Proceedings of the Sixth International Conference on
Railway Technology: Research, Development and Maintenance
Edited by: J. Pombo
Civil-Comp Conferences, Volume 7, Paper 10.5
Civil-Comp Press, Edinburgh, United Kingdom, 2024
ISSN: 2753-3239, doi: 10.4203/ccc.7.10.5

Extended Optimisation of Periodic Urban Train Operation Considering Regenerative Braking Energy Utilisation Between Adjacent Two Trains

X. Meng, W. Ohnishi and T. Koseki

**Department of Electrical Engineering and Information Systems,
The University of Tokyo, Japan**

Abstract

Energy-efficient train operation in electric railways has been focused on the single train aspect. However, further optimisation can be achieved by exchanging regenerative braking energy between trains. In this study, trains are considered in pairs consisting of a braking train as provider and a powering train as receiver arranged in temporal order. Firstly, the operation of the provider train is fixed, and then the operation of the receiver train is adjusted according to the dynamics of the provider train. The operation adjustment is achieved by a method based on neighbourhood search proposed in our work. The case studies demonstrate that the proposed method is effective in reducing the net energy consumption of multiple trains and can be easily generalised to the case of multi-train periodic operation of a long term since in every pair of trains, only the receiver train is adjusted.

Keywords: direct current electric railway, periodic train operation, regenerative braking energy, numerical optimisation, multi-fidelity modeling, neighbourhood search.

1 Introduction

The optimisation of train operations aiming at reducing energy consumption has been extensively studied since the 1950s. As for the analytical solution of energy-saving train control, Pontryagin's maximum principle was applied to derive optimal patterns [1]. Considering a more comprehensive model of electric railway, the inherent non-linearity in train motor characteristics, resistance forces, and power supply dynamics presented significant challenges, leading to the attempts of applying dynamic programming method [2] and other heuristic methods like genetic algorithms to more integrated train operation optimisation problems.

In pursuit of further energy savings, the utilisation of regenerative braking energy (RBE) has become a critical consideration. In AC electric railways, regenerative braking energy can be returned to the power grid with bidirectional substations. Urban railways with DC systems have shorter train intervals and braking happens more frequently, but the DC system faces limitations in utilisation of RBE. In DC railways, RBE can only be stored in energy storage systems (ESS), used by other trains powering in the same power supply section, or dissipated by electric resistance. The additional cost incurred by ESS implementation prompts researchers to explore alternative methods centered around adjusting train operations and run curves.

One fundamental approach to utilising RBE is the synchronization of the powering and regenerating phases of multiple trains. In previous research, the optimisation of the timetable is discussed in reference [3, 4, 5]. Besides timetable optimisation, adjustment of train operation is also found to contribute to energy saving. Su et al. [6] divided the optimisation into the timetable level and the single-train level. Chen et al. [7] managed to minimize the total energy consumption of two trains by optimising the operation strategy of one train at first and then optimising the other one based on the previous result. Similarly, Sun et al. [8] managed to increase the utilisation of RBE by adding a coasting section into the speed profile to increase the overlapping time of adjacent trains.

Although the adjustment of train operation to increase the amount of regenerative energy utilisation has been discussed in previous studies, two main issues remain. Firstly, the impact of the time sequence of the two trains exchanging energy on the overall energy consumption has not been adequately analysed, and thus the adjustment of train operation has not been formed into a general methodology. Secondly, most of the discussions are limited to two trains, ignoring the dynamic and interconnected nature of urban railway systems. In real-world scenarios, trains in urban systems usually operate in a tightly scheduled, ongoing sequence, influencing the performance and energy consumption of each other. Due to its cyclic nature, the roles of trains in energy exchange are not fixed but variable. A train that departs and provides RBE in one cycle can become a receiver of RBE in the next cycle, depending on its position in the sequence and timing. To address these issues is vital for enhancing the efficiency and sustainability of urban railway systems. In our previous work [9], a less computationally intensive train operation optimisation algorithm based on neighbourhood search is proposed. In this study, we apply it to a more general model of periodic train operation, discussing the energy-efficient operation pattern of trains utilising RBE from each other.

The outline of this paper is as follows. The first section introduces the research background of the train energy efficient operation and summarizes some newest progress on this topic in recent studies. In the second section, a mathematical model of train energy efficient operation problem is defined, including the objective function, constraints, and multi-fidelity models for evaluation. In the third section, we analyze the optimal pattern of operation for trains to increase RBE utilization and save energy. Then, a neighbourhood-search-based algorithm to solve train energy efficient operation problem is explained. In the fourth section, the proposed method is applied to a model with periodically operated trains in the numerical case study and its results are discussed. Finally, a conclusion and plan of future work are given in the last section.

