Finally, similar observation as in the two previous cases, the mass distribution of third topology appears as in a regular form with 3rd half waves.

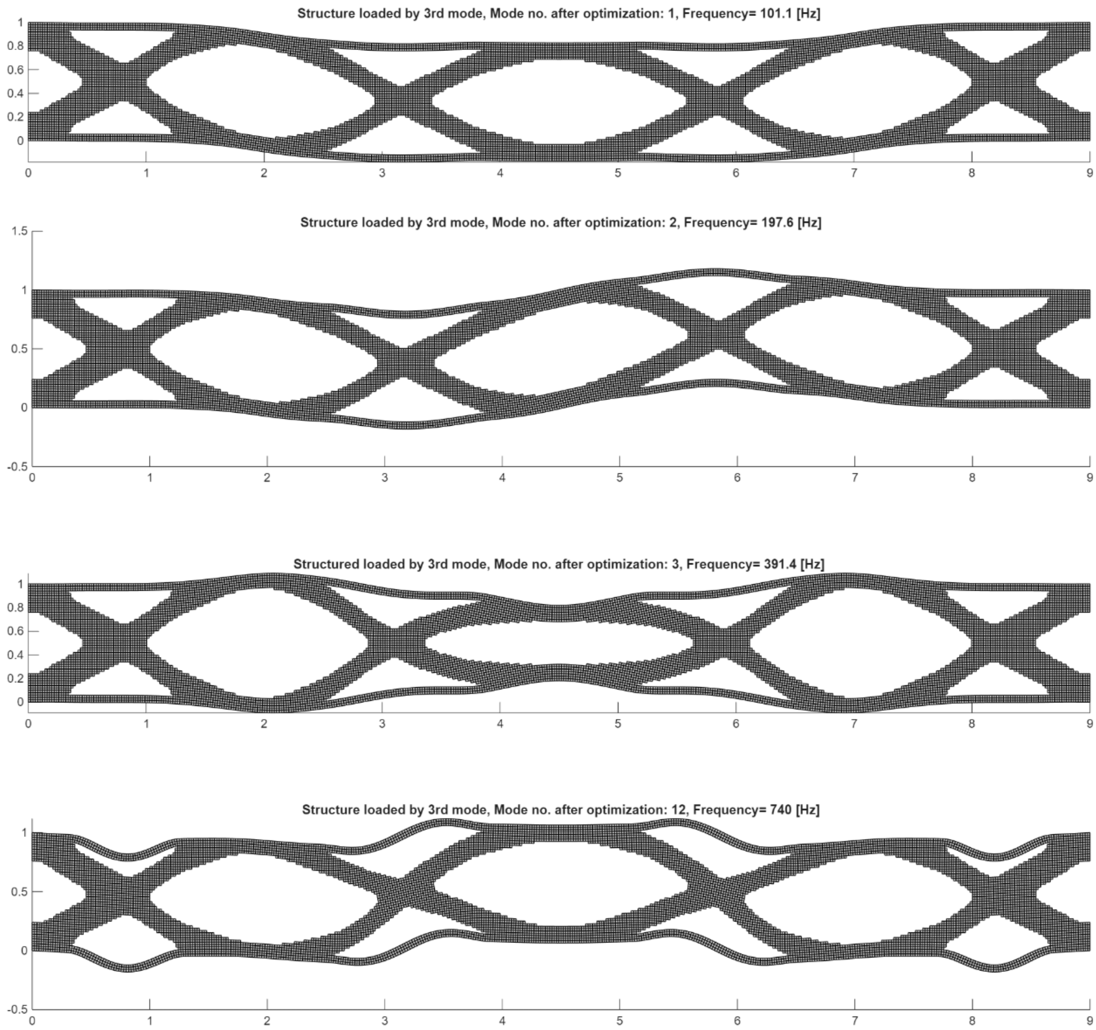


Figure 6: Mode shapes of the 3rd optimal topology

3.5. Modal Assurance Criterion (MAC)

To monitor the improvement of the design during optimization one has to pay attention to the mode switching phenomenon. It exhibits in a reordered way of modes. To keep tracking this modification during optimization process we can use so called Modal Assurance Criterion (MAC) which give us an information about spatial correlation of two pair of modes.
This criterion is usually express by the following formula

