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Crack-Safe Design Through PeriDynamic-Based SIMP Approach

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Abstract

During the service period of engineering equipment, it is very easy to suffer from the effect of accidental factors occurring local failure, and then evolved into catastrophic macro-failure. Although the current fail-safe design can resist the local failure caused by corrosion, fatigue and manual damage, it cannot resist the one caused by overloading. To solve this problem, this paper proposes a novel crack-safe design through peridynamic-based solid isotropicmaterial with penalization (SIMP) method. This method firstly utilizes the fracture simulation capability of peridynamics to obtain the most dangerous crack case of the structure through continuous iteration and fracture analysis, and then by maximizing the stiffness of the most dangerous crack case to obtain the crack-safe structure. Numerical examples show that this strategy can obtain the structural design with crack-safe effect.

Keywords: local failure, fail-safe design, overloading, crack-safe design, peridynamics, solid isotropicmaterial with penalization (SIMP) method.

1 Introduction

When the structure undergoes local failure, the structural load-bearing performance will be significantly reduced, which will lead to catastrophic consequences in engineering practice. Therefore, how to use topology optimization technology to improve the structural load-bearing performance after the occurrence of local failure has become a key concern of researchers, which is the problem of local failure safety.

Two types of local failure are generally distinguished. The first is caused by overloading. The second type is by accidental factors. At present, researchers are mainly concerned with the local failure safety caused by accidental factors, i.e., the fail-safe problem [1-3], but few researchers are concerned with the local failure safety caused by overloading.

Since the local failure caused by overloading is generally in the form of crack, we define such local failure safety problem as crack-safe problem in order to distinguish it from fail-safe problem. In crack-safe problem, the local failure (i.e., crack) is generated under the effect of external load. Therefore, solving the crack-safe problem requires, firstly, the support of fracture mechanics theory, and secondly, the setting of a reasonable optimization process. In recent years, an emerging theory of fracture mechanics—peridynamics (PD)—has attracted extensive attention from researchers because of its particular suitability for dealing with complex fracture mechanics problems [4, 5]. Therefore, the combination of PD with topology optimization provides the feasibility of solving crack-safe problem.

In this study, we combine the PD theory and SIMP method to propose a feasible optimization process for solving the crack-safe problem. In this process, we first give a non-designable domain based on the force transfer path. The purpose of setting this domain is to ensure the existence of a most dangerous crack case in the crack-safe problem. Next, we will find the most dangerous crack case through continuous iterations and fracture analysis, and then maximize the stiffness of the most dangerous crack case to obtain the crack-safe structure. Numerical examples show that the optimization process can obtain the structural design with crack-safe effect.

The rest of the paper is organized as follows: in Section 2, we give a brief introduction to the PD model and SIMP method, followed by the optimization formulation and solution procedure of the crack-safe problem in the framework of PD and SIMP. A numerical example is provided to demonstrate the effectiveness of the proposed method in Section 3. Section 4 gives the conclusion.

2 Methods

2.1 Bond-based peridynamic model (BBPD)

As illustrated in Figure 1, the BBPD model considers the internal force on a point $\boldsymbol{x}$ in a material as the set of forces on it from all points in the domain of influence of that point, and researchers express this set in terms of an integral, which leads to the integral equilibrium equation:

$$\int_{H_\delta(\boldsymbol{x})} \boldsymbol{f}(\boldsymbol{x}', \boldsymbol{x}) \mathrm{dV}_{\boldsymbol{x}'} + \boldsymbol{b}(\boldsymbol{x}) = \boldsymbol{0}, \forall \boldsymbol{x} \in \Omega, \tag{1}$$

where Ω represents the solution domain; $\boldsymbol{b}(\boldsymbol{x})$ represents the body force density; $H_\delta(\boldsymbol{x}) = \{\boldsymbol{x}' | 0 < \|\boldsymbol{x} - \boldsymbol{x}'\| \leq \delta, \delta > 0\}$ is the neighbourhood of the point x, where δ is called horizon which is the radius of the neighborhood. $\mathrm{dV}_{\boldsymbol{x}'}$ represents the infinitesimal volume associated to point $\boldsymbol{x}'$. $\boldsymbol{f}(\boldsymbol{x}', \boldsymbol{x})$ denotes the nonlocal long-range force that point $\boldsymbol{x}'$ exerts on point $\boldsymbol{x}$, and under the assumption of small deformation, the definition is:

$$\boldsymbol{f}(\boldsymbol{x}',\boldsymbol{x}) = c(\boldsymbol{x}',\boldsymbol{x})\boldsymbol{\eta}(\boldsymbol{x}',\boldsymbol{x}) \cdot \boldsymbol{e}_{\xi} \otimes \boldsymbol{e}_{\xi}, \tag{2}$$

where $\boldsymbol{\xi} = \boldsymbol{x}' - \boldsymbol{x}$ is the connection vector between the two points and it is called "bond" in PD; $\boldsymbol{e}_{\xi}$ represents the unit vector in the direction of $\boldsymbol{\xi}$; $\boldsymbol{\eta}(\boldsymbol{x}',\boldsymbol{x}) = \boldsymbol{u}(\boldsymbol{x}') - \boldsymbol{u}(\boldsymbol{x})$ represents the relative displacement of point $\boldsymbol{x}'$ to point $\boldsymbol{x}$; $c(\boldsymbol{x}',\boldsymbol{x})$ is called the micromodulus function, which corresponds to the mechanical property of material.

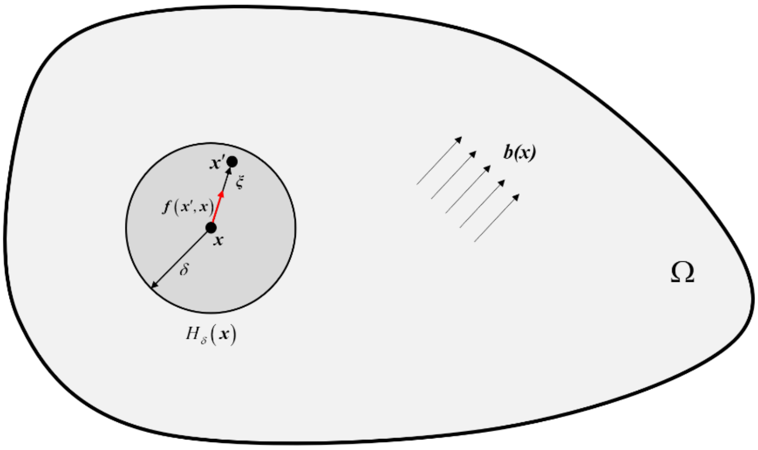


Figure 1: Schematic of the BBPD model.

In PD, bond break is adopted to represent the fracture failure. A general bond break criterion can be expressed as:

$$\mu(\boldsymbol{\xi},t) = \begin{cases} 1, \text{if } s(\boldsymbol{\xi},\tau) \le s_0, \text{for all } 0 \le \tau \le t, \\ 0, \text{otherwise}, \end{cases} \tag{3}$$

where μ is a parameter related to deformation history. When $\mu = 1$, it indicates that the bond $\boldsymbol{\xi}$ is intact. When $\mu = 0$, it indicates that the bond $\boldsymbol{\xi}$ is broken and irrecoverable. The t and τ represent loading time and s_0 is the critical bond stretch, which is generally considered to be related to the critical energy release rate G_0. The s is the stretch of the bond $\boldsymbol{\xi}$ defined as:

$$s = \frac{\|\boldsymbol{\xi} + \boldsymbol{\eta}\| - \|\boldsymbol{\xi}\|}{\|\boldsymbol{\xi}\|}. \tag{4}$$

Once s exceeds s_0, the bond is broken and irrecoverable. Correspondingly, $\boldsymbol{f}(\boldsymbol{x}',\boldsymbol{x})$ will also disappear.

Finally, based on μ, the effective damage for each point $\boldsymbol{x}$ is defined as:

$$\varphi(\boldsymbol{x}) = 1 - \frac{\int_{H_\delta(\boldsymbol{x})} \mu \mathrm{dV}_{\boldsymbol{x}'}}{\int_{H_\delta(\boldsymbol{x})} \mathrm{dV}_{\boldsymbol{x}'}}, \tag{5}$$

which can be used to indicate the failure of the structure.

