Proceedings of the Sixth International Conference on
Railway Technology: Research, Development and Maintenance
Edited by: J. Pombo
Civil-Comp Conferences, Volume 7, Paper 15.2
Civil-Comp Press, Edinburgh, United Kingdom, 2024
ISSN: 2753-3239, doi: 10.4203/ccc.7.15.2

On the Dynamic Response of Soil-Structure Coupled Railway Infrastructures: The Case of a Portal Frame Bridge

J. Chordà Monsonís[1], J. C. Sánchez-Quesada[1], E. Moliner[1], A. Romero[2], P. Galvín[2] and M. D. Martínez-Rodrigo[1]

[1] Department of Mechanical Engineering and Construction, Universitat Jaume I, Castelló de la Plana, Spain
[2] Escuela Técnica Superior de Ingeniería, Universidad de Sevilla, Spain

Abstract

Predicting the dynamic response of portal frame bridges is challenging because of the strong influence of the soil-structure interaction. This aspect adds complexity to the vibratory phenomenon, and, as a result, accurate numerical analyses are difficult to conduct and time-consuming. For this reason, the soil-structure interaction mechanism is seldom considered in the numerical models. However, as evidence shows, this constitutes an important source of discrepancy between numerical and experimental results. In this investigation, a study on an existing portal frame railway bridge is carried out. First, the identification of its modal parameters from experimental data is addressed. Then, a 3D finite element numerical model considering the track-bridge-soil system is implemented. Perfectly matched layers are considered at the model boundaries. The soil-structure interaction is evaluated and the results are used to implement a simplified model, on which the soil is substituted by a series of spring-dampers. After calibration, an experimental-numerical comparison is performed on the bridge

modal parameters to validate the followed approach. Finally, the simplified model is used to predict the dynamic response of the portal frame measured under operating conditions.

Keywords: vibrations, acceleration, underpass, partially-buried structure, experimental measurements, high-speed railway bridge.

1 Introduction

As the transition towards more sustainable ways of transportation is seen as priority, expanding railway lines involves the necessity to adapt the infrastructure to the terrain and manage the interaction and integration among other transportation networks in a coherent way. In this regard, portal frame bridges constitute a recurrent solution on modern railway lines. These structures are utilised as underpasses that allow regular roads to pass under the railway tracks. This type of bridge, which is partially buried in the ground, consists of a reinforced concrete rigid frame flanked by integral wing walls and surrounded by an embankment. Because of most of the surface is in direct contact with the soil, the dynamic response of portal frames is strongly influenced by the soil-structure interaction (SSI) [1]. Despite this is known with certainty, simulating the SSI is complex, and it is not always considered in the numerical models dedicated to analysing the dynamic response of railway bridges due to the high associated computational cost [2]. In the case of portal frames, they count with a notable energy dissipation capacity due to the large contact area with the soil. This affects in a significant way their modal properties and dynamic response under train passages. Consequently, neglecting the dynamic stiffness of the surrounding soil in the numerical models may be the source of the divergences found between numerical and experimentally identified modal parameters [3]. Moreover, the incorrect assessing of SSI effects could be a cause of imprecision when determining the resonance speed on railway bridges, which is dependent on the damping provided by the soil [4]. In the end, this could result into inefficient sructural designs [5]. On the other hand, including SSI in the numerical models could lead to a more realistic evaluation of the Serviceability Limit States of existing bridges when changing the traffic conditions is required and to more efficient structural designs. Therfore, it would be desirable to include this effect in the dynamic analyses of portal frames to predict their response in a precise way. On this matter, previous research has found that there are still implications of the soil effects on portal frames that are not yet well known. A reason to this may be the lack of reliable simple models to simulate the SSI. In consequence, few are the works found in the literature that perform experimental-numerical validations of the results obtained [1].

In the present work, a study on an existing portal frame is conducted with the following objectives: (i) to identify the modal parameters of the bridge from experimental data and (ii) to implement a numerical model of the bridge and predict its real behaviour in an accurate manner with an assumable computational cost. To this end, a 3D finite element (FE) model of the structure is configured considering the track-bridge-soil system. The soil domain is padded with perfectly matched layers (PML) acting as absorbing boundaries to reproduce the radiation of incident waves through the soil layers. This model is used to obtain frequency-dependent impedance functions that will simulate the soil-bridge interaction in a subsequent simplified model. Then, the track and bridge parameters are calibrated and an experimental-numerical comparison of the bridge modal parameters is conducted to assess the validity of the model. Finally, a train passage is simulated to evaluate the predicted dynamic response of the portal frame.

