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Analysis of the Negative Effect of High Damping on the Instability of Moving Inertial Objects

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Abstract

In this paper, layered track models are investigated in terms of detecting cases that may be dangerous for track design. Usually, track design is guided by recommendations to avoid exceeding the critical velocity, which is determined as the lowest velocity of waves propagation in the structure, which in turn is equal to the critical velocity of a single moving constant force. Such reasoning does not take into account the anomalous Doppler effect causing a moving inertial object to become unstable. It has been shown in the author's previous works that this effect becomes more dangerous when two close objects are considered, and is generally exacerbated by increased damping, which is contrary to common sense. This paper sheds more light on this deleterious effect and discusses its implications for track design.

Keywords: layered track models, critical velocity, instability, moving proximate masses, integral transforms, instability lines.

1 Introduction

Promoting rail transport stands out as a fundamental strategy in combating climate change and controlling carbon emissions. Consequently, there is a growing call to enhance rail infrastructure, focusing on augmenting track capacity through measures such as increasing travel speed and frequency of passing trains, and accommodating higher axle loads. In track design, a fundamental consideration is the critical velocity of the moving force, reflecting the lowest velocity of wave propagation within the supporting structure, as a result of dynamic interaction among all components.

However, for the above reasons, the problem of instability becomes more significant. Historically, this challenge was circumvented as a single moving mass typically veered into unstable behaviour solely within the supercritical velocity domain, preventively addressed in design protocols. However, a new complication arises with the dynamic interaction of two proximate moving masses, a complication aggravated paradoxically by increased damping. This issue has already been covered in the author's prior research. Nonetheless, the fact that such masses ought to be linked by a rigid bogie reveals new perspectives on the matter.

A considerable amount of work has already been presented in this field, proving that it is a field of significant importance and still very active. Published research can be classified using several criteria: according to the structure type into finite or infinite; or according to moving objects to structures subjected to moving force(s) or moving inertial object(s). The separation can also be made according to supporting structure arrangement into continuous (2D or 3D), or discrete, which usually consists of several layers. The moving force problem is generally much simpler, implying that fully analytical solutions can be derived in several cases.

Among pioneering works on the instability of moving inertial objects one can mention [1-3]. In [1,2], the problem of instability is exemplified on several masses moving on a finite beam. In [3], vibrations induced by single mass moving on a viscoelastically supported infinite beam are solved by integral transforms and numerical integration. The instability of single moving mass is further detailed in [4-5], where the solution is presented with the help of the D-decomposition method. Several inertial objects traversing finite structures have been recently analyzed in [6-8]. Other works on infinite structures are also implementing the D-decomposition method, which is then combined either with the dynamic Green´s function [9-12] or integral transforms [13,14]. It is commonly assumed that the mass is in permanent contact with the beam, [13,14], however, in some works a contact spring is introduced, [9-12]. None of the works on instability of moving inertial object(s) is making reference and connection to the critical velocity of the moving force. A new approach to identify instability by tracing the so-called instability lines and connection with the critical velocity of the moving force which is essential to understanding instability is given in author's works [15,16]. This approach is also suitable for the problem of two moving proximate masses, where a strong dynamic interaction can significantly alter the onset of instability. Using the mentioned approach, the conditions under which the results can be superposed can also be derived and cases where the dynamic interaction induces instability at a velocity lower than the lowest critical velocity of the moving force can be identified. Additionally, this approach can be readily extended to moving oscillators. A summary of several conclusions about layered models is presented in [17], and methods for identifying the necessary parameters for such models are described in detail in [18]. Some of the irregular cases are already summarized in [19], and the issue of instability of a single moving mass on a three-layer model is extensively detailed in [20], especially with respect to the damping level. A recent work [21] brought the need to consider the full bogie, but the instability is again determined by the D-decomposition method and the essential link to the critical velocity of the moving force is again neglected. Therefore, the results presented in this paper are novel and shed new light on the problem.

In this paper, layered models of railway track are under consideration. First, the problem is specified alongside with simplifying assumptions for the analysis. The new approach introduced in previous author's works [15-17,19,20] is used to narrow the range of parameters yielding to unstable cases of proximate masses in the subcritical range of velocities. Then such cases are extended by adding the missing part of the two-axle bogie, the results are compared, and conclusions are drawn. All results are presented in dimensionless parameters to cover a wide range of possible scenarios.

2 Problem formulation

One version of the problem deal in this contribution is depicted in Figure 1. In more detail, in this figure the three-layer model is traversed by two moving proximate masses. Reductions to one or two-layer models are obvious and can be consulted in [15-17]. Extension to two-axle bogie is exemplified in Figure 2.