2 Mathematical Model [11]

In this section, the mathematical definition of the problem and models used to evaluate train operation are introduced.

2.1 Definition of problem

This study aims to find optimal operations for one single train when there exists another train in the same power supply section generating RBE during its braking. Based on the discussion of the interaction between two trains, the method can be further applied to a more realistic system with multiple trains operating simultaneously by combining them into pairs in temporal sequence.

In this paper, the train to regenerative brake and provide RBE is called the provider train, and the train to power and absorb RBE is called the receiver train. They make up the base of energy interaction in periodic train operations. The proposed method is based on the following rules and assumptions. Firstly, the operation of the provider train is fixed, and the run curve of the receiver train will vary with its operations adjusted. Secondly, the optimal operation of a single train under given route conditions has been already acquired through offline calculations.

2.2 Objective function

The objective of this study is to minimize the net energy consumption supplied by substations, which equals to the sum of their traction energy minus utilised RBE, and can be formulated as Equation (1):

$$\min\ (E_{t1} + E_{t2} + E_{loss} - E_{RBE}) \tag{1}$$

where E_{t1} denotes the traction energy of the provider train, and is a fixed value, E_{t2} denotes the traction energy of the receiver train, and it changes with the operations adjusted, and E_{RBE} denotes the amount of RBE utilised when the regenerative braking of the provider train and the traction of the receiver train are synchronized.

2.3 Constraints

The constraints of this problem consist of arrival time, stop position and speed limit. To address these constraints, an extended objective function is employed, which converts the constraints into penalty forms, demonstrated by Equation (2):

$$f(\mathbf{p}) = E + a_p f_p(e_p) + a_t f_t(e_t) + a_v f_v(e_v) \tag{2}$$

where $f_p(e_p)$, $f_t(e_t)$ and $f_v(e_v)$ represent the error function of the stop position error, arrival time error and over-speeding error respectively, defined as the square of the absolute value of the error. a_p, a_t, and a_v denote penalty factors for errors in stop position, arrival time, and exceeding over the speed limit, respectively. They modify the allowable tolerance for constraint violation.

2.4 Simulation models for objective function evaluation

Simulation models for train operation with high-fidelity (HF) and low-fidelity (LF) models are used to evaluate the objective function values of various train operating strategies. With intelligent utilisation of both models, an equilibrium of efficiency and efficacy can be attained.

2.4.1 High-fidelity model

The complexity of the electrical network, the strong interaction between electrical and mechanical factors, and the non-linearity [12] make it impossible to solve the problem with analytical equations. The high-fidelity model calculates the dynamics of train motion and power supply circuit simultaneously by dividing the total running time by very short time steps and updating the states of the train, its location and speed, and the states of the power supply circuit at each time step based on the states in the previous step. The power supply network is modelled as a DC circuit with substations equivalent to DC voltage sources of a constant voltage and resistance. The energy consumption is calculated by integrating substation power. The configuration of high-fidelity model is shown in Figure 1.

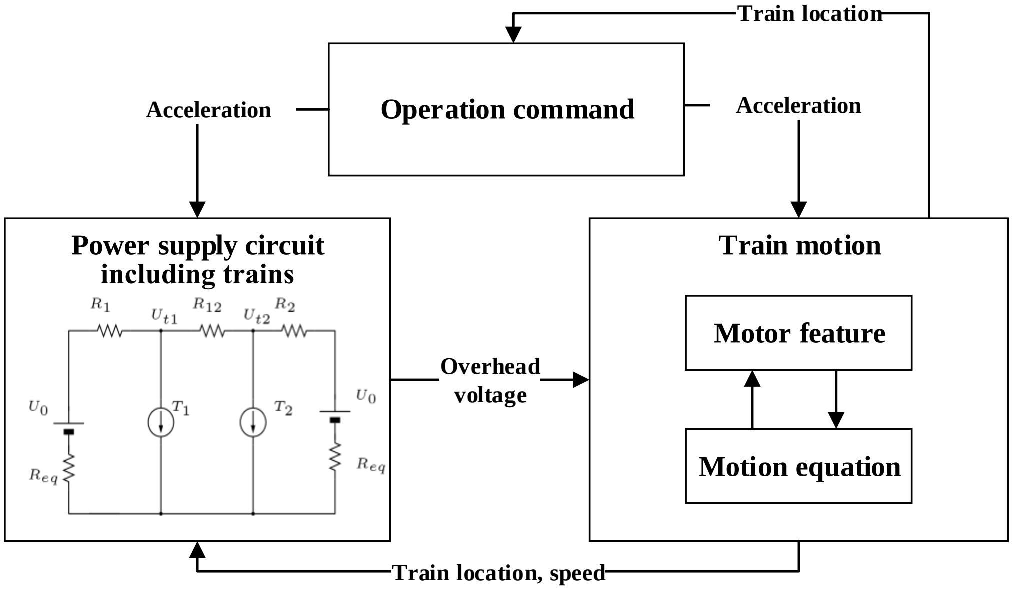

Figure 1: Configuration of high-fidelity model.