The work herein presented is organised as follows. In Section 2, the portal frame under study is described in detail. In Section 3, the modal parameters of the bridge are investigated. Section 4 addresses the numerical approach and the models implemented. On Section 5, the simplified model is calibrated and the numerical and experimental mode shapes are compared. Section 6 covers the study of the dynamic response of the portal frame, where a circulating train is simulated on the simplified model. Eventually, the main conclusions of are summarised in Section 7.

2 Bridge under study and modal identification

The portal frame under study is located on the high-speed (HS) line Madrid-Sevilla, in the Ciudad Real - Brazatortas section at the kilometric point 31+200. The bridge has 8 m of span length and a platform of 22.1 m width. The underpass consists of a reinforced concrete rectangular box integral section of 5.7 m height. Three tracks pass over the bridge. One for conventional traffic and two for HS services. The most distinctive characteristic of the bridge is that it is divided along its width in two sections by means of a longitudinal joint. In this way, two coupled structures can be found: one below the conventional traffic track and another under the HS tracks. Figure 1 shows two images of the portal frame.

Figure 1: Two views of the portal frame.

The complete set of bridge dimensions can be seen on Figure 2, where the dashed line represents the longitudinal joint.

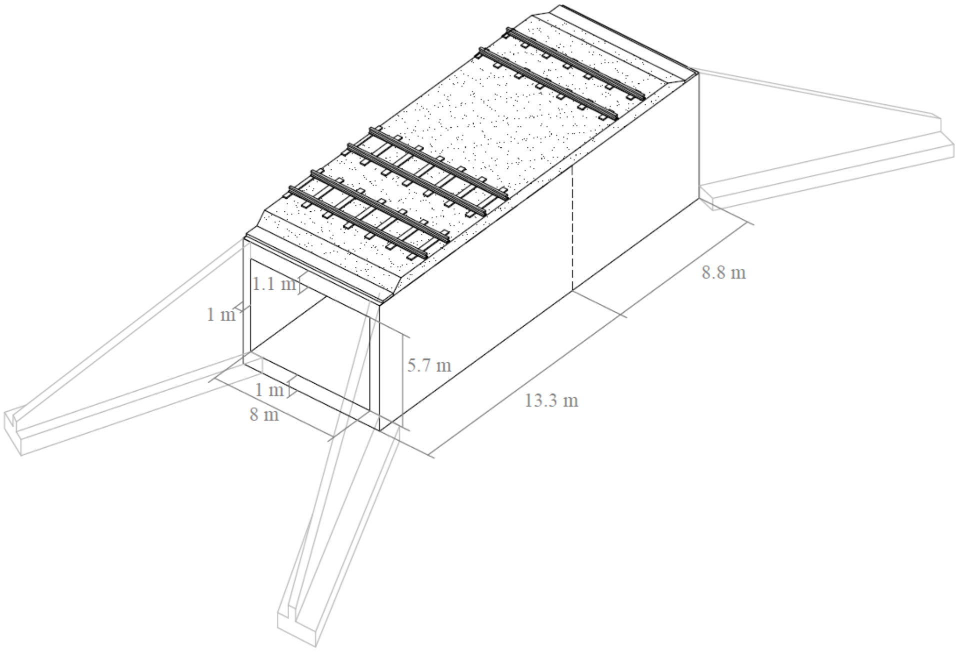


Figure 2: Dimensions of the portal frame.

3 Modal identification

An experimental campaign was carried out by the authors in September 2022 with the aim to characterise the modal parameters of the bridge. A total of 27 piezoelectric accelerometers were used to record the dynamic response of the structure with a nominal sensitivity of 10 V/g. The accelerometers were distributed over the inner face of the bridge slab as indicated in Figure 3: 18 on the top slab (shown in red); 6 at the walls at a height of 2.6 m (shown in yellow); and 3 on the floor (shown in blue). The obtained data comprehended the brigde dynamic response under the passage of 27 trains and also from the ambient vibration. This latter recording, acquired at a sample ratio of 4096 Hz and of 3600 s of duration, was then used to determine the modal parameters of the bridge. The signal was decimated to 256 Hz and a high-pass Chebyshev filter of 1 Hz was applied. Then, an operational modal analysis (OMA) was carried out.

