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Comparative Analysis of Multi-Objective and Single-Objective Optimization Approaches in Structural Engineering

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Abstract

This study investigates the effectiveness of multi-objective optimization versus single-objective optimization in structural engineering design. Through a comparative analysis, employing the same objective functions in both approaches across various scenarios, we assess their performance in balancing conflicting objectives while maintaining solution constraints. Single-objective optimization strategies involve formulating constraints based on one objective or constructing a scalar objective function with a single weight coefficient. Our findings reveal that while both approaches yield similar results, they differ significantly in complexity. Multi-objective optimization poses challenges in balancing competing objectives, while single-objective optimization with scalarization requires careful construction of the scalar objective function and weight parameter selection. However, single-objective optimization simplifies the optimization process when one objective is reduced to constraints. Additionally, the inclusion of auxiliary objective functions aids in solution refinement. Overall, our analysis highlights the potential for employing single-objective optimization as an alternative to multi-objective optimization, facilitating problem definition and enabling the incorporation of auxiliary objectives for enhanced optimization outcomes.

Keywords: finite element analysis, multi-objective optimization, scalarization, genetic algorithms, surrogate model, dynamics.

1 Introduction

Multi-objective optimization (MOO) involves simultaneously optimizing multiple conflicting objective functions while adhering to various constraints [1, 2]. This approach aims to find a set of solutions that represents a trade-off between different objectives. On the other hand, scalarization is a technique that simplifies a multi-objective optimization problem by combining multiple objectives into a single scalar objective [3,4]. Doing so transforms the problem into a more manageable single-objective optimization task.

Scalarization offers several benefits when applied in multi-objective optimization scenarios. Firstly, it simplifies the problem by converting it into a single-objective optimization task, facilitating the application of conventional optimization algorithms [3, 5–9]. Additionally, scalarization provides means to obtain a discrete representation of the set of Pareto optimal solutions, which are solutions that cannot be improved in one objective without sacrificing performance in another [1, 4]. Despite its advantages, the scalarization approach has some drawbacks. One notable drawback is the time-consuming nature of the method, as it requires running multiple single-objective algorithms for each possible solution [6]. This process can become particularly burdensome for complex optimization problems with many objectives. Additionally, although scalarization can be parallelized to expedite execution, it still necessitates running multiple single-objective algorithms without the ability to share knowledge between runs [6].

Single-objective optimization can replace multi-objective optimization not only through scalarization but also by treating one of the objective functions as the primary optimization target and introducing the other as a constraint. Single-objective optimization without constraints focuses on optimizing a single objective function without any imposed limitations [10, 11]. In this scenario, the goal is to find the optimal value of the objective function without considering any external factors. Conversely, single-objective optimization with constraints involves optimizing a single objective function while considering certain constraints that restrict the feasible solution space [11]. These constraints introduce additional challenges to the optimization process, limiting the range of possible solutions.

In conclusion, multi-objective optimization involves simultaneously optimizing multiple conflicting objectives, while scalarization simplifies the problem by transforming it into a single-objective optimization task. Single-objective optimization with constraints introduces additional challenges by considering constraints that limit the feasible solution space. Scalarization offers advantages such as simplification of the problem and obtaining Pareto optimal solutions. Still, it also has drawbacks, such as time-consuming execution and the need to run multiple single-objective algorithms. Different optimization algorithms have been employed to assess the efficacy of scalarization in multi-objective optimization, especially in constrained scenarios where effectively managing constraints is crucial. The surrogate models offer significant advantages in optimizing e.g. the dynamic properties of engineering structures; challenges such as the curse of dimensionality and the need for accurate and computationally efficient models remain areas of focus for further research and development in this field.

This paper compares results between two-objective optimization and single-objective optimization approaches. The same objective functions were examined in each of the three considered cases. In the single-objective optimization approaches, one of the objective functions was utilized to formulate constraints for optimizing the second objective function, or a single scalar objective function was constructed using a single weight coefficient.

The possibility of replacing multi-objective optimization with a single-objective optimization approach, without sacrificing the constraints and goals imposed on the solution by both considered objective functions, provides an opportunity for a more straightforward definition of the problem at hand and the potential inclusion of additional auxiliary objective functions.

2 The model and the approach applied

2.1 The model investigated

The investigated structure is a shell of the revolution created by rotating a line connecting two points. The length of the line (along the revolution axis) equals $L = 6.0$ m, the upper radius $R_{up} = 61.03$ cm, the lower radius $R_{down} = 1.3R_{up}$. The thickness of the shell is equal to $t = 1.6$ cm and is divided into eight composite layers of equal thickness. The end of the shell under analysis (with $R_{down} = 1.3R_{up} = 79.34$ cm) is fixed — all its displacements are blocked.