2.4.2 Low-fidelity model

In the low fidelity model, simplified analytical solutions of train motion equations are acquired by setting running resistance as a constant average of speed dependency. Also, the low fidelity model ignores the effects of the power supply network by setting the overhead voltage of trains as constant values instead, thus the variation of motor output is ignored. Also, transmission loss of electricity is not considered. In a journey, the parameters are assumed to be constant in each subsection and the dynamics of the train are calculated using simplified equations of motion. This model can give a quick approximation of the results.

3 Algorithm Description

In this section, the proposed algorithm to design train operation will be described. The basic idea of neighbourhood search method is to find local optimal solution in the neighbourhood of an initial solution by keeping generating new sample solutions based on a certain structure of neighbourhood [13]. The heuristic has been applied widely in optimisation problems of different fields. As for railway system, examples of its application include timetable optimisation [14, 15], transit system design [16], and so on.

As for the energy-efficient operation problem in this study, we make use of the analytical optimal pattern based on Pontryagin's principle and low-fidelity model to design an initial solution and limit search space. Then, several different operators are applied to perturb current best solution to move to next solution in the neighbourhood search, and multi-fidelity models are used for solution evaluation. The process of generating new solutions in local solution space is also called perturbation. In the following sections, the two steps of the proposed method, initial solution design and neighbourhood search, will be explained.

3.1 Initial solution design

While real-world scenarios may involve energy exchange between multiple trains within one power supply section, the focus of this study is on the simplest case of interaction between two trains. This minimal unit of analysis serves as a foundational framework that can be extended to address more complex and realistic situations. In this paper, trains are considered in pairs and the train to generate regenerative energy is called provider train, while the train to absorb the regenerative energy is called receiver train. As for train run curve adjustment, we choose to adjust the receiver train only, which departs later. The main reason is that, from a cause-and-effect perspective, it is more adaptable to modify the train operation following a temporal sequence. Even if a delay or a sudden operation change occurs in the provider train, the receiver train can be adjusted accordingly before its departure, accommodating the altered conditions. Besides, the key of utilising RBE is to synchronize two phases of two trains, and adjusting the provider train will lead to an increase of its own traction energy consumption, which is not energy efficient considering loss of energy transition.

Optimal train operation strategies considering RBE utilisation can be approximated with a constant absorb ratio [10] or a variant one [8]. It can be inferred that when the departure time interval is extremely short, indicating nearly simultaneous braking of the preceding and tracking trains, there is no need to adjust the curve, as such adjustments may lead to increased energy consumption. Conversely, when the interval is excessively long, there is also no need for curve adjustments, as the original curve allows for the utilisation of most regenerative braking energy. This deduction suggests that the optimal run curve pattern of the receiver train changes only when regenerative braking occurs while it is coasting or holding speed for the original curve. In such cases, the optimal strategy involves coasting earlier than the original pattern and a second acceleration when RBE becomes available. This insight helps narrow down the region where the optimal run curve may exist. In this study, we use this rule to generate the initial solution and decide the solution structure for the subsequent neighbourhood search.

Suppose the original optimal operation pattern for single train is to switch from maximum traction to coasting at the position l_1, and to switch from coasting to maximum braking at the position l_2. Then the initial solution for optimising search can be generated by adding two switching points between l_1 and l_2. The time when the train reaches the position l_1 can be calculated by $t_1 = \sqrt{\frac{2l_1}{a_1}}$ where a_1 represents the maximum acceleration. Time when the provider train starts braking t_{b1} is known according to its fixed curve and known departure interval time. Determine the back-and-forth relationship between t_1 and t_{b1}, and if $t_1 < t_{b1}$, insert a coasting between these two moments. The length of the inserted coasting is determined by $l_c = \sqrt{2a_1 l_1}(t_{b1} - t_1) + \frac{1}{2}a_2(t_{b1} - t_1)^2$. Since the second phase is coasting and the control input is zero, the acceleration applied in this phase a_2 is only resistance, which can be calculated by average speed. Notice that the optimal pattern of this case does not consist speed holding phase. However, the proposed method can also handle cases where speed holding phase exists.