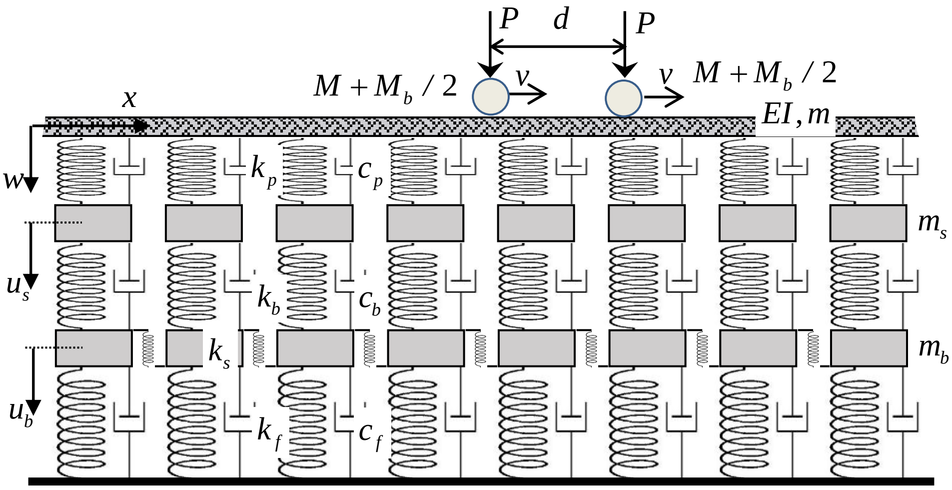

Figure 1: Three-layer model of the railway track traversed by two moving proximate masses, adapted from [19].

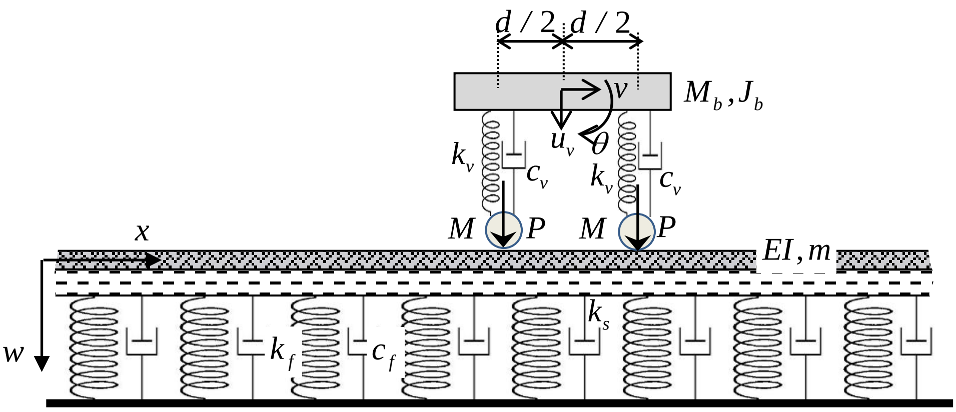

Figure 2: One-layer model of the railway track traversed by a two-axle bogie.

Assumption for the analysis of induced vibrations as well as the governing equations can be consulted in [15-17]. Exception is the bogie introduction, which needs the adaptation of the right-hand side and additional equations. Thus, for the sake of completeness the right-hand side of the beam equilibrium in the case of two moving masses is:

$$\left(P-\left(M+\frac{1}{2}M_b\right)w_{01,tt}(t)\right)\delta(x-x_1)+\left(P-\left(M+\frac{1}{2}M_b\right)w_{02,tt}(t)\right)\delta(x-x_2) \quad (1)$$

This equation for the case with bogie, it changes to:

$$\begin{aligned}&\left(P-Mw_{01,tt}(t)-k_v\left(w_{01}-u_v+\theta\frac{d}{2}\right)-c_v\left(w_{01,t}-u_{v,t}+\theta_{,t}\frac{d}{2}\right)\right)\delta(x-x_1)\\&+\left(P-Mw_{02,tt}(t)-k_v\left(w_{01}-u_v-\theta\frac{d}{2}\right)-c_v\left(w_{01,t}-u_{v,t}-\theta_{,t}\frac{d}{2}\right)\right)\delta(x-x_2)\end{aligned} \quad (2)$$

and the additional equations are:

$$M_b u_{v,tt}(t)-k_v\left(w_{01}+w_{02}-2u_v\right)-c_n\left(w_{01,t}+w_{02,t}-2u_{v,t}\right)=0 \quad (3)$$

$$J_b\theta_{,tt}+k_v\left(w_{01}-w_{02}+\theta d\right)\frac{d}{2}+c_v\left(w_{01,t}-w_{02,t}+\theta_{,t}d\right)\frac{d}{2}=0 \quad (4)$$

where x_1 and x_2 mark the fixed coordinate of the rear and front wheel mass, respectively. Derivatives are designated by the variable in subscript position preceded by a comma and the meaning of other symbols is exemplified in Figures 1 and 2.

