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Improved TLBO Algorithm for Truss Size Optimization Considering Geometric Nonlinearity

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Abstract

This study introduces a novel enhancement to the Teaching–Learning-Based Optimization (TLBO) algorithm for structural size optimization of trusses by embedding geometrically nonlinear analysis into the design evaluation process. While traditional TLBO-based truss optimizations typically rely on linear finite element analysis, the proposed framework integrates a Newton–Raphson solver to more accurately capture large-displacement behaviour. This modification allows for a more realistic representation of structural response, especially in flexible or slender truss systems. The optimization aims to minimize the total structural weight by adjusting the cross-sectional areas of the truss members, subject to stress and displacement constraints. Constraint violations are addressed using a quadratic penalty formulation. The classical 10-bar truss problem is employed as a benchmark to validate the method. The results demonstrate that incorporating geometric nonlinearity within the TLBO framework significantly improves the robustness and realism of the optimized designs. This enhanced approach provides a practical alternative for realistic truss design without changing the underlying topology.

Keywords: geometric nonlinearity, TLBO, metaheuristics algorithms, truss optimization, nonlinear finite element analysis, Newton–Raphson.

1 Introduction

Structural optimization has become an indispensable tool in modern engineering design, enabling the development of structures that are not only safe and reliable but also economical and sustainable. By systematically refining structural parameters, engineers can achieve designs that optimally balance performance requirements—such as strength, stiffness, and stability—against constraints related to material consumption, construction cost, and serviceability [1,2]. Over the past few decades, this field has evolved significantly, driven by both theoretical advances and computational capabilities.

Structural optimization encompasses three main categories: size, shape, and topology optimization. Size optimization refers to the process of determining the optimal cross-sectional areas or dimensions of structural members to minimize an objective function. Shape optimization focuses on determining optimal boundary geometries to minimize mass while meeting structural constraints. Topology optimization [3,4], on the other hand, seeks the optimal material distribution within a design domain. Rozvany et al. [5] emphasized the importance of layout or topology optimization as an economically rewarding design task, highlighting both exact analytical methods and approximate discretized approaches. Habashneh and Rad [6] developed a topology optimization framework that integrates reliability-based design with geometrically nonlinear analysis accounting for imperfections. Furthermore, Hsu [7] provided a comprehensive overview of structural shape optimization, highlighting the latest advancements in the field.

Over the years, a broad spectrum of optimization algorithms has been developed to solve these problem classes. Gradient-based methods, while computationally efficient for well-posed, differentiable problems, often struggle with complex, non-convex design spaces and can become trapped in local minima. To overcome these limitations, numerous metaheuristic algorithms have been introduced, including genetic algorithms, simulated annealing, particle-based swarms, and differential evolution are commonly applied in structural design optimization, especially when dealing with nonlinear and constrained optimization spaces, all of which have demonstrated robustness in handling nonlinear, multi-modal, and constrained problems [8–14].

Within this group of metaheuristics, the Teaching–Learning-Based Optimization (TLBO) algorithm, first formulated by Rao et al. [15], has gained considerable attention due to its simple structure, parameter-free nature, and effective convergence characteristics. Inspired by classroom dynamics, TLBO simulates the teacher's role in enhancing student performance, with learners iteratively improving their solutions through teacher-guided and peer-interactive phases. In structural applications, TLBO has proven to be a competitive alternative to traditional evolutionary algorithms. Camp and Farshchin [16] demonstrated the effectiveness of TLBO in optimizing three-dimensional trusses using a modified version of the algorithm. Other studies have extended TLBO for use in multi-objective optimization of structures, stress-based design, and vibration control [17,18].

Despite its widespread adoption, most existing TLBO-based structural optimization frameworks rely on linear finite element analysis (FEA) to evaluate design performance. While this simplification reduces computational burden, it fails to capture the true structural behavior under conditions involving large displacements or rotations. In practical applications—such as long-span, slender, or flexible truss systems—geometric nonlinearity can significantly influence internal force distributions and global stability. Neglecting these effects may lead to suboptimal or even unsafe designs.