The enhanced frequency domain decomposition (EFDD) method was used to detect the bridge structural modes. It was found that due to the partial decoupling of the two structures by the longitudinal joint, the bridge has a complex modal behaviour on which both bridge sections participate in a different proportion. Consequently, coupled or mixed modes appear, on which both bridge sections (HS and conventional) have a noticeble deformation. However, in some cases, uncoupled modes are also detected. Figure 4 lists the identified modes of the bridge. The first mode corresponds to the longitudinal bending of the HS section. In this particular case, the participation of the conventional section is negligible. In the second and third modes, the defor-

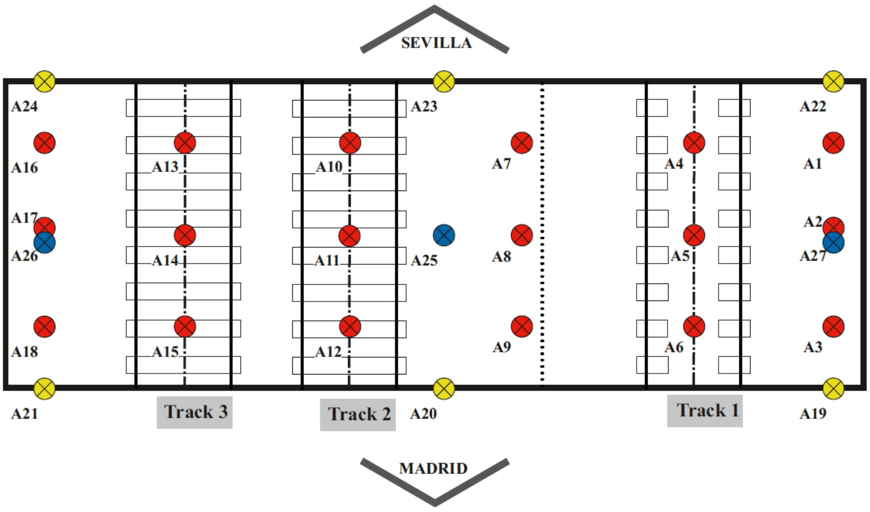
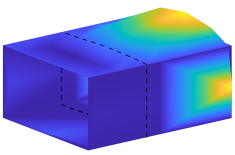
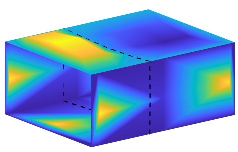


Figure 3: Schematic representation of the accelerometer layout.

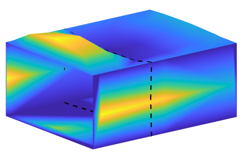
mation on the conventional part is predominant. This is more clearly visible on the third mode, which constitutes the fundamental bending mode of this bridge section. The fourth one is a transverse bending mode of the conventional region. Finally, the fifth mode represents a coupled transverse bending mode of the whole structure with a higher deformation on the HS part.



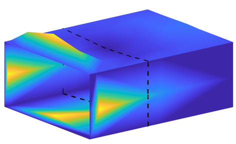
(a) $f_1 = 22.407$ Hz



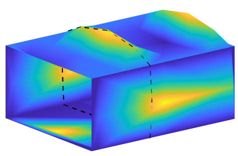
(b) $f_2 = 25.440$ Hz



(c) $f_3 = 26.802$ Hz



(d) $f_4 = 36.228$ Hz



(e) $f_5 = 44.266$ Hz

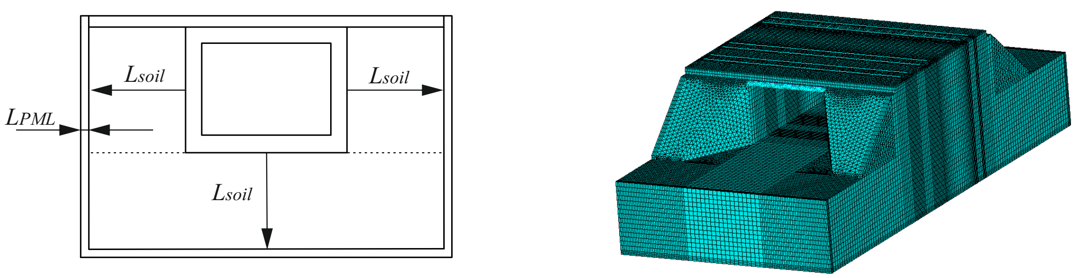
Figure 4: Identified modes of the portal frame.