The solution method follows these steps: at first, equations are transformed from fixed coordinates to the moving ones. Then, dimensionless parameters are introduced. For them, a Winkler beam characterized by EI, m, k_f and moving force P is selected as a reference beam. Before the switch to dimensionless counterparts, all parameters are assumed in their distributed version. The range of possible values, based on formulas presented in [18] can be consulted in [20].

3 Results

It has already been proven that the problem of instability of single moving mass and two moving proximate masses is substantially different, [15-17]. The main difference lies in respecting or not respecting the critical velocity of the moving force. In one-, two- and three-layer models, there are one, two, and three critical velocities in regular cases. In irregular cases, the missing velocities are replaced by so-called pseudo-critical velocities. In the regular cases of one moving mass, all such delimited regions are respected, and the instability lines are contained in these regions. In irregular cases, some internal critical velocities may be crossed by instability lines, but this never happens for the lowest one. When two proximate masses are moving, critical velocities are not respected. Therefore, instability can occur in the subcritical velocity range. However, not all such cases are considered viable, as the crossing can occur at a very high moving mass ratio and is therefore not physically possible.

After omitting the contribution of shear and damping, the one-layer model has no variable parameters except the velocity and moving mass ratios. Figure 3 shows that for certain dimensionless distances between the masses, an unstable behaviour is detected in the subcritical velocity region. Instability lines are plotted as a function of velocity ratio, and their ordinates indicate the moving mass ratio for which there is a switch between stable and unstable behaviour. The velocity ratio is defined with respect to the reference beam as:

$$\alpha = \frac{v}{v_{ref}}, \text{ with } v_{ref} = \sqrt[4]{\frac{4k_f EI}{m^2}} = \frac{1}{\chi}\sqrt{\frac{k_f}{m}} \text{ and } \chi = \sqrt[4]{\frac{k_f}{4EI}} \tag{5}$$

and the moving mass and damping ratios, and dimensionless distance between masses as:

$$\eta_M = \frac{M\chi}{m}, \ \eta_f = \frac{c_f}{2\sqrt{mk_f}}, \ \tilde{d} = d\chi \tag{6}$$

For comparison with the case of moving bogie, the mass M must be increased by $M_b/2$. Considering Eq. (5), the critical velocity ratio of one-layer model with no shear contribution is equal to unity.

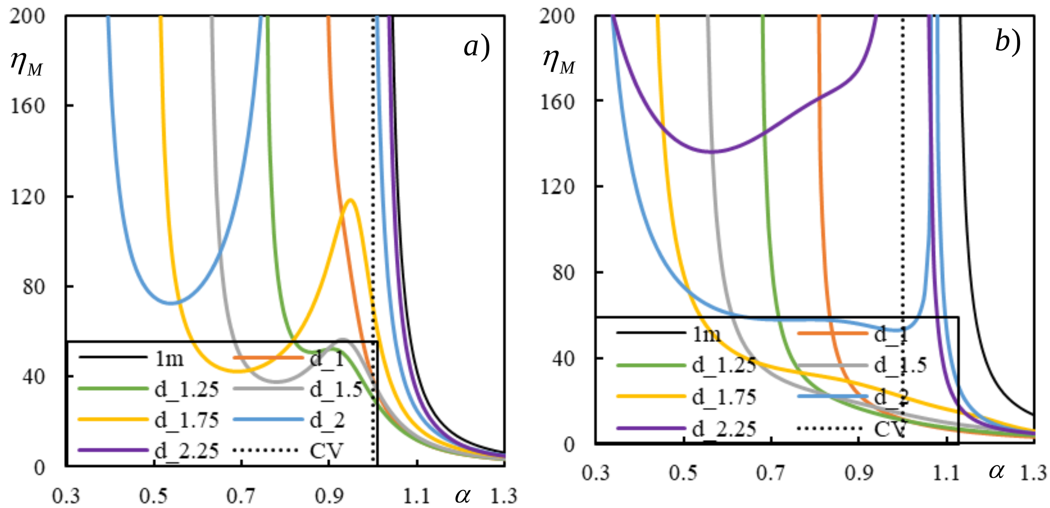

Figure 3: One-layer model: aggravation due to the increased damping a) $\eta_f = 0.05$; b) $\eta_f = 0.3$ (1m – single moving mass, d_x – two moving masses at dimensionless distance x).