Motivated by this limitation, the present study proposes an enhanced TLBO framework that incorporates geometrically nonlinear finite element analysis into the optimization loop , hereafter referred to as GNTLBO. A Newton–Raphson-based solver is embedded within the TLBO evaluation phase, enabling the accurate assessment of design candidates under large-displacement conditions. This integration ensures that the algorithm produces solutions that are not only optimal in terms of weight but also realistic in their structural performance. Unlike conventional approaches, the proposed method enhances the physical fidelity of the optimization process without altering the underlying topology.

The effectiveness of the presented approach was examined through the classical 10-bar space truss problem is used as a benchmark. This widely studied problem provides a standardized basis for evaluating optimization algorithms and allows for direct comparison with traditional TLBO implementations. The goal is to minimize the total structural weight by optimizing the cross-sectional areas of the truss members while satisfying allowable stress and displacement constraints. Constraint violations are penalized using a quadratic penalty function, and all computations are performed in MATLAB.

2 Methods

The focus of this research is to optimize the truss member areas in order to minimize the total structural weight, subject to stress and displacement constraints. The structural layout, including node coordinates and element connectivity, is assumed to be fixed throughout the optimization process, i.e., only size variables are considered. This approach follows the classical size optimization framework adopted in earlier studies such as Camp and Farshchin [16], but introduces a significant enhancement through the use of geometrically nonlinear finite element analysis:

$$\mathbf{A} = [A_1, A_2, ..., A_i] \tag{1}$$

with A_i representing the cross-sectional area of the i-th truss element. The total weight W of the structure can be calculated as:

$$W(\mathbf{A}) = \sum^{n}_{i=1} \rho_i A_i L_i \tag{2}$$

In this expression, ρ_i stands for the material's density and L_i denotes the undeformed length of the i-th element. The formulation also includes the following constraints:

$$\sigma_i = |F_i| / A_i \leq \sigma_{allo} \tag{3}$$

$$|u_j| \leq u_{max} \tag{4}$$

The stress constraint ensures that the axial stress in each truss member remains within the allowable limit specified by the material properties. For each member i, the axial stress σ_i is computed as the absolute value of the internal axial force F_i divided by the corresponding cross-sectional area A_i. This value must not exceed the allowable stress σ_{allo}, ensuring structural safety under the applied loading conditions. In addition to stress constraints, displacement constraints are imposed to maintain serviceability and limit excessive deflections. Specifically, the magnitude of displacement u_j at each degree of freedom j must remain below a prescribed maximum allowable displacement u_{max}. These constraints collectively ensure that the optimized truss design is both structurally sound and serviceable. To enforce the constraints, a penalty-based approach is adopted as:

$$f_p(\mathbf{A}) = W(\mathbf{A}) \times (1 + P_\sigma + P_u)^2 \tag{5}$$

The optimization framework developed in this study is based on the Teaching–Learning-Based Optimization (TLBO) algorithm introduced by Rao et al. [15], which simulates the interaction between a teacher and learners in a classroom. The algorithm comprises two main phases: the teaching phase, where learners gain knowledge from the teacher, and the learning phase, where learners interact and share information to improve their understanding.
During the teacher phase, every learner attempts to improve their knowledge based on the gap between the teacher's performance and the current class average. Mathematically, this process is governed by

$$X_k^{(new)}(_j) = X_k^{(old)}(_j) + D(_j) \tag{6}$$

$$D(_j) = TF \cdot r \cdot (T(_j) - M(_j)) \tag{7}$$

Here, $X_k(_j)$ corresponds to the j-th variable of the k-th solution, with $T(_j)$ indicating the teacher's value and $M(_j)$ the population average. The teaching factor TF , often set to 1 or 2, scales the influence of the teacher, while r is a uniformly distributed random number in the range [0,1]. The computed update direction $D(_j)$ encourages each learner to move toward the teacher's position.