4 Numerical approach

The numerical procedure followed in this work is intended to assess the viability of reproducing the modal behaviour and the dynamic response of the portal frame with an admissible computational cost. To this aim, a comprehensive 3D FE numerical model considering the track, the bridge and the surrounding soil is implemented. In this step, the effect of the SSI is evaluated and the results obtained are used to configure a simplified version of the previous model.

4.1 The soil-bridge-track model

Initially, a complete 3D FE numerical model of the portal frame is implemented with ANSYS(R) v.17.1. This model considers the tracks, the bridge structure and the surrounding soil. In order to avoid wave reflections at the model boundaries, solid PML elements are defined to absorb propagating waves. This feature allows the reduction of the soil domain and therefore permits a lower computation time. Sufficient soil and PML lengths have been determined to ensure the adequacy and stability of the results, as $L_{soil} = 6.0\,\mathrm{m}$ and $L_{PML} = 0.5\,\mathrm{m}$, as indicated in Figure 5(a). In total, three layers of PML elements are introduced. The meshing in the PML region is discretised based on the frequency to fit between 5 and 20 elements in a single wave length $\lambda = 2\pi{\cdot}C_s/\omega$, being ω the highest identified natural frequency of the bridge [5]. The different parts of the model are meshed as follows: solid elements with isotropic elastic behaviour are used to mesh the soil domain (SOLID185). The bridge structure is meshed with shell elements (SHELL181). With regard to the track, ballast and sleepers are also meshed with (SOLID185) finite elements. Rail pads are represented by means of spring-dampers (COMBIN14), and rails are conceived as Timoshenko beams (BEAM188). Besides, non-structural elements such as handrails are also considered as lumped masses (MASS21). The soil volume under the bridge and the backfill forms a prism of dimensions 24 m × 49.1 m × 7.5 m. In total, the model has 1,340,132 degrees of freedom (DOF). A view of the complete model can be seen on Figure 5(b). The mechanical properties of the track, bridge and soil are listed in Table 1. The soil stratum is considered homogeneous.



(a) (b)

Figure 5: Complete numerical model. (a): Dimensions of the soil and PML domains, and (b): 3D view of the model.

4.2 The simplified model

With the aim of reducing the complexity of the model while still preserving the added flexibility and damping introduced by the soil, a simplified version of the previous model is implemented in ANSYS(R) v.17.1. In the new configuration, the surrounding soil is substituted by a series of discrete linear spring-dampers that simulate the dynamic interaction of the soil with the portal frame. These elements are arranged on 65 uniformly distributed points over the whole area of the bridge-soil interface at the bridge walls and the bottom face of the box slab. At each point, three spring-dampers are located in the three spatial directions (X, Y, Z): one perpendicular and two tangential to the contact surface, as indicated in Figure 6(a). In this way, the DOFs of the model are reduced to 175,904. A view of the simplified model is shown in Figure 6(b).

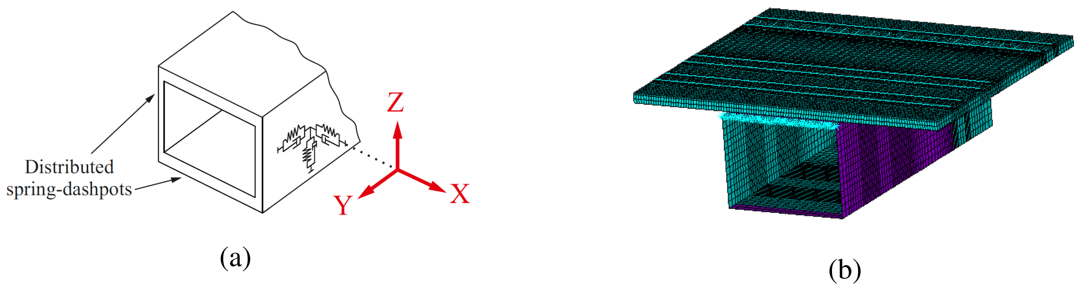


Figure 6: Simplified model. (a): Distribution and orientation of the spring-dampers, and (b): 3D view of the model.

