Validation of an automated kriging-based methodology to calibrate PSO parameters: 
application to parametric optimization of truss structures

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Abstract

For years, the Particle Swarm Optimization (PSO) algorithm has been widely studied and many improved versions have been developed: from the swarm's topologies to the addition of new parameters, including machine learning approaches. However, the tuning of the fundamental PSO parameters has been less studied, but may lead to significant improvements on the convergence accuracy of PSO. This paper aims to develop an automated methodology to calibrate PSO parameters for a given optimization problem. The process is based on the kriging estimation of the best combination of PSO parameters. In this way, the Automated Tuning parameter Calibration (ATpC) methodology gives the optimal PSO setup for each considered problem in order to lead to a better convergence accuracy. The proposed ATpC methodology is applied to parametric optimization of truss structures. ATpC methodology performance is assessed by comparison of two different PSO setups usually used in the literature. The numerical results show that the ATpC methodology allows to significantly improve the convergence accuracy of PSO.

Keywords: optimization, metaheuristics, pso, calibration parameter, kriging, truss structures.
1 Introduction

For years, meta-heuristic algorithms have been widely studied and many improved versions have been developed. From the observation of birds flocks and based on Reynolds’ research [1], Kennedy and Eberhart [2] developed the Particle Swarm Optimization (PSO) algorithm. Founded on the collective intelligence of flocks, PSO uses a swarm of particles, defined as potential solutions of the considered optimization problem, converging by smartly following each other to the global optimum of the objective function to be optimized. There are four parameters that govern PSO equation: (i) the inertia weight \( \omega \), (ii) the individuality parameter \( c_1 \), (iii) the sociability coefficient \( c_2 \) and (iv) the population size \( N \).

Many sensitivity analysis [3-6] have been performed on the PSO parameters in order to determine the best combination adapted to the considered problem to be solved. However, these references demonstrate that the choice of PSO parameters strongly depends on the optimization problem to be solved. Performing a sensitivity analysis on PSO can allow to find an efficient parameter configuration but is highly time-consuming. Based on these observations, it is interesting to develop a methodology that allows to automatically tune the PSO parameters adapted to the considered optimization problem.

This short paper investigates a new automated methodology based on the development of a kriging-based meta-model in order to determine the best PSO parameter combination allowing the efficient convergence of the algorithm on the considered problem. The kriging meta-model is a geostatistical methodology developed by D. Krige [7] for the mining industry, used to predict the spatial distribution of ore. Then, in the late 80's, kriging has been adapted to the prediction of surrogate models of deterministic functions [8]. The kriging methodology allows to interpolate the values of a function \( f \) at undetermined points \( x^0 \) from the knowledge of \( f \) on determined points \( x^{(i)} \). The process is based on the hypothesis that the deterministic function \( f \) is the realization of a Gaussian Process (GP) [9] computed with the deterministic function \( f \) and a Gaussian noise modelled by a normal distribution.

This short paper aims at presenting Automated Tuning parameters Calibration (ATpC) methodology that determine the optimal PSO setup leading to a better convergence accuracy, in a low number of fitness evaluation, applied to a truss structure optimization problem.

2 Methods

In this paper, PSO performance has to be optimized, and is defined as the capability of the algorithm to efficiently converge to the global optimum of the considered objective function \( g \). PSO performance is a function of the PSO parameters \( (\omega, c_1, c_2, N) \). The PSO algorithm is applied to a mechanical truss structure optimization problem described as follows [10]:
\[
\begin{aligned}
\min g(S_1, S_2, \ldots, S_p) &= m \\
u_{y1}, u_{y2}, u_{y3}, u_{y4} &< \pm D_{\text{max}} \\
\sigma_e &< Re
\end{aligned}
\]

with \((S_1, S_2, \ldots, S_p)\) the \(p\) design variables corresponding to beam sections, \(m\) the mass of the truss structure, \(u_{yi}\) the vertical displacement of node \(i\), \(D_{\text{max}}\) the vertical displacement allowed, \(\sigma_e\) the axial stress in beam \(e\), \(Re\) the elastic limit of the considered material. The displacements and stress are computed by a finite element model composed of \(j\) nodes and \(p\) beams representing the truss structure (see Fig.1). Forces \(P\), and boundary conditions are applied to nodes according to the description of the problem. A penalty method is applied to deal with optimization constraints [11]. In PSO process, the number of iterations is set to 100.

