Various uncertainties of diverse sources are apparent in structural analysis. Aleatoric uncertainty on the one hand and epistemic uncertainty on the other hand are distinguished. Uncertainty models based on probability theory, such as random variables, random processes and random fields, are capable to formulate uncertain quantities that describe aleatoric uncertainties. Contrarily, uncertainty models based on possibility theory, such as intervals or fuzzy variables, can be used to describe epistemic uncertainties by uncertain quantities. Polymorphic uncertainty models are used in this contribution, to incorporate the aleatoric and epistemic uncertainty characteristics that are present in structural analyses. Therefore, methods to combine fuzzy and stochastic variables are applied to model the uncertain quantities of a structural analysis. This combination results in fuzzy probability-based randomness as polymorphic uncertainty model.

Dependencies of uncertain quantities can be distinguished in intradependencies and interdependencies. Intradependencies can be described based on autocorrelation structures, which are the basis for the definition of random processes (time dependency) or random fields (spatial dependency). Time-dependent structural problems are addressed and the influence of the uncertainty of the process on the structural reliability is investigated. Therefore, novel methods are developed to model polymorphic uncertain time intradependency in fuzzy probability-based random processes. Additionally, the extension to join polymorphic uncertain time and space interdependencies is discussed. Methods are developed to model polymorphic uncertainty...
uncertainties in cyclic load processes. Hence, fuzzy parametrized autocorrelation structures for cyclic load processes are introduced.

A numerical example is computed to show the applicability and advantages of the novel developments. A cyclic load of an oscillating machine affects the subjacent machine foundation, which is built of a compressed wood plate. In this contribution, simulation of compressed wood is performed, based on the finite element method. In order to realistically simulate timber structures consisting of compressed wood components, highly elaborated material models with extensive simulation effort are required. The production process of a compressed wood plate is simulated in a preliminary investigation. The resulting density distribution of the simulated compressed wood component is examined. The data of density distribution is the basis for an uncertainty modelling of the respective spatial intradependency.

For the main investigation, the compressed wood component is exposed to a cyclic load function. This basic solution it is used as basis for a polymorphic uncertainty quantification. Resulting from the polymorphic uncertainty quantification, fuzzy-valued reliability measures are evaluated from the stresses and strains of the structure.

**Keywords:** polymorphic uncertainty, uncertain time-dependency, timber structure, compressed wood, uncertainty quantification, fuzzy probability-based random process.

1 Introduction

Since sustainable building materials are increasingly requested, the applicability of timber components in advanced fields of application and novel wood products are investigated in recent researches. In [1, 2], compressed wood is introduced as a novel timber product. The idea of compressed wood is based on the improvement of the mechanical properties of timber components. This is similar to the idea of timber products like glulam, plywood or oriented strand boards, but compressed wood is produced from timber in natural condition and initially produced without synthetic adhesives. A thermo-mechanical process is applied to produce compressed wood from natural timber boards, by compressing them in a hot press under a specifically defined temporal heating, pressing, cooling and unloading process, see [1]. According to [2], dimensional accuracy, surface hardness, elastic modulus, and shear strength of the component can be improved by this process. The resulting compressed wood can be used as a basis for timber products, e.g. by agglutinating to large timber components in desired physical dimensions with synthetic adhesives or other subsequent finishing processes, see e.g. [1].

Various uncertainties of diverse sources are apparent in structural analysis. According to [3, 4, 5], aleatoric uncertainty on the one hand and epistemic uncertainty on the other hand are distinguished. Aleatoric uncertainty is resulting from natural variability and, therefore, is irreducible. Uncertainty models based on probability theory, such as random variables, random processes and random fields, are capable to formulate uncertain quantities that describe aleatoric uncertainties, see [4, 5, 6]. Contrarily, epistemic uncertainty is resulting from lack of knowledge, incertitude or
bad data. Hence, epistemic uncertainty is theoretically reducible. Uncertainty models based on possibility theory, such as intervals or fuzzy variables, can be used to describe epistemic uncertainties by uncertain quantities, see [4, 5, 6].

According to [3, 4, 5, 6], the underlying uncertainty in parameters and coefficients of structural problems can be characterized by both, aleatoric and epistemic uncertainty, in combination. Such combined uncertainty characteristics are referred to as polymorphic uncertainty models, in which it is possible to define aleatoric uncertain quantities and epistemic uncertain quantities for different parameters of the same structural analysis and, moreover, it is possible to combine both uncertainty characteristics within a single parameter of a structural problem. This is possible, for instance, by fuzzy-parametrized distribution functions of random variates, which is referred to as fuzzy probability-based randomness, see [3, 4, 5].

2 Methods

In order to realistically simulate timber structures consisting of compressed wood components, highly elaborated material models with extensive simulation effort are required. In this contribution, simulation of compressed wood is performed, based on the finite element method, by the extensive material and element formulations of [7, 8]. The constitutive relations incorporate the coupling of thermal, hygrical and mechanical behaviour of wood. With this model, the elastic, inelastic, temperature- and moisture-dependent behaviour of timber structures can be simulated at large deformations. Moreover, realistic simulations are facilitated by taking the anisotropic nature of wood into account. This is permitted by the local-orthotropic material formulation, which is based on an individual evaluation of the grain dependent material properties at each integration point with respect to the respective tree trunk axis.