3.2 Neighbourhood search and operators for perturbation [9]

The process of neighbourhood search is described by Figure 3. To generate new solutions within the neighbourhood of the initial solution or the current best solution, the algorithm employs operators. The process of generating new solutions is called perturbation, which refers to adjusting the current solution with a relatively small step in diverse directions. The optimal pattern of train run curve should be made up by maximum acceleration, coasting, re-acceleration whose optimal rate is unsolvable analytically, coasting and maximum braking. This limits the neighbourhood structure of solutions and decides the rules of perturbation. In this study, four types of operators as shown in Figure 2 are applied to perturb the current solution and generate new solutions.

The functions of operators are explained as follow:

1. Operator 1: This operator extends or shortens the distance of the first powering phase. Figure 2a illustrates an example of shortening the length of acceleration, leading to a later arrival.

2. Operator 2: This operator extends or shortens the distance of coasting after the first acceleration. Figure 2b illustrates an example of extending the length of coasting, leading to a lower average speed and later arrival.

3. Operator 3 and Operator 4: The operator3 extends or shortens the length of the second powering phase, and the operator4 increases or decreases the acceleration of this phase. Given that the available regenerative power from the provider train is non-increasing over time, and the traction power required by the receiver train is non-decreasing, a trade-off exists between traction energy consumption and RBE utilisation. The length and acceleration of the second powering phase is adjusted. Figure 2c illustrates an example of extending the second powering phase length. Figure 2d illustrates an example of increasing the acceleration of the second powering phase. Both examples show an earlier arrival.

Considering the distinct effects of four operators, they are applied to the current best solution in pairs, creating combinations of two operators at a time. Subsequently, a solution generated by each combination of operators undergoes slight perturbation

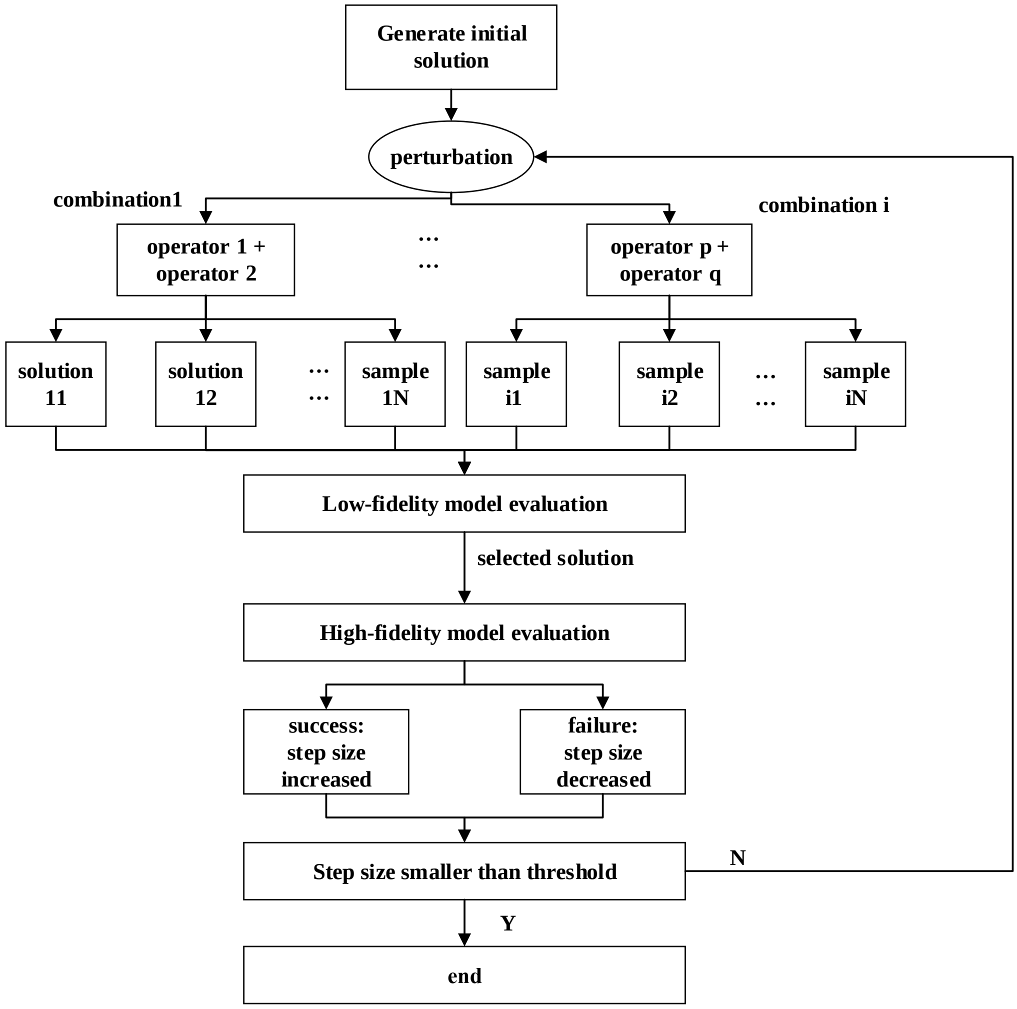

Figure 2: Process of neighbourhood search algorithm with multi-fidelity models.

