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A Physics-Informed Neural Network Approach to Estimating the Coefficient of Consolidation in Geotechnical Engineering

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Abstract

The coefficient of consolidation is a critical parameter in geotechnical engineering, influencing the design and safety of various infrastructure projects. Traditionally, coefficient of consolidation is estimated through laboratory consolidation tests, which are time-consuming, subject to operator variability, and may not accurately reflect in-situ conditions. The challenge becomes more significant when site-specific information is incomplete or unavailable. With the increasing integration of Physics-Informed Neural Networks (PINNs) in geotechnical modeling, this study proposes a novel PINN-based framework that incorporates the Mikasa's one-dimensional consolidation equation to estimate site-specific coefficient of consolidation values under varying data availability.

In this study, we developed a PINN-based model to estimate the coefficient of consolidation using only subsidence data, without requiring explicit information on embankment surcharge loading history**.** The results demonstrate that the proposed approach can reliably infer coefficient of consolidation capturing the essential features of the consolidation process. This work highlights the adaptability, efficiency, and physical consistency of the PINN framework, particularly in data-scarce geotechnical settings. By reducing dependence on traditional laboratory testing and prior loading records, this approach offers a scalable and interpretable alternative for consolidation analysis in both research and practical applications.

Keywords: soil consolidation, Physics Informed Neural Networks (PINNs), coefficient of consolidation, artificial neural network, soil subsidence., geotechnical modeling.

1 Introduction

Soil consolidation is a fundamental phenomenon in geotechnical engineering, describing the gradual compression of saturated soils under external loading due to the dissipation of excess pore water pressure. Accurate understanding of this process is crucial for predicting settlement behavior in foundations, embankments, and other infrastructure, especially in fine-grained soils such as clays and silts. For that effective coefficient of consolidation (c_v) estimation is a crucial parameter.

The classical one-dimensional (1D) consolidation theory introduced by Terzaghi (1925) [1] laid the foundation for modern consolidation analysis. However, it has limitations, particularly in accurately estimating the coefficient of consolidation (c_v)[2]. Because the coefficient of consolidation (c_v) plays a vital role in estimating the rate and extent of settlement. Terzaghi's theory uses the coefficient of consolidation (c_v) to model pore pressure dissipation, indirectly linking it to settlement over time. To address this limitation, Mikasa (1963) proposed a more realistic approach that defines coefficient of consolidation (c_v) based on strain rather than pore water pressure, allowing a more direct representation of settlement behavior [3].

Traditionally, coefficient of consolidation (c_v) is estimated through laboratory tests such as the Casagrande logarithm-of-time fitting method (Casagrande and Fadum, 1940) [4] and the Taylor square-root-of-time method (Taylor, 1948)[5]. Among these, the Casagrande method is most used in soil mechanics' practice [6] . However, these conventional approaches are often time-consuming, susceptible to human error, and constrained by limited data availability—particularly in remote locations or post-disaster conditions where timely and accurate soil assessment is critical.

With advances in deep learning, data-driven methods, especially Physics-Informed Neural Networks (PINNs), have emerged as powerful tools for solving forward and inverse problems governed by partial differential equations (PDEs). PINNs embed physical laws such as governing equations and boundary conditions directly into the learning process, enabling models to generalize well even with sparse or noisy data [7] . Recent applications of machine learning in geotechnics include regression-based prediction of the coefficient of consolidation (cv) [8], pore pressure estimation [9] , and stratified ground consolidation [10]. However, most of these studies are based on Terzaghi's 1D equation. Very few have incorporated Mikasa's formulation into the PINN framework, despite its improved realism in capturing soil behavior. Furthermore, the challenge of estimating cv when embankment loading history is unavailable, a common issue in post-disaster environments or un-instrumented sites, remains underexplored.