In the original TLBO formulation, the mean M(j) is calculated as a simple average across the population. However, to improve search efficiency, a fitness-weighted mean is sometimes used, which emphasizes the influence of high-performing individuals:

$$M(_j) = (\sum_{k=1}^{n} F_k \cdot X_k(_j)) / (\sum_{k=1}^{n} F_k) \tag{8}$$

The Learner Phase of the TLBO algorithm simulates the collaborative learning process among students. In this phase, each learner attempts to enhance their performance by interacting with another randomly selected learner. The underlying principle is that learners can learn from each other, particularly when one has better performance. The update rule depends on a fitness comparison between two randomly selected individuals, say p and q. If learner p performs better than learner q, the new design is updated by moving p further in the direction away from q; otherwise, p moves toward q. This process is mathematically expressed as:

$$X_p^{(new)}(j) = X_p(j) + r \cdot (X_p(j) - Xq(j)), \quad \text{if } F_p < Fq \tag{9}$$
$$X_p^{(new)}(j) = X_p(j) + r \cdot (Xq(j) - X_p(j)), \quad \text{otherwise} \tag{10}$$

where r is a stochastic value randomly sampled from the interval [0,1], and F_p, Fq are the objective values (fitnesses) of learners p and q, respectively. This interaction helps diversify the search process and prevents stagnation by allowing information exchange across the population. The learner phase is executed N times so that every individual has the opportunity to improve through peer interaction.

Each design candidate generated during the optimization process is evaluated using a geometrically nonlinear finite element analysis based on the Newton–Raphson iterative method. Unlike linear analysis, which assumes infinitesimal displacements and a fixed stiffness matrix, geometrically nonlinear analysis captures the influence of large displacements and rotations on the structural response by updating both internal forces and the stiffness matrix in each iteration. This capability is essential for accurately assessing slender truss systems or load scenarios that induce significant geometric changes, as linear assumptions may lead to unsafe or overly conservative designs [19].

$$\mathbf{R} = \mathbf{F}_{ext} - \mathbf{F}_{int}(\mathbf{U}) \tag{11}$$

where $\mathbf{R}$ is the residual force vector, $\mathbf{F}_{ext}$ is the externally applied load vector, and $\mathbf{F}_{int}(\mathbf{U})$ is the internal force vector, which is a function of the current displacement vector $\mathbf{U}$. The internal forces are calculated based on the deformed geometry at each iteration, accounting for axial elongation and direction changes of the truss elements.

$$\mathbf{U}^{k+1} = \mathbf{U}^k + [\mathbf{K}^k]^{-1} \mathbf{R}^k \tag{12}$$

where $\mathbf{K}^k$ is the tangent stiffness matrix evaluated at the k-th iteration. This matrix represents the sensitivity of internal forces to changes in displacement and must be updated at each step to reflect the evolving structural configuration. Convergence is assumed when the norm of the displacement increment $\Delta\mathbf{U}$ falls below a predefined tolerance level. If convergence is not reached within a maximum number of iterations, the current design is penalized or discarded as infeasible.

The Newton–Raphson technique is widely adopted in structural mechanics due to its quadratic convergence behavior, provided that the initial guess is close to the true solution. Its integration into metaheuristic-based optimization frameworks—such as the TLBO algorithm presented here—enhances the physical realism of the design evaluation, particularly for load cases where geometric effects are non-negligible. Similar nonlinear solvers have been employed in previous truss optimization studies to capture the true structural response under large deformations.