2.2 SIMP method

The basic idea of the SIMP method is to describe the topology distribution of the structure through a density field function $\rho(\boldsymbol{x})$. The standard optimization formulation of the SIMP method is given by:

$$\begin{aligned} &\text{Find: } \chi(\boldsymbol{x}) \\ &\text{Minimize: } I = I(\chi) \\ &\text{subject to: } g_i(\chi) \le 0, i = 1, \cdots, m, \\ &\qquad \chi(\boldsymbol{x}) = 0 \text{ or } 1, \forall \boldsymbol{x} \in \Omega, \end{aligned} \tag{6}$$

where $I = I(\rho)$ is the objective function. $g_i(\rho)$ represents the *i*-th constraint function with *m* denoting the total number of the constraints. Ω represents the design area.

2.3 The optimization formulation and solution procedure of the crack-safe problem

In this study, the crack-safe problem is solved by maximizing the stiffness of the most dangerous crack case. In the solution process, we first set the non-designable domain based on the force transfer path to ensure the existence of the most dangerous crack case. After that, the most dangerous crack case is found by continuous iterations and fracture analysis. Finally, the crack-safe design is obtained by maximizing the stiffness of the most dangerous crack case. In addition, considering the multi-region and multi-load crack-safe problem that exists in engineering practice, we need to further maximize the stiffness of the most dangerous crack case with the smallest stiffness among all the cases in order to satisfy the multi-region and multi-load crack-safe design. In summary, the optimization formulation for the crack-safe problem in the framework of BBPD and SIMP can be obtained as follows:

$$\begin{aligned} &\text{Find: } \rho(\boldsymbol{x}) \\ &\text{Minimize: } I = \max_{i = 1, \cdots, n_{wc}} \left(-C^i\right) \\ &\text{Subject to:} \\ &\int_\Omega \int_{H_\delta(x)} \frac{1}{2} \boldsymbol{f}\left(\boldsymbol{x}', \boldsymbol{x}, \rho(\boldsymbol{x}'), \rho(\boldsymbol{x}), d^i\right) \cdot \left(\boldsymbol{v}^i(\boldsymbol{x}') - \boldsymbol{v}^i(\boldsymbol{x})\right) \mathrm{dV}_{\boldsymbol{x}'}\, \mathrm{dV}_{\boldsymbol{x}} = 0, \\ &\qquad \forall \boldsymbol{v}^i \in \mathcal{U}_{ad}, i = 1, \cdots, n_{wc}, \\ &\qquad V - \bar{V} = \int_\Omega \rho(\boldsymbol{x}) \mathrm{dV} - \bar{V} \le 0, \\ &\qquad \boldsymbol{u}^i = \bar{\boldsymbol{u}}, \text{on } \Gamma_{\boldsymbol{u}}, i = 1, \cdots, n_{wc}, \\ &\qquad \rho(\boldsymbol{x}) = 1, \forall \boldsymbol{x} \in \Omega^{ud}, \\ &\qquad 0 \le \rho(\boldsymbol{x}) \le 1, \forall \boldsymbol{x} \in \Omega \backslash \Omega^{ud} \end{aligned} \tag{7}$$

with

$$C^i = \int_\Omega \int_{H_\delta(x)} \frac{1}{2} \boldsymbol{f}\left(\boldsymbol{x}', \boldsymbol{x}, \rho(\boldsymbol{x}'), \rho(\boldsymbol{x}), d^i\right) \cdot \boldsymbol{\eta}^i(\boldsymbol{x}', \boldsymbol{x}) \mathrm{dV}_{\boldsymbol{x}'}\, \mathrm{dV}_{\boldsymbol{x}}, \tag{8}$$

where C^i denotes the compliance of the most dangerous crack case for the *i*-th combination and n_{wc} denotes the total number of the combination of the region and load. d^i denotes the most dangerous crack case for the i-th combination, which needs to be obtained through continuous iterations and fracture analysis. To facilitate the measurement of structural stiffness, displacement loading is used as the external load in this study. Therefore, maximizing the smallest most dangerous crack case stiffness in all combinations is equivalent to minimizing the negative of the largest most dangerous crack case compliance. The first constraint equation represents the virtual work principle in bond-based PD, which is used to constrain the force balance of the structure. $\boldsymbol{u}^i$ and $\boldsymbol{v}^i$ represent the i-th displacement function and the test function, respectively; $\bar{\boldsymbol{u}}$ represents the prescribed displacement on the boundary $\Gamma_{\boldsymbol{u}}$; $\mathcal{U}_{ad} = \{\boldsymbol{v} | \boldsymbol{v} \in \mathbb{H}^1(\Omega), \boldsymbol{v} = \boldsymbol{0} \text{ on } \Gamma_{\boldsymbol{u}}\}$. $\bar{V}$ represents the upper bound of available material volume. Ω^{ud} is the non-designable domain and Ω represents the total design domain.