5 SSI calibration on the simplified model

The interaction between the bridge and the soil is simulated by means of frequency-dependent lumped impedances in the simplified model. In order to calibrate the values of stiffness and damping assigned to these parameters, first, the SSI is evaluated on the complete model. To do so, a harmonic analysis is conducted, appliying a distributed vertical force $F_s(\omega)$ along both rails of track 2 on the complete span of the bridge. Then, the displacement at each spring-damper location in the three spatial directions is obtained. Following, the dynamic stiffness can be calculated as $K_{v,d}(\omega) = F_s(\omega)/U_s(\omega)$, where $U_s(\omega)$ is the displacement of the slab at each point in the corresponding direction, yielding the next properties of stiffness $K_v(\omega) = \text{Re}[K_{v,d}(\omega)]$ and damping $C_v(\omega) = \text{Im}[K_{v,d}(\omega)]/\omega$ for each spring-damper element [2]. Despite these results being dependent on the frequency, the differences among the different values of the computed stiffness and damping in the bridge frequency range [22.4-44.3 Hz] were low. Hence, within the simplified model, all spring-dampers are tuned to a consistent magnitude at the fundamental frequency of the bridge f_1.

Entity	Part	Property	Symbol	Value	Unit
Track	Rail	Elastic Modulus	E_r	$2.10 \cdot 10^{11}$	Pa
		Moment of inertia	I_r	$3038 \cdot 10^{-8}$	m^4
		Linear mass	m_r	60.34	kg/m
	Rail pad	Stiffness	K_d	$1.00 \cdot 10^{8}$	N/m
		Damping	C_d	$7.50 \cdot 10^{4}$	Ns/m
	Sleepers	Elastic modulus	E_p	$3.60 \cdot 10^{10}$	Pa
		Poisson's ratio	ν_p	0.2	[-]
		Mass	m_p	320	kg
	Ballast	Elastic modulus	E_b	$1.10 \cdot 10^{8}$	Pa
		Poisson's ratio	ν_b	0.2	[-]
		Density	ρ_b	1950	kg
		Height	h_b	0.728	m
Bridge	Slab	Elastic modulus	E_l	$35.71 \cdot 10^{9}$	Pa
		Poisson's ratio	ν_l	0.2	[-]
		Density	ρ_l	2500	[-]
	Joint	Elastic modulus	E_j	9522.50	Pa
		Poisson's ratio	ν_j	0.2	[-]
		Density	ρ_j	2500	[-]
Soil	Whole domain	Shear wave speed	c_s	350	m/s
		Poisson's ration	ν_s	0.2	[-]
		Density	ρ_s	1950	kg/m^3

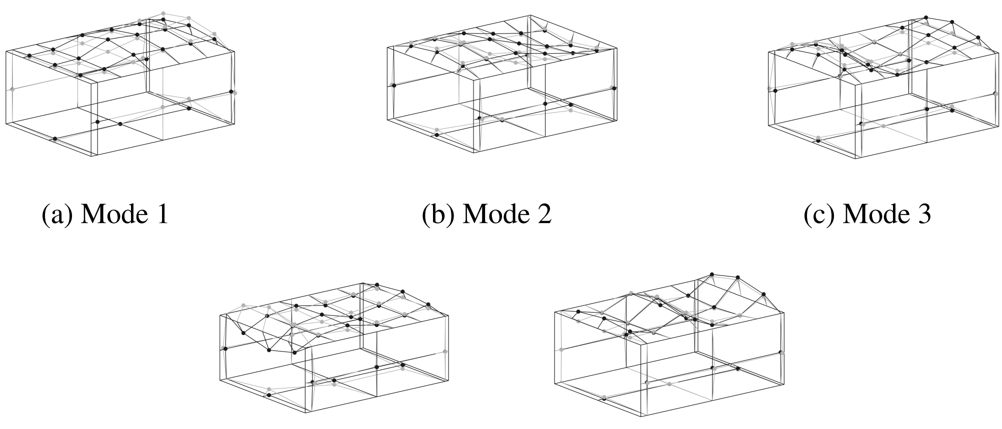
Table 1: Mechanical properties of the model.