by adjusting the length of each phase and the acceleration of the second acceleration with a random step size. This approach aims to enhance search efficiency and prevent the algorithm from becoming trapped in a local minimum. All generated solutions are then evaluated using the low-fidelity model, and one selected solution is further assessed using the high-fidelity model. The step size of adjustment of each operator is determined by multiplying a predefined step size by a random number. The step size is dynamically adjusted as neighbourhood search progresses. When three consecutive adjustments successfully lead to improved solutions, the step size is doubled. Conversely, when three consecutive adjustments fail to find a better solution, it is considered that the probability of discovering a superior solution within the current neighbourhood is low. Consequently, the step size is reduced, and the search is conducted within a smaller neighbourhood. When the step size falls below 1% of the

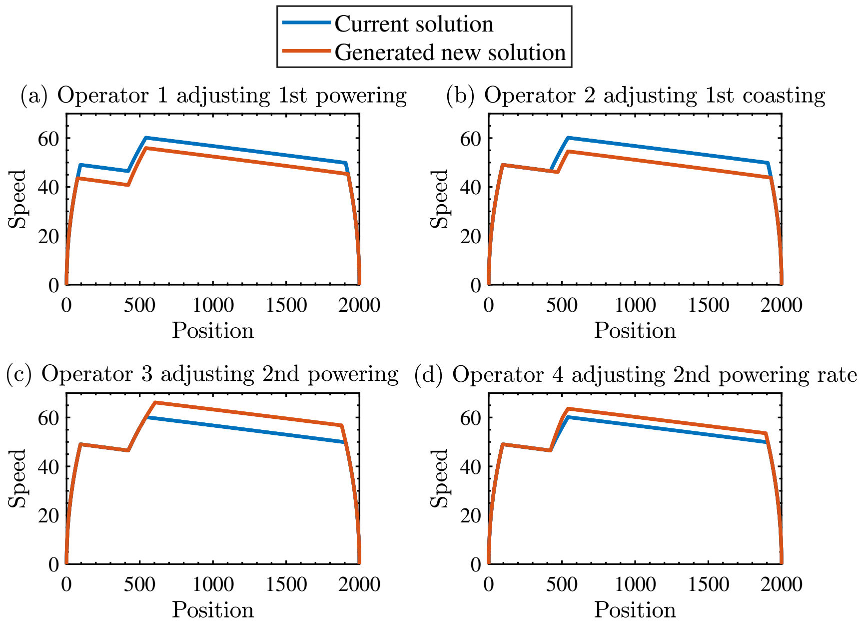

Figure 3: Four operators used in neighbourhood search.

initial step size, it is considered that the search has converged to a local minimum. At this point, the search is either restarted or concluded.

4 Case Study

In our previous study [9], the proposed method is applied to a simple model considering the operation of only two trains. The proposed method can get comparable solution as the dynamic programming method, which is expected to give the globally optimal solution numerically, but greatly reduce computational time, as shown in Figure 4.

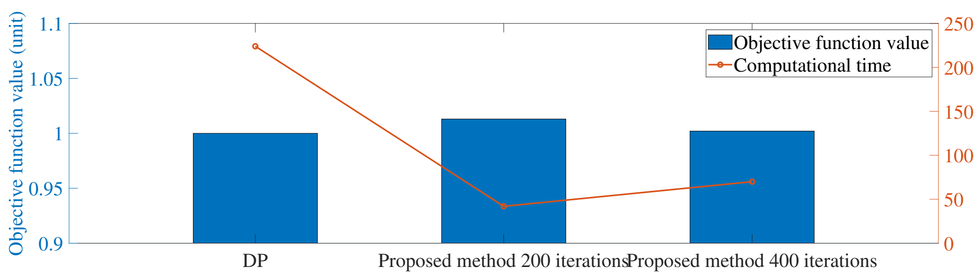

Figure 4: Solution quality and computational time comparison of proposed method and dynamic programming method [9]

Motivated by the effectiveness demonstrated in preceding work, this case study examines the performance of the method in an expanded model, which consists of two inter-station sections traversed by trains operating in opposing directions. The distance between stations is set to be the same for both sections, with substations on both sides. The configuration of the designed model is shown in Figure 4. Each train follows a periodic schedule, initiating its journey from designated starting stations and repeating the cycle upon reaching the final stations. For clarity and simplicity, we refer to the train moving from the lower end of the line to the upper end as the Up Train, and the train moving in the opposite direction as the Down Train. Here Up Train+ refers to the train on up direction in the next cycle, while Up Train- refers to the train on up direction in the previous cycle, in a periodic train operation, and the same for Down Train+ as well as Down Train-.