3 Results

In this section, a numerical example is provided to demonstrate the effectiveness of the proposed approach in designing the crack-safe structure. Uniformly distributed particles are adopted for the spatial discretization of the problem design domain. The particle space is of size $\Delta x = \Delta y = \Delta z = 1$, where Δx, Δy and Δz represent the distance between any two adjacent particles in the *x*, *y*, and *z* directions, respectively. Since, in this section, it is only intended to demonstrate the effectiveness of the proposed approach, all parameters in the examples are dimensionless. The Young's modulus and Poisson's ratio are $E_0 = 1$ and $\upsilon = 1/3$, respectively. In PD, the horizon is specified as $\delta = 3\Delta x$, the critical bond stretch is set to $s_0 = 0.06$.

4.1 The L-shape beam problem

The geometry and boundary conditions of the L-shape beam is shown in Figure 2. The black area in Figure 2 is the non-designable domain. The upper bound of the available material is set to 50%$|D|$, where $|D|$ represents the total area of design domain. For the L-shape beam example, in the case of the loading condition shown in Figure 2, we consider the crack-safe problem in two regions, where region 1 is the region containing the connection between the beam structure and the fixed end as shown in the red box in Figure 2, and region 2 is the region containing the concave corner of the beam structure as shown in the blue box in Figure 2.

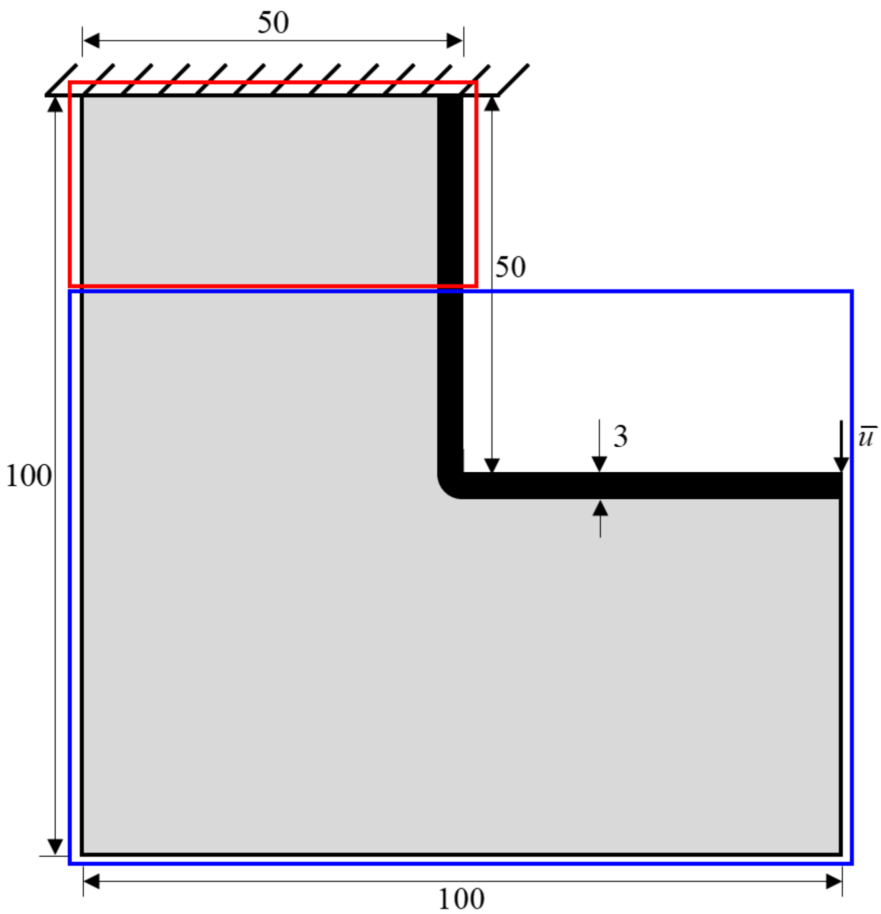
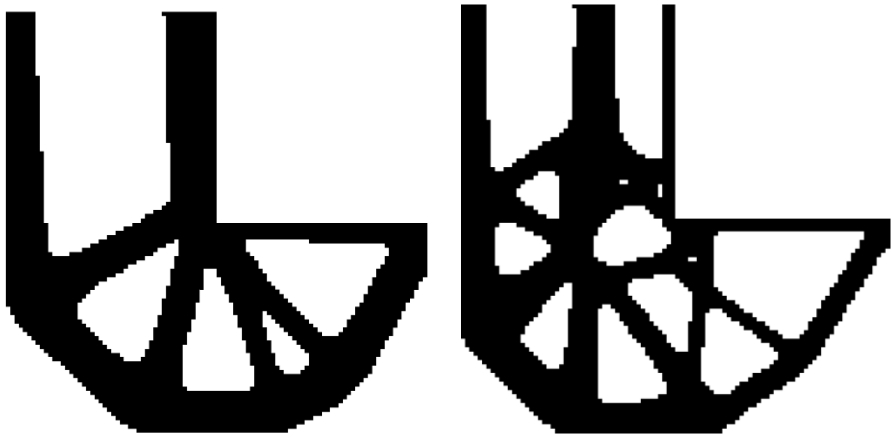