To complete the calibration, a final adjustment is carried out on the mechanical properties of the simplified model, namely on the ballast density and on the elastic modulus of the bridge slab. However, as seen on previous sections, the bridge presents an asymetric modal behaviour due to the partial decoupling exerted by the longitudinal joint. This causes different degrees of deformation in each bridge section in certain modes. Because of that, the authors have found that differentitating the main properties of the slab and the ballast from both bridge parts is necessary to replicate the experimental modal parameters of the structure in an proper way. In any case, after this calibration, the properties of the spring-dampers were accordingly recalculated. The variation with regard to the previous values was minimal, leading to stable results. Table 2 summarises the updated parameters.

The comparison of the bridge modes is provided in Figure 7. Numerical mode shapes are represented in black, whereas the experimental ones are depicted in grey. Detailed results of this comparison are listed in Table 3. The modal assurance criterion (MAC) is calculated considering the accelerometers underneath the top slab (A1 to A18) and is used to assess the similarity between the numerical and the experimental modes. As can be seen, the comparison provides reasonable agreement, particularly on the fourth and fifth modes.

Parameter	Bridge section	Nominal	Updated	Variation
Ballast density kg/m^3	Conventional	1950	1560	-20%
	High Speed	1950	2340	20%
Slab elastic modulus (GPa)	Conventional	35.71	30.35	-15%
	High speed	35.71	26.07	-27%

Table 2: Mechanical properties of the bridge before and after calibration.



(a) Mode 1 (b) Mode 2 (c) Mode 3

(d) Mode 4 (e) Mode 5

Figure 7: Experimental-numerical comparison of the bridge mode shapes.

6 Dynamic response of the portal frame

Once the simplified model has been fully calibrated, the evaluation of the dynamic response of the portal frame is carried out. This section addresses the formulation of the dynamic problem, which is solved by means of the complex modal superposition method [6]. Then, the bridge numerical response to passing trains is compared to the corresponding experimental recordings of the real structure.

Results	f_1	f_2	f_3	f_4	f_5
Exp.	22.41	25.44	26.80	36.23	44.27
Num.	22.49	25.43	27.01	34.15	37.51

Results	MAC_1	MAC_2	MAC_3	MAC_4	MAC_5
Exp.	-	-	-	-	-
Num.	0.63	0.56	0.63	0.71	0.77

Table 3: Experimental-numerical comparison of the bridge frequencies (in Hz) and MACs.

6.1 The SSI interaction problem

The equilibrium equation of the system, applied to the bridge model with N DOF is:

$$M\ddot{u}\left(t\right)+C\dot{u}\left(t\right)+Ku\left(t\right)=F\left(t\right) \tag{1}$$

with initial displacement and velocity conditions $u(0) = u_0$ and $\dot{u}(0) = \dot{u}_0$, respectively. The mass, damping and stiffness matrices are represented as M, C and K. The damping in the problem is non-proportional i.e., $(M^{-1}C)(M^{-1}K)\neq(M^{-1}K)(M^{-1}C)$, because of the SSI effect induced by the spring-dampers. In consequence, the mode shapes are complex and the position of each DOF is defined by two parameters: amplitude and phase. Therefore, a set of $2N$ equations is needed to evaluate the solution of the N DOF of the structure, that can be written from Eq. 1 as a first order differential matrix equation:

$$A\dot{y}\left(t\right)+By\left(t\right)=P\left(t\right) \tag{2}$$

where:

$$A=\begin{bmatrix}C & M\\ M & 0\end{bmatrix} \quad B=\begin{bmatrix}K & 0\\ 0 & -M\end{bmatrix} \quad P=\begin{bmatrix}F(t)\\ 0\end{bmatrix} \quad y(t)=\begin{bmatrix}u(t)\\ \dot{u}(t)\end{bmatrix} \tag{3}$$

being y the state vector and A and B two real and symmetric matrices of dimensions $2N\times 2N$. If the free vibration case is considered, Eq. 2 yields:

$$A\dot{y}\left(t\right)+By\left(t\right)=0 \tag{4}$$

From which the trial soultion can be conveyed as $y(t) = \Psi_j e^{s_j t}$, where s_j is the j-th element of a total set $2N$ eigenvalues and Ψ_j are the respective eigenvectors:

$$\Psi_j=\begin{bmatrix}\phi_j\\ s_j\phi_j\end{bmatrix} \tag{5}$$

Proceeding onwards, natural frequencies, damped natural frequencies and modal dampings are determined from the eigenvalues as $\omega_j = |s_j|$, $\omega_{dj} = |\text{Im}[s_j]|$ and $\zeta_j = -\text{Re}[s_j]/|s_j|$. In the case of forced vibration, the solution to Eq. 2 takes the form:

$$y(t) = \sum_{j=1}^{2N} \Phi_j z_j(t) \tag{6}$$

Considering the orthogonality conditions $\Psi_j^T A \Psi_k = 0$ and $\Psi_j^T B \Psi_k = 0$ for any pair of modes $j \neq k$ (where the superscript T indicates matrix tranpsonse) and normalising the eigenvectors to the matrix A (i.e., $\Psi_j^T A \Psi_j = 1$), Eq. 2 results into a set of $2N$ uncoupled equations [6], where $p_j(t) = \Psi_j^T P(t)$:

$$\dot{z}_j(t) + \alpha_j z_j(t) = p_j(t) \tag{7}$$

In order to work on Eq. 7, several parameters need to be clarified. First, to calculate the bridge response, a complex modal analysis is carried out in ANSYS(R), from wich the modal shapes of the simplified model are obtained. Then, the modal shapes are used in the complex modal superposition method to compute the bridge response under moving loads. This analysis is carried out in MATLAB(R) v.2021. Since the mode shapes computed with ANSYS are normalised to the mass matrix M, it is required to normalise them to the matrix A. To do so, it is possible to obtain the following scaling parameter δ_j by developing $\Psi_j^{T,M} A \Psi_j^M = 1$, where the superscript M indicates normalisation to the M matrix (i.e., ANSYS modes). In this process, a simplification is assumed when addressing $\Psi_j^{T,M} A \Psi_k^M = 0$: s_j does not correspond to its conjugate $s_j \neq \bar{s_k}$ but to the same eigenvalue $s_j = s_k$.

$$\delta_j = 2m_j\omega_j\zeta_j + 2m_j s_j = 2(\omega_j\zeta_j + s_j) \tag{8}$$

Thus, the parameter δ_j allows normalising the modal shapes obtained with ANSYS to the A matrix (indicated by the A superscript):

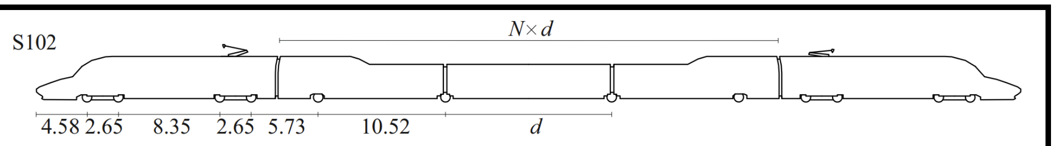
$$\Psi_j^A = \Psi_j^M / \sqrt{\delta_j} \tag{9}$$

Next, by developing $\Psi_j^{T,M} B \Psi_j^M = \alpha_j$, and scaling as indicated in Eq. 9, it can be obtained that $\alpha_j = (\omega_j^2 - s_j^2)/\delta_j$ in relation to this parameter in Eq. 7. Leading then a non stiff differential equation which resolution is carried out by means of Runge Kutta (4,5) explicit algorithm [7]. Eventually, the bridge displacements are calculated from the equation below, that takes into account that the eigenvalues and eigenvectors of structures with non-proportional damping are pairs of complex conjugates. A total of 12 modes, considering the conjugates, are normalised to the A matrix and used to compute the solution in the complex modal superposition method, ranging from 22.5 to 48 Hz.

$$u(t) = \sum_{j=1}^{N} 2\text{Re}[\phi_j z_j(t)] \tag{10}$$

6.2 Response under operating conditions

In this section, the simplified model is used to predict the dynamic response of the portal frame. To this aim, the passage of the RENFE S102 articulated train on its duplex configuration is simulated. As indicated in Table 4, this train circulated on track 2 heading to Sevilla at a speed of 242.1 km/h. Each composition was formed by two locomotive coaches and twelve carriages. Additional information is provided on this table, namely, the characteristic distance d and the average axle load P of the passenger coaches.