Each shell layer can be made of a different composite material, with a different direction of the composite reinforcement fibres. Three materials are taken into account: carbon fibre-reinforced polymer (CFRP), glass fibre-reinforced polymer (GFRP), and basalt fibre-reinforced polymer (BFRP). Their material properties and costs are taken from [12]. The material costs are unit-less since they show only the mutual relation of the costs of different materials. All properties of the applied materials are summarized in Table 1.

	E_a	E_b	E_c	ν_{ab}	ν_{ac}	ν_{bc}	G_{ab}	G_{ac}	G_{bc}	ρ	Cost
	GPa			$\times 10^3$			GPa			kg/m^3	—
CFRP	120.0	8	8	14.0	28.0	28.0	5	5	3	1536	10.20
GFRP	40.0	4	4	26.0	44.0	28.0	3	3	3	1320	1.36
BFRP	33.1	6	6	90.6	45.3	45.3	3	3	3	1765	1.00

Table 1: Material properties of three considered composite materials.

The finite element model is described by sixteen varying parameters that are subject to further optimization. The variable parameters are as follows: (a) material of each of the eight composite layers that make up the structure shell; μ_i, $i = 1, 2, \ldots, 8$, $\mu_i \in 1, 2, 3$, (b) lamination angle of the eight composite layers; λ_i, $i = 1, 2, \ldots, 8$, $-90^\circ \leq \lambda_i \leq +90^\circ$ (real values or integer ones with a step of 5°, 15° or 45°).

The finite element model comprises square-like, multilayered shell 4-node MITC4 elements (first-order shear theory). Each layer corresponds to one composite layer with possibly different material properties and lamination angles. The base size of the elements, h, is chosen to be almost equal to $h = 5$cm (it differs slightly in the circumferential and longitudinal directions; moreover, it also differs for different locations along the axis of the whole shell).

2.2 Optimization of fundamental natural frequency with regards to structure's cost

The standard formulation of the multi-objective optimization problem—find the values of the arguments collected in a 16-element vector $\boldsymbol{p}$ for which two considered objective functions yield minimal values possible—is given as:

$$\boldsymbol{p}^{\text{opt}} = \underset{\boldsymbol{p} \in \mathbb{P}^{16}}{\arg\min} \left\{ \mathrm{g}_f(\boldsymbol{p}), \mathrm{g}_c(\boldsymbol{p}) \right\}, \tag{1}$$

where $\boldsymbol{p}$ is a vector of structure parameters, and $\mathbb{P}^{16}$ is here the 16-dimensional space of the decision parameters gathered in vector $\boldsymbol{p}$.

The first objective function $\mathrm{g}_f(\boldsymbol{p})$ concerns the maximization of the structure's fundamental natural frequency and is expressed by the following equation:

$$\mathrm{g}_f(\boldsymbol{p}) = -f_1 \,. \tag{2}$$

The second objective function allows for—simultaneous with the optimization of the dynamic characteristics—minimization of the cost of materials necessary for constructing the structure:

$$\mathrm{g}_c(\boldsymbol{p}) = \mathrm{cost}(\boldsymbol{p}) \,. \tag{3}$$

The simultaneous optimization of the fundamental natural frequency f_1 and the structure's costs $\mathrm{cost}(\boldsymbol{p})$ can also be solved using scalarization approach, where the only scalar objective function $\mathrm{g}_s(\boldsymbol{p})$ is a linear combination of $\mathrm{g}_f(\boldsymbol{p})$ and $\mathrm{g}_c(\boldsymbol{p})$:

$$\mathrm{g}_s(\boldsymbol{p}) = -(1-\kappa)\, \mathrm{g}_f(\boldsymbol{p}) + \kappa\, \mathrm{g}_c(\boldsymbol{p}), \quad \text{for} \quad 0 \leq \kappa \leq 0.5 \,, \tag{4}$$

where κ is a weighting factor. The optimization process is thus given as follows:

$$\boldsymbol{p}^{\text{opt}} = \underset{\boldsymbol{p} \in \mathbb{P}^{16}}{\arg\min} \left\{ \mathrm{g}_s(\boldsymbol{p}) \right\}. \tag{5}$$

The third approach tested in the paper is a single-objective optimization of the fundamental natural frequency f_1 with some constraints on the cost of the structure:

$$\boldsymbol{p}^{\mathrm{opt}} = \underset{\boldsymbol{p}\in\mathbb{P}^{16}}{\arg\min}\,\{\mathrm{g}_f(\boldsymbol{p})\} \quad \text{for} \;\; c_{min} \leq \mathrm{cost}(\boldsymbol{p}) \leq c_{max} \;, \tag{6}$$

where c_{min} and c_{max} are carefully selected limits of a particular cost bound.