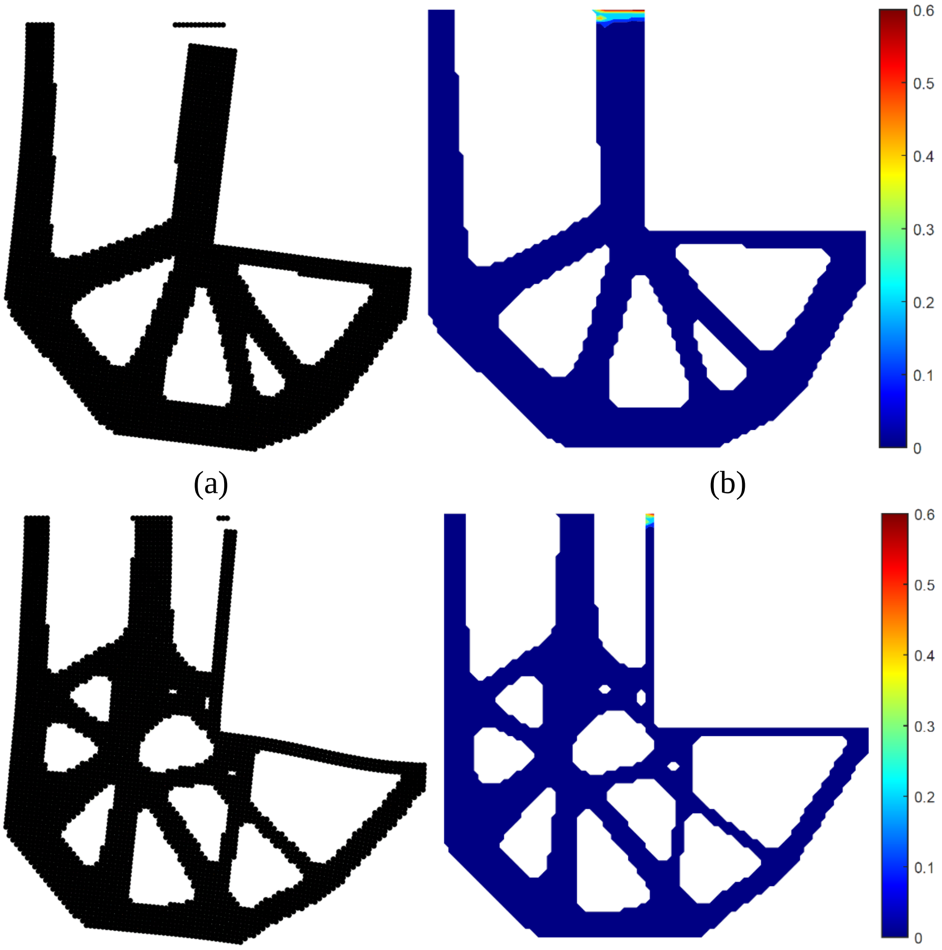
Figure 2: The geometry and boundary conditions of the L-shape beam problem.