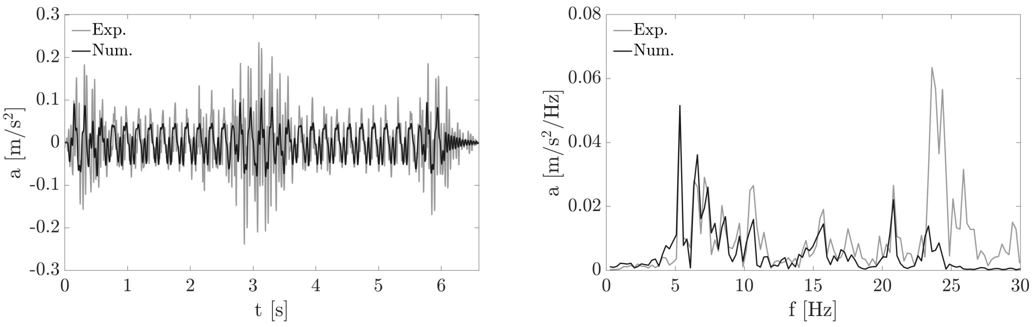


Train	Track	Ride	Scheme	V [km/h]	N	d [m]	P [kN]
S102 duplex	2	M-S	L-12C-L//L-12C-L	242.1	12	13.14	165

Table 4: RENFE S102 train passage information.

The experimental response was acquired at the accelerometer labelled as A11, located at midspan under track 2 (see Figure 3). Below, Figure 8 shows the experimental-numerical comparison of the vertical acceleration at the A11 location after simulation, both in the time domain (a) and in the frequency domain (b). The grey line stands for the experimental data, whereas the black one corresponds to the numerical signal. The responses are filtered applying two Chebyshev filters with high-pass and low-pass frequencies of 1 Hz and order 3 and 30 Hz and order 10, respectively.

As can be seen, in the time domain the numerical response keeps a good ressemblance with the experimental data. Regarding the frequency content of the acceleration, the signal presents important contributions at low frequencies related to the characteristic distance of the train d between shared axles: $f = V/d =$ 242.1/3.6/13.14 $\approx$ 5 Hz and subsequent harmonics. Two frequency peaks nearing 20 and 25 Hz approach the first frequencies of the bridge. However, in the case of the 25 Hz maximum, the bridge response is notably underestimated by the numerical prediction.



(a) (b)

Figure 8: Experimental-numerical comparison of the passage of the S102 duplex train at the location of the A11 accelerometer.

7 Conclusions

In this paper, the modal and dynamic behaviour of an existing portal frame are investigated. To this aim, a 3D FE numerical model of the structure, considering the track, the bridge and the soil system is implemented. PML elements are used on this first version to reduce the reduced computational cost. The SSI data collected with this model is then used to configure a simplified version of it, in which the effect of the soil is simulated with distributed linear spring-damper elements at the soil-bridge interfaces. After additional calibration on the bridge parameters, the numerical mode shapes are compared to the experimental ones, obtaining satisfactory results. Then, the dynamic response of the bridge under operating conditions is evaluated. In summary, the following can be concluded:

1. The identification process was a key part of the bridge study, as, because of the partial decoupling between bridge sections, the portal frame presented an unusual modal behaviour that resulted into separated modes (those on which the deformation of one of the bridge sections was predominant) and mixed modes (on which both sections participated). This was helpful for later calibrating steps.

2. The use of PML elements allowed a faster SSI evaluation on the complete model and the reduction of the soil domain necessary to compute an adequate solution.

3. The implementation of the simplified model permitted a fast method to analyse the modal parameters of the bridge, obtaining numerical counterparts with reasonable accuracy. With respect to the simulation of train passages, the numerical response matched the experimental one in the frequency range from 0 to 22-23 Hz. Then, the predicted response lost accuracy and underpredicted frequency peaks related to higher structural modes.

The results obtained could be useful in future research dedicated to predict the dynamic behaviour of partially buried structures such as portal frames. Further improvement can be achieved on the techniques dedicated to this goal, that will led to a better assessment of the integrity, safety and sustainability of the railway infrastructure.

Acknowledgements

The authors acknowledge: Universitat Jaume I (PREDOC/2022/26); Spanish Ministry of Science and Innovation, Agencia Española de la Investigación (AEI), FEDER EU Funds (PID2022-138674OB-C2); Regional Government of Economic Transformation, Industry, Knowledge and Universities of Andalusia (PROYEXCEL 00659); European Union EU-Rail JU (InBridge4EU), and the Andalusian Scientific Computing Centre (CICA).

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