The optimization problems given by Equation (1), Equation (5) and Equation (6) are herein solved using non-dominated sorting genetic algorithm II (NSGAII) [13], a GA-based multi-objective search method that is not derivative-based. During the optimization of the dynamic properties of the investigated structure, the number of calculations of dynamic properties corresponding to different values of the model parameters reaches at least several hundred or, more probably, several thousand. Applying the finite element model leads to highly time-consuming numerical simulations. A neural network-based surrogate model (or metamodel) is proposed to overcome this problem, before the application the surrogate model is trained on examples obtained applying finite element model.

3 The results

Figure 1 depicts results obtained from optimization defined according to the three approaches specified in Equation (1), Equation (5) or Equation (6) (yellow, blue and red lines, respectively). In each case, four distinct approaches were considered, where the values of lamination angles were treated as continuous variables (symbolically represented as interval 0^o in Figure 1a) or as integer variables with steps of 5^o (Figure 1b), 15^o (Figure 1c), or 45^o (Figure 1d).

The analysis of Figure 1 demonstrates that in each of the described approaches to optimization, very similar results were obtained for both fundamental natural frequency and structure's cost. The Pareto fronts shown in the figures were obtained either directly from multi-objective optimization or through a series of successive calculations (for different values of the κ coefficient in scalarization or different values of prescribed limits imposed on the overall structure's cost), each of them corresponding to a single-case single-objective optimization.

Despite obtaining similar results, the complexity of the considered problems varies significantly. In the case of multi-objective optimization, the challenge lies in balancing two conflicting objectives. When scalarization is employed, particular attention must be paid to constructing the scalar objective function and the appropriate selection of κ. Only in the last approach, where one objective function has been reduced to constraints imposed on the obtained solution, is the approach straightforward, and the resulting outcome is independent of user-defined parameters.