In this study, we propose a PINN-based consolidation model that incorporates Mikasa's one-dimensional governing equation to predict the soil coefficient of consolidation (cv) and one-day subsidence. By integrating known embankment deformation into the Mikasa PDE, the model provides a valuable alternative to traditional laboratory testing methods.

2 Methods

2.1 Laboratory test:

One-dimensional consolidation tests were conducted on a silty clay specimen using an oedometer under controlled laboratory conditions. The testing protocol involved incremental loading and soaking to replicate in-situ behavior. On Day 1, the vertical deformation was recorded as 4.5×10^{-3} cm. The pre-consolidation pressure (σ_c) or the sandy soil was identified at 0.05 kgf/cm². The coefficient of consolidation (cv) was estimated using the Casagrande logarithmic time fitting method based on test results over the first three days. Relevant data from Day 1, used in the subsequent PINN modeling, are summarized in Table 1.

2.2 Mikasa Governing Equation:

The governing equation for one-dimensional consolidation adopted in this study is based on the classical formulation by Mikasa, under the assumption of small deformations:

$$\frac{\partial \varepsilon}{\partial t} = c_v \frac{\partial^2 \varepsilon}{\partial z^2} \quad (0 \leq z \leq H;\ t > 0) \tag{1}$$

Here, ε represents strain, which is a function of time t and depth z. For generality and numerical stability, Equation (1) was nondimensionalized using:

Depth: $Z = \frac{z}{H}$ and Time factor: $T_v = \frac{c_v t}{H^2}$

The resulting nondimensional form is:

$$\frac{\partial \varepsilon}{\partial T_v} = \frac{\partial^2 \varepsilon}{\partial Z^2} \quad ; \ [0 \leq Z \leq 1,\ T_v > 0] \tag{2}$$

2.3 Initial and Boundary Conditions:

In this study, one-dimensional consolidation of a single soil layer was modeled with the equation (1). The soil layer was defined from z= 0 at the top surface to z=H at the bottom. The top boundary (BC_t) at z=0 was set to allow water to drain out, while the bottom boundary (BC_b) at z=H was treated as a no-flow boundary, representing a plane of symmetry. This setup allowed us to model only half the soil layer, simulating double drainage conditions with less computational effort.The initial condition (*IC*) assumed that the excess pore water pressure was the same throughout the layer right after loading, representing an immediate application of load.

2.4 Physics-Informed Neural Networks (PINNs)

A PINN framework was developed to solve the nondimensionalized consolidation problem. The model integrates data-driven observations with physics-based constraints derived from Equation (2), enabling both forward predictions and inverse parameter estimation.

a. Loss Function

The total loss function $\mathcal{L}_{Total}(\theta;\ \tau)$ comprises four components:

- Data Loss $\mathcal{L}_{OBS\_DEFORM}$: Measures discrepancy between observed and predicted deformation.
- PDE Residual $\mathcal{L}_{PDE}$: Enforces conformity to the governing equation.
- Boundary Condition Loss $\mathcal{L}_{BC}$: Enforces boundary conditions at soil surface and base.
- Initial Condition Loss $\mathcal{L}_{IC}$: Enforces the initial strain distribution.

$$\mathcal{L}_{Total}(\theta;\ \tau) = \mathcal{L}_{OBS\_DEFORM} + \mathcal{L}_{PDE} + \mathcal{L}_{BC} + \mathcal{L}_{IC} \tag{3}$$

Model accuracy was evaluated using the root mean square error (RMSE):

$$\mathcal{L}_{RMSE}(\theta;\ \tau) = \frac{1}{N_{obs}} \sum_{i=1}^{N_{obs}} (y_{z,i}^{predicted} - y_{z,i}^{observed})^{2} \tag{4}$$

b. Network Architecture and Training

The PINN architecture consisted of a fully connected network: 2×15×20×1, where the input corresponds to nondimensional depth and time, and the output is vertical strain. The *tanh* activation function was used across all hidden layers. Weights were initialized via the Glorot normal method. A two-stage training strategy was adopted:

1. Stage 1: The coefficient of consolidation (*cv*) was fixed at 3.05×10^{-8} m²/s (from laboratory tests). The network was trained to reproduce observed subsidence and strain profiles.
2. Stage 2: The fixed coefficient of consolidation (*cv*) was released as a trainable parameter, enabling the model to infer a spatially and temporally varying coefficient of consolidation (*cv*) profile based on observed data.