3 Results

To validate the performance of the proposed TLBO framework enhanced with geometrically nonlinear analysis, the classical ten-bar truss problem is adopted as a benchmark case study. This structure is widely utilized in structural optimization research due to its simplicity, well-defined constraints, and availability of reference solutions. The ten-bar truss consists of six nodes and ten members, arranged in a planar configuration forming two stacked triangles connected at the middle. The nodal layout and member connectivity are fixed throughout the optimization process. The design domain is symmetric, and the structure is subjected to vertical point loads applied at two nodes on the bottom chord. The design variables are the cross-sectional areas of the ten truss members, Ai, for i = 1, 2, ..., 10, which are to be optimized to minimize the total structural weight. The coordinates of the nodes, element connectivity, material properties, and loading conditions are listed below.
Steel was selected as the material for the truss, characterized by a Young's modulus of 10,000 ksi and a density of 0.1 lb/in^3. The allowable axial stress is limited to 25 ksi, and the maximum nodal displacement is constrained to 2.0 inches. The loading conditions involve two vertical point loads of 100 kips applied downward at nodes 2 and 3. Nodes 1 and 4 are fully constrained in both the X and Y directions, providing support to the structure. Each cross-sectional area Ai is bounded between 0.1 in^2 and 35.0 in^2 to ensure stability and prevent singularities in the stiffness matrix. All members are assumed to have circular cross sections for simplicity in analysis and practical constructability. The configuration of the ten-bar truss, including node and member numbering, loading conditions, and support constraints, is illustrated in Figure 1.

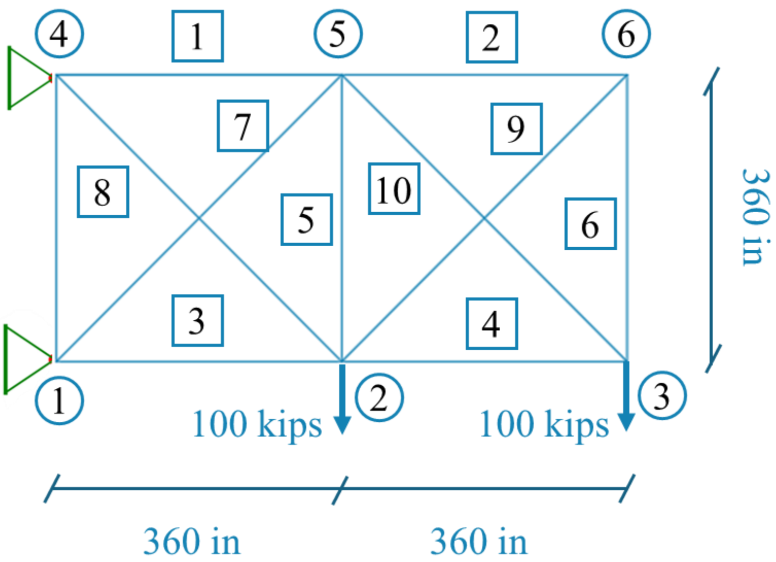

Figure 1. 10-bars truss layout

The main goal of the optimization process was to minimize the overall weight of the truss by adjusting the cross-sectional areas of its ten structural members. The best design obtained through the proposed approach resulted in a total weight of 504.801 lb, highlighting the method's effectiveness. Details of the optimal cross-sectional areas are listed in Table 1. Notably, the solution tends to assign smaller areas to the diagonal elements, while larger sections are preserved for the vertical and bottom chord members, indicating a well-balanced material distribution in response to the applied loads and boundary conditions.

Member	**Area (in^2)**
1	28.31
2	5.45
3	12.12
4	20.15
5	25.56
6	31.97
7	20.91
8	10.42
9	26.10
10	3.35

Table 1: Optimized areas for the truss members.

These values satisfy all imposed stress and displacement constraints under large-displacement behavior. The incorporation of geometrically nonlinear analysis ensures that member deformations are accurately captured, enhancing the reliability of the final design. Compared to the results reported by Camp and Farshchin [16], who obtained a final weight of 545.175 lb using the classical TLBO method based on linear finite element analysis, the proposed nonlinear-enhanced framework offers a notable improvement in structural efficiency. The reduction in weight reflects the advantage of evaluating member responses under actual geometric nonlinearity, which avoids over-conservatism associated with linear assumptions. Despite using the same initial problem setup, the refined solution space and improved constraint handling allow for a more realistic and lighter design.