After iterative solving, the obtained optimization results are shown in Figure 3. Figure 3(a) shows the classical stiffness maximization result. Figure 3(b) shows the crack-safe result. Next, we perform a quasi-static analysis of the above structure under the considered combinations of region and load to compare the crack-safe effect. The corresponding fracture cases and damage contours of the structures under the first combination are shown in Figure 4. The corresponding fracture and damage contours for the second combination are shown in Figure 5. The force-displacement curves and stiffness-displacement curves of the structures under the first and second combination are shown in Figure 6 and Figure 7, respectively. The stiffness values of the structures before and after the occurrence of fracture and the percentage of the remaining stiffness after fracture with respect to the initial stiffness for the two combinations are given in Table 1. From Figure 6, Figure 7 and Table 1, it can be seen that the crack-safe structure have significant crack-safe effect under both combinations compared to the pure stiffness design. This proves that the proposed method is capable of designing a structural design with crack-safe effect.



(a) (b)

Figure 3: (a) the stiffness maximization result and (b) the crack-safe result.



(c) (d)

Figure 4: The fracture results and damage contours of (a, b) the stiffness maximization result, (c, d) the crack-safe result under the first combination.

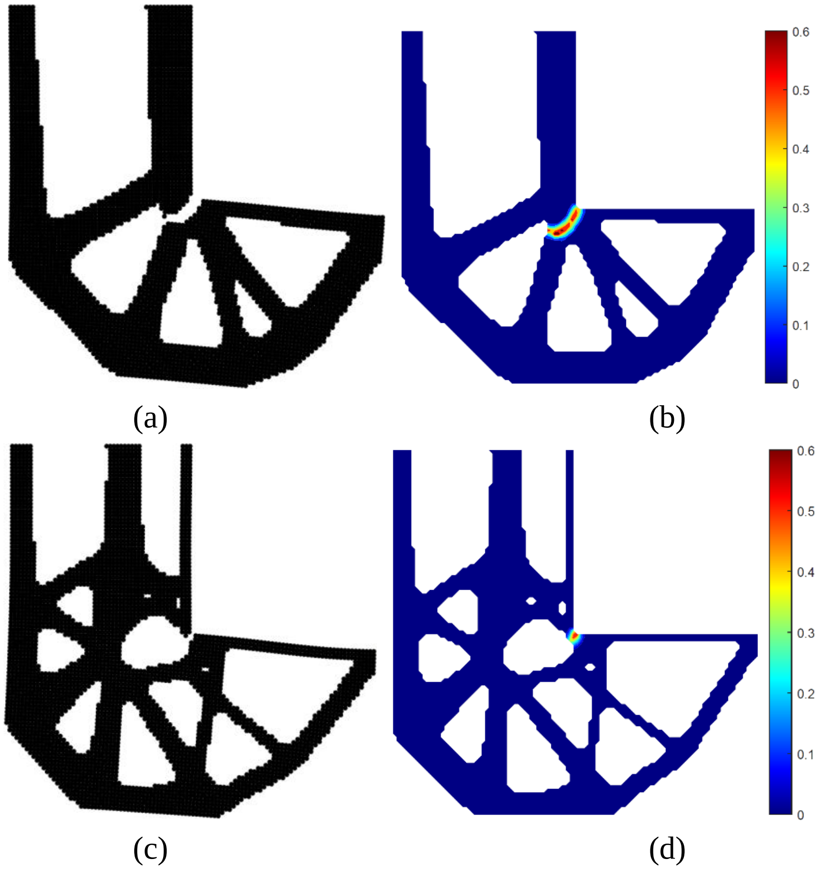
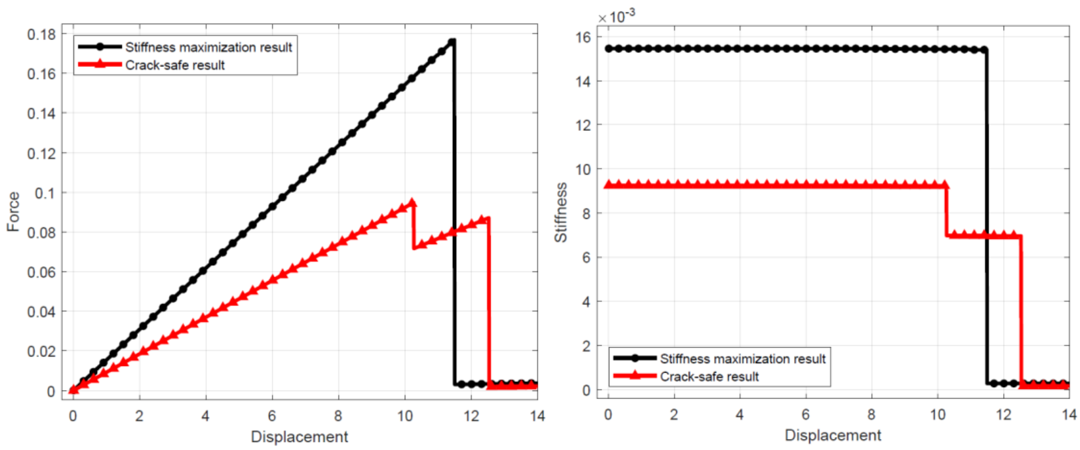
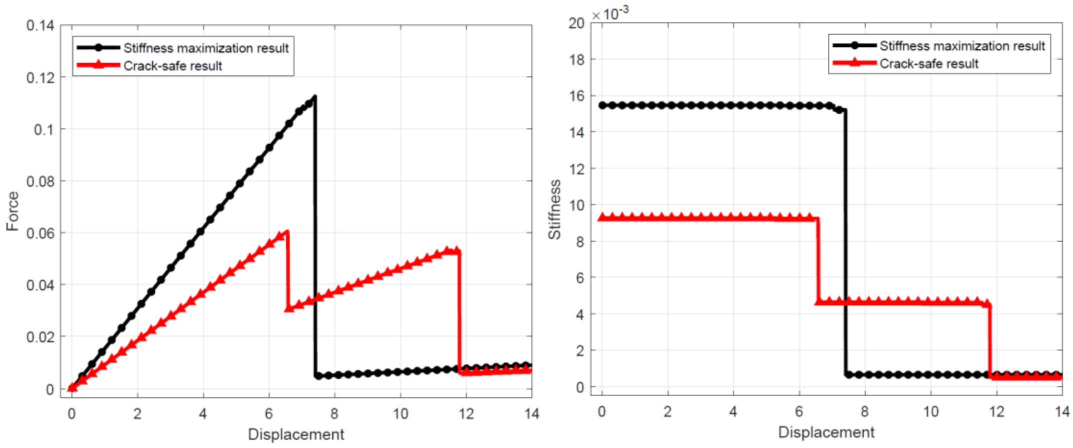