In the two-layer model there are more parameters that can be varied, namely there are additional mass, stiffness and damping ratios related to the railpads and sleepers, and are defined as:

$$\kappa_p = \frac{k_p}{k_f}, \ \mu_s = \frac{m_s}{m}, \ \eta_p = \frac{c_p}{2\sqrt{mk_f}} \tag{7}$$

Extensive parametric analysis concluded that instability could occur in the subcritical range of velocities for realistic moving mass ratios only when the dimensionless distances are within $\tilde{d} \in \langle 1.25;1.75 \rangle$. For this analysis, μ_s was tested with steps of 0.1 and $\tilde{d}$ with steps of 0.25. Additionally, such a situation also depends on κ_p, for which the initial (lowest) value can be determined, and then all higher values are also critical. The starting values ranged from 10 to 66. Some cases are shown in Figure 4. For this analysis $\mu_s = 1$ and thus the critical velocity ratio does not depend on κ_p and equals $1/\sqrt{2}$. κ_p was chosen as 300 and $\eta_p = 0.05$.

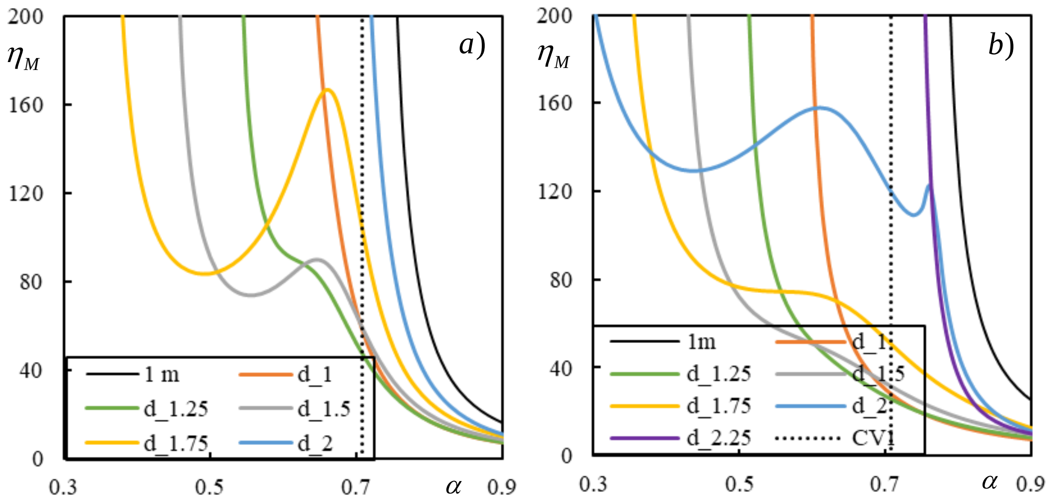

Figure 4: Two-layer model: aggravation due to the increased damping a) $\eta_f = 0.05$; b) $\eta_f = 0.3$ (1m – single moving mass, d_x – two moving masses at dimensionless distance x).

In the three-layer model, there are again more parameters that can be varied because the ballast layer is included, and therefore there are additional mass, stiffness and damping ratios, defined as:

$$\kappa_b = \frac{k_b}{k_f}, \ \mu_b = \frac{m_b}{m}, \ \eta_b = \frac{c_b}{2\sqrt{mk_f}} \tag{8}$$

By extensive parametric analysis, it is also possible to find scenarios for which instability occurs in the subcritical velocity range for realistic moving mass ratios. This analysis cannot be performed in a rigorous way because the critical velocity is not well-defined in all cases, and when it occurs, it must be replaced by a pseudocritical value, which does not correspond to the true resonance and may be ambiguous, [17, 20]. In Figure 5 two cases are shown. It is seen that the region of 5 resonances is difficult to predict. When there are 5 resonances, then the 3 critical velocities are well-defined, however in all the other cases the lowest critical velocity can only be determined by parametric analysis. Moreover, when the case is closer to the region with 5 resonances, then the pseudocritical value it is dominant, while in the middle region it is ambiguous. Nevertheless, given a proper estimate of the lowest critical velocity, a parametric analysis can be performed and cases with realistic moving mass ratios indicating unstable behaviour in the subcritical velocity range can be identified. One of them in shown in Figure 6.