The convergence behavior of the GNTLBO algorithm throughout the optimization process is illustrated in Figure 2. The figure depicts the penalized objective value (total weight) at each iteration. As seen, the optimization process exhibits a smooth and rapid decline in the objective value within the first few iterations, followed by a gradual tapering as it approaches the optimal solution. The convergence curve highlights the algorithm's ability to balance exploration and exploitation effectively. The final objective stabilizes at approximately 504.801 lb, confirming the robustness of the proposed approach. The efficient convergence trend also reflects the proper functioning of the penalty function in handling constraint violations during early iterations.

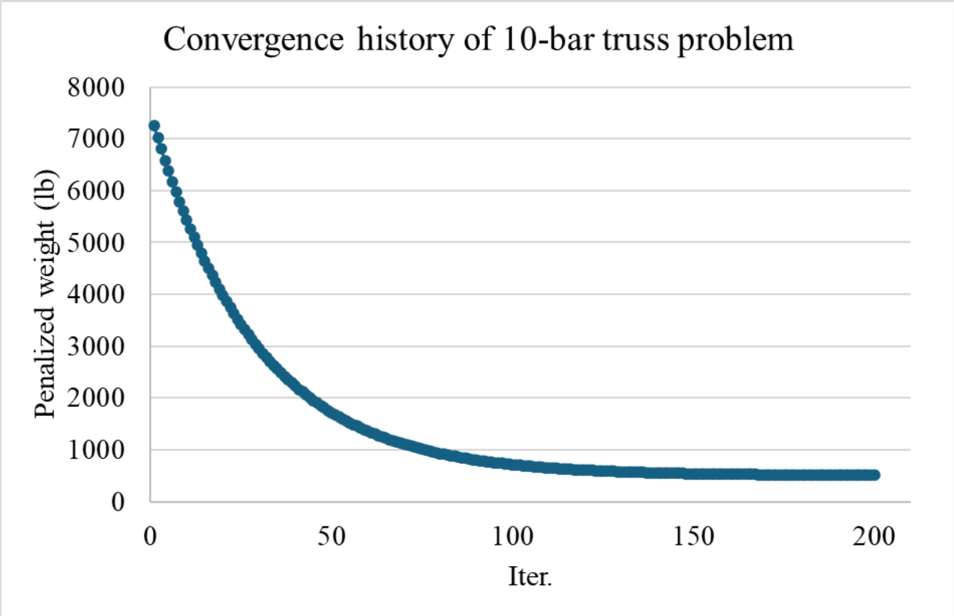

Figure 2. Convergence history of GNTLBO

4 Conclusions and Contributions

This study presented a geometrically nonlinear extension of the Teaching–Learning-Based Optimization algorithm, referred to as GNTLBO, for structural size optimization of trusses. The proposed approach incorporates a Newton–Raphson-based nonlinear analysis procedure directly within the optimization loop, allowing for the accurate evaluation of structural response under large displacements. By doing so, the framework improves upon conventional TLBO implementations that rely on linear assumptions and are therefore prone to inaccuracies in flexible or slender structures. The effectiveness of the proposed method was demonstrated using the classical 10-bar truss benchmark problem. The GNTLBO algorithm successfully identified a design with a final structural weight of 504.801 lb, which satisfies all stress and displacement constraints. Compared to existing linear TLBO results from the literature, the GNTLBO framework produced a lighter and more realistic design, emphasizing the importance of considering geometric nonlinearity in structural optimization.

Furthermore, the convergence history revealed that the algorithm maintains strong performance in terms of exploration and convergence, with the penalized objective value rapidly declining and stabilizing around the optimal design. This behavior indicates that the integration of geometric nonlinearity does not compromise the convergence capability of the TLBO algorithm but rather strengthens its applicability to real-world structural problems. Future work may extend this approach to more complex three-dimensional structural systems or integrating plastic materials.

References

[1] Vanderplaats GN. Structural Optimization-Past, Present, and Future. Https://DoiOrg/102514/351158 2012;20:992–1000. https://doi.org/10.2514/3.51158.

[2] Bendsøe MP. Optimization of Structural Topology, Shape, and Material. Optimization of Structural Topology, Shape, and Material 1995. https://doi.org/10.1007/978-3-662-03115-5.