Training employed the *Adam optimizer* with a learning rate of 10^{-3} over 400 epochs. Test loss curves (e.g., for Day 1 in Figure 1) confirmed convergence, with low residuals indicating strong generalization.

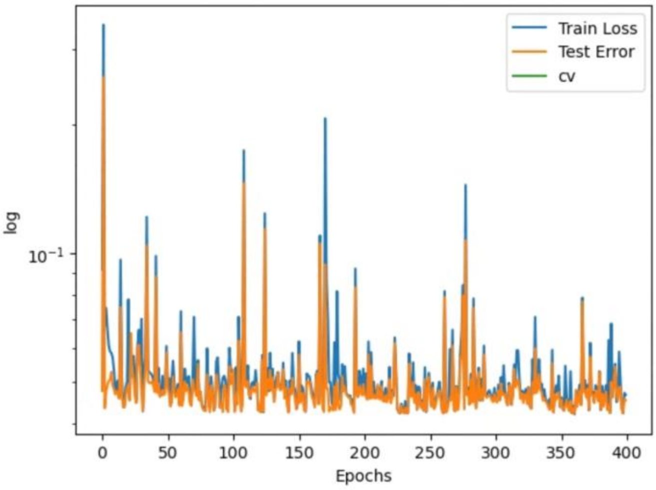

Figure 1: The evolution of the testing errors as a function of the training epochs for day one

c. Boundary Filtering Strategy

During the enforcement of the governing PDE in the PINN training, collocation points near critical spatial and temporal boundaries were filtered out. Specifically, points close to the maximum depth ($z = z_{max}$) at the earliest ($t = t_{min}$) and at the 1-day mark (t = 2400), were excluded from the set of random spatiotemporal collocation points where the PDE residual is evaluated. This collocation point filtering avoids numerical stiffness and boundary-induced artifacts, enabling the model to focus on accurately learning the interior solution of the consolidation PDE and ensuring stable and robust PDE enforcement throughout training.

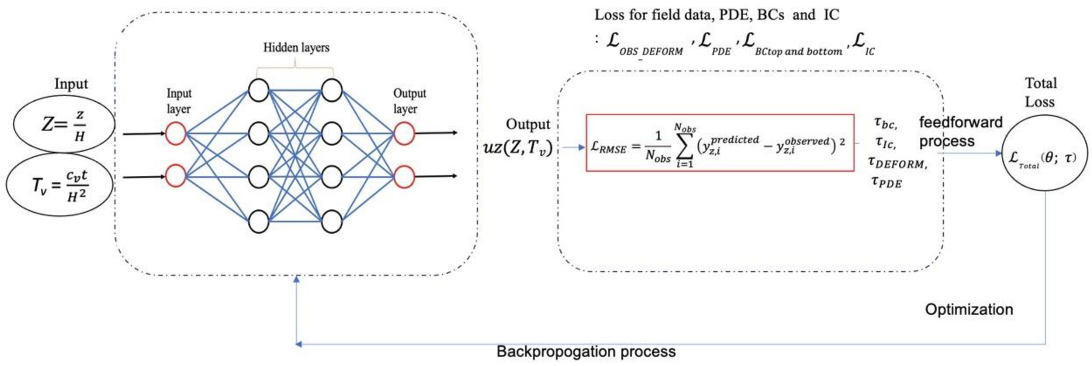

Figure 2: Flow diagram of network.