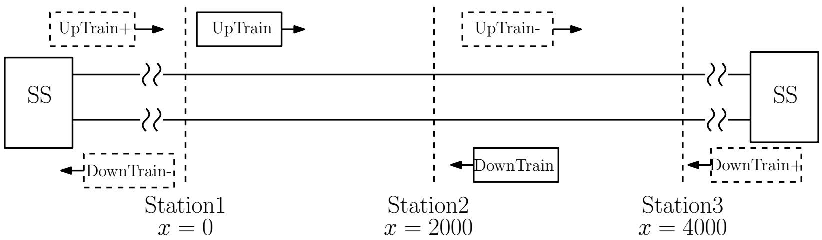

Figure 5: Configuration of the case study model consisting of two substations, three stations and trains operated periodically on both directions.

The cyclical nature of operation ensures a continuous flow of trains along the route. The interaction between trains is governed by adjusting the run curve of the following train while maintaining the operation of the preceding train fixed. The kinetic parameters and electric parameters of the designed model are listed in the table below.

time	distance	speed limit	train weight
150s	2000m	80km/h	353000kg
voltage	line resistance	motoring efficiency	regenerating efficiency
1500V	0.0327Ω/km	0.9	0.8

Table 1: Kinetic and electrical parameter settings of case study.

The timetable of the train operation is shown in Figure 5. The following description is based on the up direction train, and the period from Up Train departure at Station 1, until its arrival at Station 3, is considered to be a complete cycle. In one cycle, utilisation of RBE happens 4 times, and the relative location relationships of provider train and receiver train are different and analysed as follow:

1. Up Train is a receiver train and Down Train is a provider train. The distance between them is relatively long but decreasing with time.

2. Down Train is a receiver train and Up Train is a provider train. The distance between them is relatively short.

3. Up Train is a receiver train and Down Train is a provider train. The distance between them is relatively long and increasing with time.

4. Down Train+ is a receiver train and Up Train is a provider train. The distance between them is relatively short.

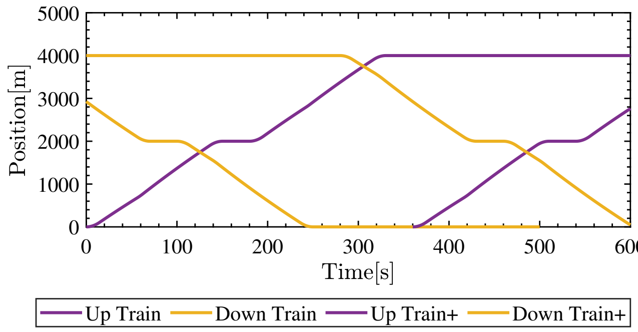

Figure 6: Timetable and corresponding location of trains on both directions.

By giving a certain run curve to one train at any time, subsequent trains running periodically can be paired with their previous trains and their energy-efficient operations can be acquired through neighbourhood search. In this case study, we tested three different cases. The run curve of Up Train is set as the optimal run curve of single train without considering RBE utilisation, and adjustment of run curve starts from Down Train. Neighbourhood search is iteratively carried out based on the results of previous iteration for 12 cycles. In the first case, no delay happens and all trains depart following the timetable. In the second case, a delay of 10 seconds is inserted to Up Train in the fifth cycle, which means it departs 10 seconds later than the standard time. In the third case, a delay of 10 seconds is inserted to Down Train in the fifth cycle. The run curves of all trains in cycle 1, 2, 5, 6, 11, and 12 are shown in Figure 7, with run curves in other cycles omitted because they are of similar shapes. We assume that when a train is delayed, the prior goal is to arrive the next station on time, so here the delayed train run curve is also set as the optimal run curve of a single train.

The energy consumption of adjusted train operations in above three cases are compared in Figure 8.