[3] Habashneh M, Cucuzza R, Aela P, Movahedi Rad M. Reliability-based topology optimization of imperfect structures considering uncertainty of load position. Structures 2024;69:107533. https://doi.org/10.1016/J.ISTRUC.2024.107533.

[4] Grubits P, Cucuzza R, Habashneh M, Domaneschi M, Aela P, Movahedi Rad M. Structural topology optimization for plastic-limit behavior of I-beams, considering various beam-column connections. Mechanics Based Design of Structures and Machines 2024. https://doi.org/10.1080/15397734.2024.2412757.

[5] Rozvany GIN, Bendsø MP, Kirsch U. Layout Optimization of Structures. Appl Mech Rev 1995;48:41–119. https://doi.org/10.1115/1.3005097.

[6] Habashneh M, Movahedi Rad M. Plastic-limit probabilistic structural topology optimization of steel beams. Appl Math Model 2024;128:347–69. https://doi.org/10.1016/J.APM.2024.01.029.

[7] Hsu YL. A review of structural shape optimization. Comput Ind 1994;25:3–13. https://doi.org/10.1016/0166-3615(94)90028-0.

[8] Rajeev S, Krishnamoorthy CS. Discrete Optimization of Structures Using Genetic Algorithms. Journal of Structural Engineering 1992;118:1233–50. https://doi.org/10.1061/(ASCE)0733-9445(1992)118:5(1233).

[9] Fourie PC, Groenwold AA. The particle swarm optimization algorithm in size and shape optimization. Structural and Multidisciplinary Optimization 2002;23:259–67. https://doi.org/10.1007/S00158-002-0188-0/METRICS.

[10] Schutte JF, Groenwold AA. Sizing design of truss structures using particle swarms. Structural and Multidisciplinary Optimization 2003;25:261–9. https://doi.org/10.1007/S00158-003-0316-5/METRICS.

[11] Hajela P. Nongradient methods in multidisciplinary design optimization - Status and potential. J Aircr 1999;36:255–65. https://doi.org/10.2514/2.2432.

[12] Saka MP. Optimum Design of Steel Skeleton Structures. Studies in Computational Intelligence 2009;191:87–112. https://doi.org/10.1007/978-3-642-00185-7_6.

[13] Wu CY, Tseng KY. Topology optimization of structures using modified binary differential evolution. Structural and Multidisciplinary Optimization 2010;42:939–53. https://doi.org/10.1007/S00158-010-0523-9/TABLES/5.

[14] Kaveh A, Talatahari S. Particle swarm optimizer, ant colony strategy and harmony search scheme hybridized for optimization of truss structures. Comput Struct 2009;87:267–83. https://doi.org/10.1016/J.COMPSTRUC.2009.01.003.

[15] Rao R V., Savsani VJ, Balic J. Teaching–learning-based optimization algorithm for unconstrained and constrained real-parameter optimization problems. Engineering Optimization 2012;44:1447–62. https://doi.org/10.1080/0305215X.2011.652103.
[16] Camp C V., Farshchin M. Design of space trusses using modified teaching–learning based optimization. Eng Struct 2014;62–63:87–97. https://doi.org/10.1016/J.ENGSTRUCT.2014.01.020.
[17] Wu D, Wang S, Liu Q, Abualigah L, Jia H. An Improved Teaching-Learning-Based Optimization Algorithm with Reinforcement Learning Strategy for Solving Optimization Problems. Comput Intell Neurosci 2022;2022:1535957. https://doi.org/10.1155/2022/1535957.
[18] Farshchin M, Camp C V., Maniat M. Multi-class teaching–learning-based optimization for truss design with frequency constraints. Eng Struct 2016;106:355–69. https://doi.org/10.1016/J.ENGSTRUCT.2015.10.039.
[19] de Borst R, Crisfield MA, Remmers JJC, Verhoosel C V. Non-Linear Finite Element Analysis of Solids and Structures: Second Edition. Non-Linear Finite Element Analysis of Solids and Structures: Second Edition 2012. https://doi.org/10.1002/9781118375938.