Figure 5: The fracture results and damage contours of (a, b) the stiffness maximization result, (c, d) the crack-safe result under the second combination.



(a) (b)

Figure 6: (a) Force-displacement curves and (b) Stiffness-displacement curves for the first combination.



(a) (b)

Figure 7: (a) Force-displacement curves and (b) Stiffness-displacement curves for the second combination.

	Initial stiffness	First combination		Second combination	
		Remaining stiffness	Remaining percentage	Remaining stiffness	Remaining percentage
Stiffness maximization result	0.01545	0.00027	1.75%	0.00064	4.14%
Crack-safe result	0.00925	0.00698	75.46%	0.00463	50.05%

Table 1: The stiffness of the two structures.

4 Conclusions and Contributions

In this study, a new solution procedure based on PD model and SIMP method is proposed to design crack-safe structures capable of resisting overloading. In the specific process, we first give an undesignable domain based on the force transfer path to ensure the existence of the most dangerous crack case. Next, the most dangerous crack case is found through continuous iterations and fracture analysis. Finally, the crack-safe structure is obtained by maximizing the stiffness of the most dangerous crack case. Numerical examples show that the optimization process given in this study can obtain structural designs with crack-safe effects. The research in this paper fills the gap in local failure safety design.

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Conflict of interest

The authors declared no potential conflicts of interest with respect to the research, author-ship, and/or publication of this article.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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