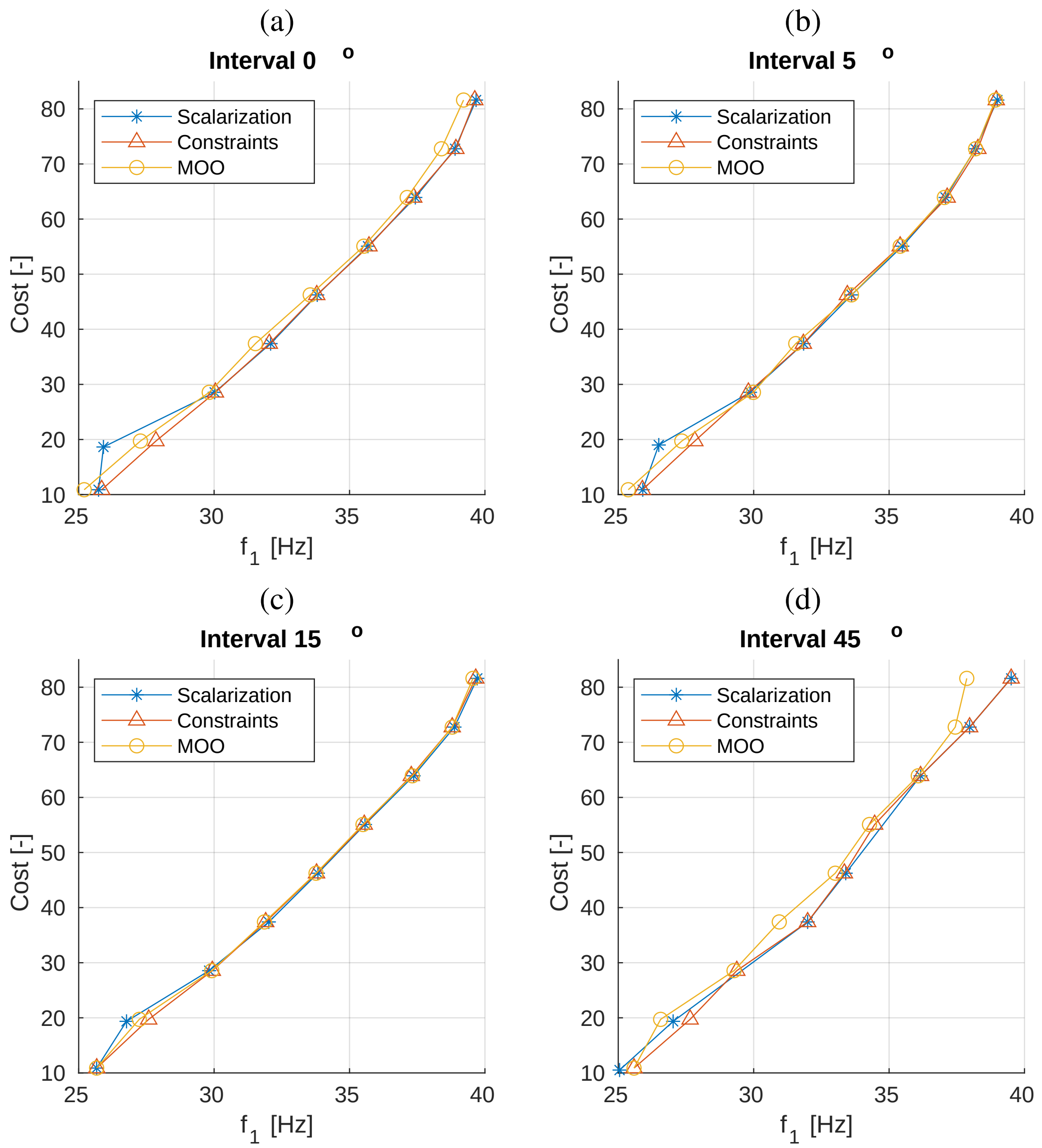

Figure 1: Three approaches to optimizing fundamental natural frequency f_1, simultaneously with cost minimization. Different lamination angles intervals verified: (a) real value, interval of 0^o, (b) 15^o, (c) 15^o, (d) 45^o.

Auxiliary objective functions can aid the search for an optimal solution. For example, in the considered problem, after encoding the construction cost as constraints, a second objective function supporting the optimization of frequency f_1 can be added. In the discussed task, such a supporting function could, for instance, minimize the difference between the first two natural frequencies. It has been observed that f_1 reaches its maximum value at mode shapes crossing, precisely when $f_2 = f_1$. The formula describes the optimization task:

$$\boldsymbol{p}^{\text{opt}} = \underset{\boldsymbol{p}\in\mathbb{P}^{16}}{\arg\min} \{ \mathrm{g}_f(\boldsymbol{p}), \mathrm{g}_{f2}(\boldsymbol{p}) \} \quad \text{for} \quad c_{min} \leq \text{cost}(\boldsymbol{p}) \leq c_{max} \ , \tag{7}$$

where

$$g_{f2}(\boldsymbol{p}) = -|f_2 - f_1|. \tag{8}$$

The results of applying MOO with supporting objective functions are shown in Figure 2. There is no visible difference between the classical MOO approach and the MOO with supporting functions. However, the MOO with supporting functions seems more reliable and stable.

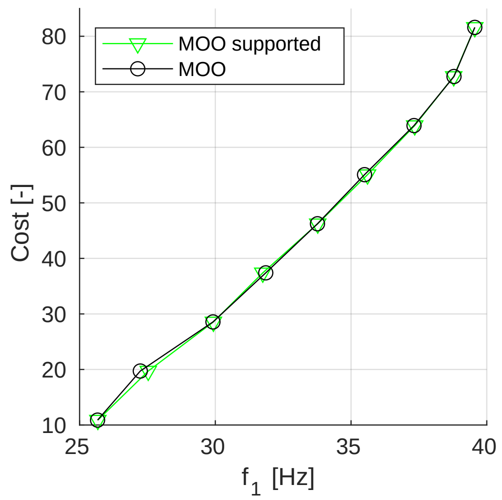

Figure 2: Maximization of fundamental natural frequency f_1 with supporting objective function; 15^o interval.

4 Conclusions

This study compared the outcomes of two-objective and single-objective optimization approaches, each considering the same objective functions in three cases. In single-objective optimization, either one objective function was utilized to formulate constraints for optimizing the second objective function, or a single scalar objective function was constructed using a single weight coefficient.

The potential replacement of multi-objective optimization with a single-objective optimization approach, while retaining the constraints and objectives imposed by both considered objective functions, offers the advantage of simplifying problem definition and allowing for additional auxiliary objective functions. Despite achieving similar results, the complexity of the problems varied significantly. Multi-objective optimization presents a challenge in balancing conflicting objectives, while scalarization requires careful construction of the scalar objective function and appropriate selection of weight parameters. However, in cases where one objective function is reduced to constraints, the optimization approach becomes more straightforward, yielding outcomes independent of user-defined parameters.

Utilizing auxiliary objective functions can aid in searching for an optimal solution. For instance, in the considered problem, a supporting objective function minimizing the difference between the first two natural frequencies was introduced after encoding construction costs as constraints. This approach maximizes the first natural frequency while minimizing the discrepancy between the first two frequencies, particularly at mode shapes crossing. The results indicate that multi-objective optimization with supporting objective functions produces outcomes similar to classical multi-objective optimization but with potentially increased reliability and stability.

Acknowledgments

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