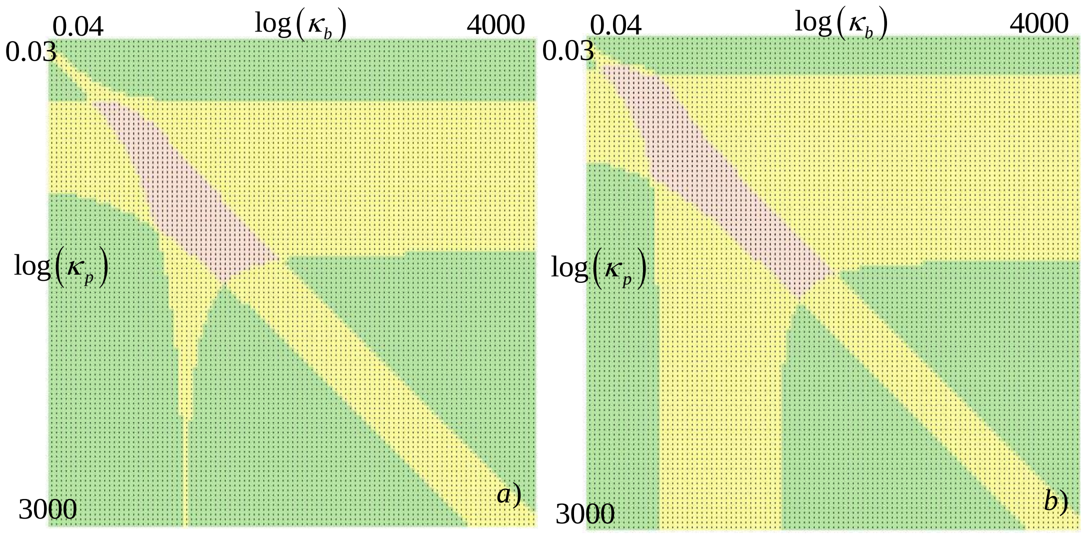

Figure 5: Number of resonances for $\mu_s = 1$, the indicated range of κ_b and κ_p: a) $\mu_b = 2$; b) $\mu_b = 4$ (1-red,3-yellow,5-green).

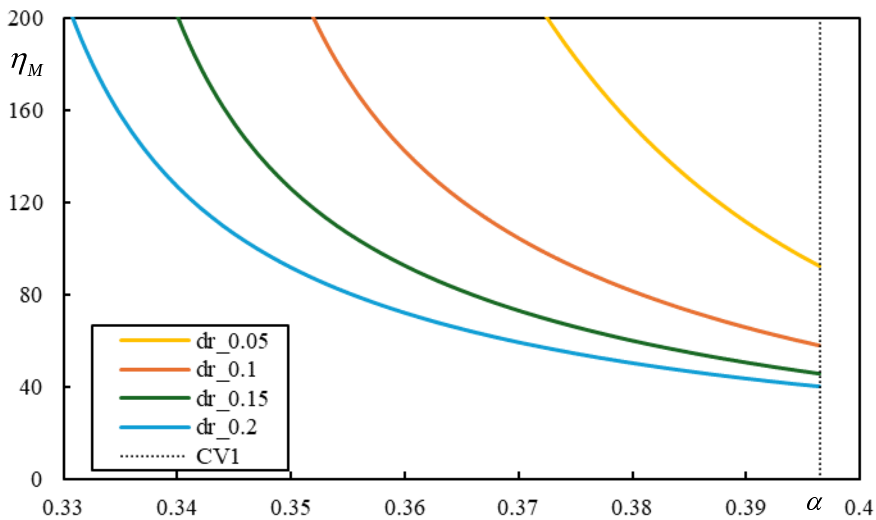

Figure 6: Three-layer model: aggravation due to the increased damping for $\mu_s = 1$, $\mu_b = 2$, $\kappa_p = 30$, $\kappa_b = 0.113$ and $\tilde{d} = 1.75$ (CV1 – the lowest critical velocity, dr_x – $\eta_p = \eta_b = \eta_f = \text{x}$).

In Figure 6, instability lines are only plotted in the subcritical range. In this case, values for $\kappa_p = 30$ and $\kappa_b = 0.113$ are chosen from the logarithmic scale of the figure as $\kappa_p = 0.03 \times 10^{0.05\times 60}$ and $\kappa_p = 0.04 \times 10^{0.05\times 9}$, which yields a well-defined value for the critical velocity.

4 Conclusions and Contributions

In this paper, it was shown that the instability of moving inertial objects can change the perception of the critical velocity, which for practical applications is understood as the speed that moving trains cannot exceed to keep their passengers in safe conditions. Since this speed is commonly understood as the lowest velocity of wave propagation in a structure that corresponds to the critical velocity of a single moving constant force, this paper clearly shows that this perception must be corrected.

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