As can be seen from the figure, the total net energy consumption of trains is reduced in first several cycles. Run curves of Up Train+, Up Train++ and Up Train+++ and the total energy consumption amount remain the same in cycle 2, 3 and 4, implying a stable converging solution for trains is acquired. It offers energy-saving benefits and reinforces the viability of the proposed method on a larger scale. In cycle 5, Train1 is delayed for 10 seconds, and the run curves of following trains in this cycle and subsequent cycles are re-adjusted based on this change. But the shape of run curves

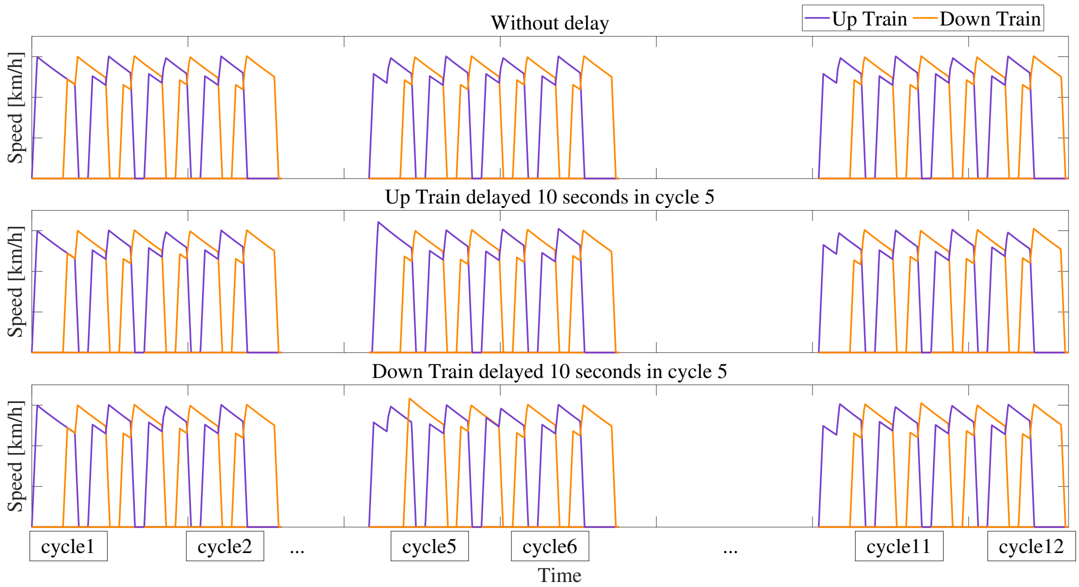

Figure 7: Train run curves in all cycles with Train1 delayed 10 seconds in cycle 5.

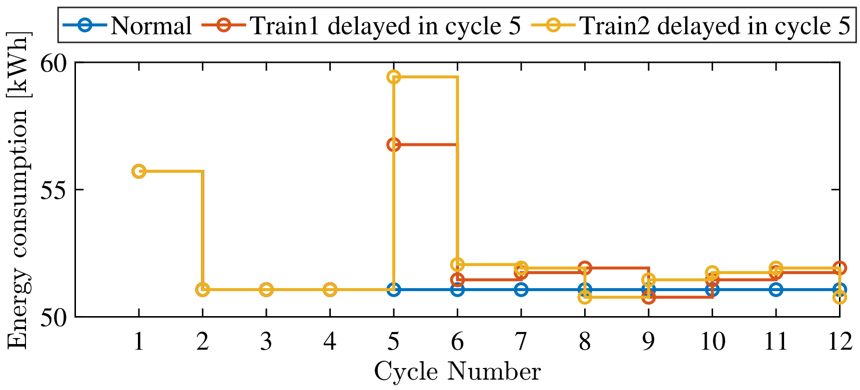

Figure 8: Objective function variation in iterative calculations

of following trains do not change much. Since trains tend to operate purely during the braking phase and the train stopping curves basically overlap, no matter how the operation of the first half of the train changes, the impact on the subsequent trains will be relatively small. This is also one of the important reasons for this study to adopt the approach of taking the operation of the provider train as a fixed condition and optimizing only the operation of the receiver train. Also, the tendency of results after a delay occurred proves that disturbance brought by delay can be removed through iterative calculation and a stable state can be reached again.

5 Conclusion

Optimising train operation holds significant importance in saving energy of DC electric urban railways, particularly for enhancing regenerative braking energy utilisation. The neighbourhood search algorithm proposed in our work is used to optimise run curves of adjacent two trains to reduce their net energy consumption. The high time-efficiency of the algorithm enables it to solve problems with multiple trains that run periodically. Based on braking and powering timing, trains are combined into a pair as provider train and receiver train, and the receiver train operation is adjusted based on the provider train movement. A stable solution for all train operations in one cycle is acquired by iteratively optimising train operations in temporal sequence, and the stable solution saves total energy consumption in the long term. The convergence of optimised train operation yields the reliability and applicability of the proposed method. Besides, numerical case studies with delay are carried out, and the proposed method is proven to be effective in reducing the effect of disturbance on energy consumption. Adjusting only the operation of the receiver train, instead of both trains in a pair, can still significantly reduce net energy consumption in a system with multiple periodically running trains.