3 Results

On Day 1, the Physics-Informed Neural Network (PINN) model yielded an initial estimate for the coefficient of consolidation (c_v), of 2.32 × 10⁻⁴ cm²/s. This prediction slightly overestimated the benchmark analytical value of 1.97 × 10⁻⁴ cm²/s, which was obtained using Casagrande's logarithmic time fitting method, a standard technique in geotechnical engineering for interpreting laboratory consolidation data. The relative error between the PINN prediction and the benchmark value was approximately 17.77%, indicating a minor numerical discrepancy. This level of accuracy is acceptable for preliminary geotechnical assessments, where small deviations in coefficient of consolidation (c_v) have minimal impact on consolidation time predictions. Nonetheless, this early prediction highlights the capability of the PINN framework to capture the temporal evolution of consolidation behavior. The close agreement between the PINN-derived coefficient of consolidation (c_v), and the analytical reference supports the model's ability to approximate physics-governed deformation responses from limited data inputs, even in the absence of direct loading information.

Parameters	PINN [cm²/s]	Lab test [cm²/s]	Relative error [%]
c_v	2.32×10^{-4}	1.97×10^{-4}	17.77

Table 1: '*Cv*' Estimation by PINN vs Lab Test and Relative Error (%)

Further insights were obtained from the PINN-predicted subsidence profile [see (Figure 3)] under the applied load during Day 1, which demonstrated excellent agreement with the laboratory-measured subsidence observed under identical loading conditions. The model accurately captured the soil deformation, corresponding to the onset of *primary consolidation*, which predominantly occurred within the first 200 minutes. This behavior is typical of near-surface soils, where higher permeability facilitates faster pore pressure dissipation, leading to more immediate settlement following load application. After this initial phase, the subsidence profile stabilized, indicating a transition to a slower deformation regime.

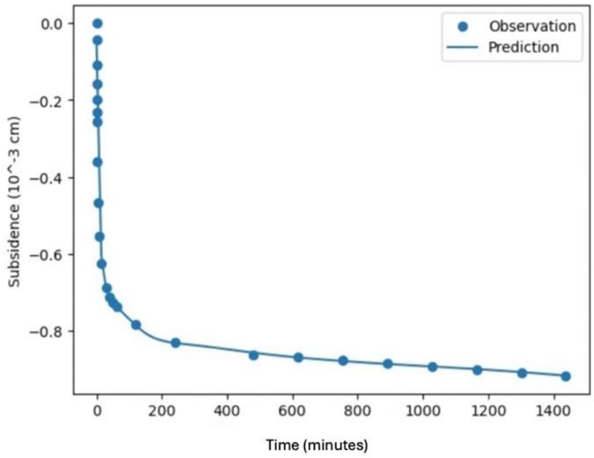

Figure 3: PINNs predicted and observed soil subsidence for one day

4 Conclusions and Contributions

This study demonstrated the effectiveness of Physics-Informed Neural Networks (PINNs) in modeling the one-dimensional consolidation behavior of soft soils. By integrating observational data with governing physical laws, the PINN model was able to estimate the coefficient of consolidation (c_v) and replicate the subsidence profile without requiring explicit loading information.

The model's early prediction of coefficient of consolidation (c_v), closely approximated the value obtained using Casagrande's logarithmic time fitting method, with a relative error of 17.77%. Moreover, the PINN successfully captured the initial rapid contraction of near-surface soils during the primary consolidation phase, followed by a stable settlement response, consistent with both laboratory observations and theoretical expectations. The results confirm the potential of PINNs as a robust and data-efficient framework for inverse analysis and predictive modeling in geotechnical engineering. By enforcing physical consistency through partial differential equations and boundary conditions, PINNs offer a powerful alternative to conventional empirical approaches, particularly in scenarios where field data are limited or incomplete. Future work will focus on extending the framework to multi-layered soil systems and incorporating time-varying loading conditions to further enhance the applicability of PINNs in real-world geotechnical problems.

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