$$\mathrm{MAC}_{ik} = \frac{\left[\boldsymbol{\varphi}_j^{(i)}(l)\right]^T \boldsymbol{\varphi}_j^{(k)}(l-1)}{\left\|\varphi_j^{(i)}(l)\right\| \left\|\varphi_j^{(k)}(l-1)\right\|}$$

where $\boldsymbol{\varphi}_j^{(k)}(l)$ is k-th mode shape ($k = 1,2,3$) calculated from optimal topology for inertia forces generated j-th mode of the design domain ($j = 1,2,3$) at l-th iteration.

In tables below columns are related to mode shapes of the optimal topology while rows design domain forms.

We can easily recognize that the highest correlation for mode 1 of optimal topology occurs at 1st mode of design domain. Exception is topology no. 2 which has the third mode correlated with 2nd mode of the design domain.
However, the most discrepancy can be observed for the third mode of the design domain, which is correlated with 6th, 5th and 12th mode of the optimized topology no.1, 2 and 3, respectively.

Optimized topology for 1st mode ------------- ***Design domain mode***	***1***	***2***	***3***	***4***	***5***	***6***
1	**0.99**	0.00	0.00	0.00	0.00	0.10
2	0.00	**0.83**	0.00	0.00	0.53	0.00
3	0.017	0.00	0.00	0.00	0.00	**0.80**

Table 1: Comparison of the mode shapes correlation for the 1st optimal topology

Optimized topology for 2nd mode ------------- ***Design domain mode***	***1***	***2***	***3***	***4***	***5***
1	**0.96**	0.00	0.00	0.00	0.29
2	0.00	0.00	**0.43**	0.00	0.00
3	0.24	0.00	0.00	0.00	**0.84**

Table 2: Comparison of the mode shapes correlation for the 2nd optimal topology

Optimized topology for 3rd mode ------------- ***Design domain mode***	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***	***12***
1	**0.98**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00	0.03
2	0.00	**0.88**	0.00	0.00	0.00	0.00	0.00	0.13	0.12	0.00	0.42	0.00
3	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.22	0.00	**0.87**

Table 3: Comparison of the mode shapes correlation for the 3rd optimal topology

3.6. Natural frequencies before and after optimization

Finally, table 4 provides an information about effectiveness of the proposed optimization process. It can be seen that the first topology optimized for 1st mode of the design domain achieves the highest increase in natural frequency. It move from 60.24 Hz for the first mode of the design domain to 190.4 Hz, which gives increase by factor 3.16, while the natural frequencies of other modes are magnified by factor 1.38-1.93. Similar phenomena can be observed for the second and third natural frequency of the topology no.2 and 3, respectively.

Design domain	*Mode no.*	*1st inertial force*	*Multiplier*	*Mode no.*	*2nd inertial force*	*Multiplier*	*Mode no.*	*3rd inertial force*	*Multiplier*
60,24	1	190,4	**3,16**	1	106,4	1,77	1	101,1	1,68
152,7	2	211,1	1,38	3	399,7	**2,62**	2	197,6	1,29
273,4	6	527,2	1,93	5	414,4	1,52	12	740,0	**2,71**

Table 4: Comparison of the natural frequencies (design domain vs optimal topologies) [Hz]

4 Conclusions and Contributions

In this study we proposed simplified procedure for topology optimization of beams supporting deck of the ramp structures. It was shown that selecting a given mode from the set of modes of the initial design domain we can obtain significant increase in the corresponding frequency by a factor of 3. It can be achieved using quasi-static approximation of the original eigenfrequency maximization problem.

Acknowledgments

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