Future works will enhance the applicability of the proposed method to more realistic situations by discussing optimal operation modes suitable for energy interaction between two trains with varying gradients or speed constraints on the route, for example. The dynamic effect of the occurrence of delays on the formation of periodic stable solutions and how to respond to them will also be part of the discussion in future work.

References

[1] P. Howlett, G. Philip, I.P. Milroy, P.J. Pudney, “Energy-efficient train control”, Control Engineering Practice, 2, 2, 193-200, 1994.

[2] H. Ko, T. Koseki, M. Miyatake, “Application of dynamic programming to the optimization of the running profile of a train”, WIT Transactions on The Built Environment, 74, 2004.

[3] Y. Bai, Y. Cao, Z. Yu, T.K. Ho, C. Roberts, B. Mao, “Cooperative control of metro trains to minimize net energy consumption”, IEEE Transactions on Intelligent Transportation Systems, 21, 5, 2063-2077, 2020.

[4] R. Liu, S. Li, L. Yang, J. Yin, “Energy-efficient subway train scheduling design with time-dependent demand based on an approximate dynamic programming approach”, IEEE Transactions on Systems, Man, and Cybernetics: Systems, 50, 7, 2475-2490, 2020.

[5] W. Zhou, Y. Huang, L. Deng, J. Qin, “Collaborative optimization of energy-efficient train schedule and train circulation plan for urban rail”, Energy, 263, 125599, 2023.

[6] S. Su, X. Wang, Y. Cao, J. Yin, "An energy-efficient train operation approach by integrating the metro timetabling and eco-driving", IEEE Transactions on Intelligent Transportation Systems, 21, 10, 4252-4268, 2019.
[7] M. Chen, X. Feng, Q. Wang, P. Sun, "Cooperative eco-driving of multi-train under dc traction network", IEEE Transactions on Transportation Electrification, 7, 3, 1805-1821, 2021.
[8] X. Sun, Z. Yao, C. Dong, D. Clarke, "Optimal control strategies for metro trains to use the regenerative braking energy: A speed profile adjustment approach", IEEE Transactions on Intelligent Transportation Systems, 24, 6, 5883-5894, 2023.
[9] X. Meng, T. Sun, W. Ohnishi, T. Koseki, "Two-Step Multi-Fidelity Approaches for Optimizing Railway Operations through Collaborative Energy Exchange between Powering and Regenerating Trains: Comparison of Computational Load and Energy Saving Performance", Joint Technical Meeting on Transportation and Electric Railway/Micromachine and Sensor System, IEE Japan, 27-32, 2024.
[10] A. Albrecht, P. Howlett, P. Pudney, X. Vu, P. Zhou, "The key principles of optimal train control—part 1: Formulation of the model, strategies of optimal type, evolutionary lines, location of optimal switching points", Transportation Research Part B: Methodological, 94, 482-508, 2016.
[11] X. Meng, T. Koseki, "Proposal of fast calculation of energy-saving train run curve based on surrogate method", Joint Technical Meeting on Transportation and Electric Railway/Linear Drives, IEE Japan, 65-70, 2022.
[12] P. Firpo, S. Savio, "Optimized train running curve for electrical energy saving in autotransformer supplied ac railway systems", 23-27, 1995.
[13] H.R. Lourenço, O.C. Martin, T. Stützle, "Iterated local search", Handbook of Metaheuristics, 320-353, 2003.
[14] T. Zhang, D. Li, Y. Qiao, "Comprehensive optimization of urban rail transit timetable by minimizing total travel times under time-dependent passenger demand and congested conditions", Applied Mathematical Modelling, 58, 421-446, 2018.
[15] X. Dong, D. Li, Y. Yin, S. Ding, Z. Cao, "Integrated optimization of train stop planning and timetabling for commuter railways with an extended adaptive large neighborhood search metaheuristic approach", Transportation Research Part C: Emerging Technologies, 117, 102681, 2020.
[16] D. Canca, A. De-Los-Santos, G. Laporte, J.A. Mesa, "An adaptive neighborhood search metaheuristic for the integrated railway rapid transit network design and line planning problem", Computers & Operations Research, 78, 1-14, 2017.