Dependencies of uncertain quantities can be distinguished in intradependencies and interdependencies, see [4]. Interdependencies are defined between different uncertain quantities. Interdependencies are modelled e.g. by interactions of fuzzy variables or cross-correlation and multivariate copula definitions between random variates. Contrarily, intradependencies are defined within the same uncertain quantity, but on the domain of the quantity, which commonly is the physical space or time in engineering tasks. For instance, intradependencies can be described based on autocorrelation structures, which are the basis for the definition of random processes (time dependency) or random fields (spatial dependency).

Polymorphic uncertainty models are used in this contribution, to incorporate the aleatoric and epistemic uncertainty characteristics that are present in structural analyses. Therefore, methods to combine fuzzy and stochastic variables are applied to model the uncertain quantities of a structural analysis. This combination results in fuzzy probability-based randomness as polymorphic uncertainty model for the input and output quantities of the structural analysis. Time-dependent structural problems are addressed and the influence of the uncertainty of the process on the structural reliability is investigated. Therefore, novel methods are developed to model
polymorphic uncertain time intradependency in fuzzy probability-based random processes. Additionally, the extension to join polymorphic uncertain time and space interdependencies is discussed. In [6], fuzzy parametrized random processes are defined for the simulation of concrete additive manufacturing. Additionally, in this contribution, methods are developed to model polymorphic uncertainties in cyclic load processes. Hence, fuzzy parametrized autocorrelation structures for cyclic load processes are introduced.

3 Results

A numerical example is computed to show the applicability and advantages of the novel developments. A cyclic load of an oscillating machine affects the subjacent machine foundation, which is built of a compressed wood plate. It is assumed, that a sustainable material and the reduced weight of the compressed wood foundation in comparison to a concrete foundation is desired.

A finite element method model is defined as basic solution for the structural problem, incorporating the material and element formulations of [7, 8]. The finite element simulation is computed by time step integration. Those extensive formulations are required to simulate the production process of a compressed wood plate in a preliminary investigation. The resulting density distribution of the simulated compressed wood component is examined. The data of density distribution is the basis for an uncertainty modelling of the respective spatial intradependency. For the main investigation, the compressed wood component is exposed to a cyclic load function. This basic solution has deterministic input and output parameters and it is used as basis for the polymorphic uncertainty quantification.

A random process is assumed for the variability of the cyclic load and the process is defined by the time intradependency in the autocorrelation structure. It is assumed, that the parameters of the autocorrelation structure and parameters of the probability distribution function are not precisely known and, therefore, are characterized by fuzzy parameters. This results in a fuzzy probability-based random process for the cyclic load. Based on this polymorphic uncertainty modelling of the time intradependency, a nested uncertainty quantification algorithm is required. A stochastic analysis is performed for the random variates, based on the subset sampling method. This stochastic analysis is embedded in a fuzzy analysis for the fuzzy variables, which is computed by an α-level optimization, see [4]. This nested analysis is required, since deterministic samples of the fuzzy analysis are used as parameters of the random process in the embedded stochastic analysis. Because of that, the complete uncertainty quantification framework is referred to as fuzzy probability-based stochastic analysis.

Resulting from the polymorphic uncertainty quantification, fuzzy-valued reliability measures are evaluated from the stresses and strains of the structure. Since the focus is on the uncertain cyclic load process, results are not only investigated at the end of simulation, but also over time. The influence of the polymorphic uncertain time
intradependency is examined and, therefore, methods for visualization of this influence over time are developed.

4 Conclusions and Contributions

In this contribution, compressed wood is investigated as a sustainable material with improved mechanical properties in comparison to natural timber components. Novel methods are developed to model uncertainties in a cyclic load process, which is defined by a fuzzy probability-based random process. Therefore, fuzzy parametrizations of probability distribution function and corresponding autocorrelation structure are formulated. Advanced methods for uncertainty quantification of polymorphic uncertain time intradependency are suggested, aiming at a reliability analysis of a structural problem. Methods for visualization and interpretation are developed for the time dependent polymorphic uncertainty and results of the reliability analysis.

The applicability of the suggested methods and novel developments is shown in a numerical example of a compressed wood machine foundation under cyclic load. The cyclic load process is characterized by aleatoric and epistemic uncertainties. The advantages of the research are discussed in the results with focus on the influence of the polymorphic uncertain time dependency on the fuzzy valued reliability measures. Extensibility to mixed time and spatial intradependency is discussed and the appropriate methods are presented, for instance, to incorporate the spatial variability of material properties of the compressed wood components. For this spatial variability, random fields can be introduced, which extend the domain of the time intradependency to spatial intradependency. The formulation of this extension as combined fuzzy-probability based random field and process is introduced. The results could furthermore be facilitated, e.g. to support decision making in design optimization processes.

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References