The ATpC methodology can be described with the following 5 steps:
1. Latin Hypercube Sampling of \(n\) PSO parameters setup \((\omega, c_1, c_2, N)\)
2. Initialize the seed
3. Apply PSO with the \(n\) PSO setup to the truss problem (1)
4. While the number of iterations < \(\lambda = 20\) do:
   a. Compute the surrogate model by kriging methodology [12] with the \(n\) known points : PSO parameters and PSO response shape a response hypersurface. On this hypersurface, there is a minimum corresponding to a combination of PSO parameters minimizing PSO response for the problem (1)
   b. Compute the Expected Improvement (EI) criterion [13].
   c. Create a new point where the EI is maximum, i.e. where the probability to find a better minimum is the highest.
5. Identify the minimum value of the kriging meta-model obtained at the last iteration. This minimum is obtained for the best parameter setup.

In order to compare ATpC performance, a base-set of PSO parameters is considered based on the classical PSO parameter values found in the literature [14,15]. These 3 calibrations of PSO parameters are applied to the truss optimization problem (1), and PSO performances are compared to ATpC calibration.

The ATpC methodology and the classical PSO algorithm are performed 12 times. Mean and standard deviation are computed and compared.

3 Results

The truss structure optimization problem to be solved is composed of \(j = 6\) nodes and \(p = 10\) beams as represented in Fig.1. Beams material are assumed to be elastic-isotropic with a Young modulus \(E\) set to 68.9e3 MPa, and a density \(\rho\) set to 2770 kg/m³. The elastic limit \(Re\) is 172.37MPa. The minimal and maximal beam areas are set to 64.52mm² and 22.58e3mm² respectively. The force \(P\) applied is set to \(-4.448e5N\) at nodes n°2 and 4 in the Y direction. The maximum displacement allowed \(D_{\text{max}}\) is set to 50.8mm at nodes n°1 to 4 in the Y direction.
The optimization of the 10-beams planar truss structure presented in Fig.1 is performed with ATpC PSO parameters, Trelea PSO parameters [15] and Clerc PSO parameters [14]. The Trelea configuration corresponds to \((\omega = 0.6, c_1 = 1.7, c_2 = 1.7, N = 50)\) whereas Clerc configuration uses the following set \((\omega = 0.5, c_1 = 2, c_2 = 2, N = 50)\). The minimum mass \(m\) of the optimal structure obtained by PSO with Trelea, Clerc and ATpC parameters are given in Table 1. The mean value, and standard deviation of the 12 runs of the optimal mass \(m\) are computed.

![Figure 1: Truss structure configuration (6 nodes and 10 beams).](image)

<table>
<thead>
<tr>
<th></th>
<th>PSO Trelea parameters</th>
<th>PSO Clerc parameters</th>
<th>PSO ATpC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>2495.32 kg</td>
<td>2493.74 kg</td>
<td>2482.82 kg</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>18.93</td>
<td>18.99</td>
<td>13.14</td>
</tr>
<tr>
<td>Rank</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1: Mean and standard deviation of the 12 different fitness values obtained after the PSO optimization process using Trelea, Clerc and ATpC parameter configurations.

The best fitness value (i.e. the minimal truss mass) obtained over the 12 runs of the ATpC methodology on the truss structure optimization problem and the best parameters tuning are given in Table 2 for the 10-beams truss structure optimization problem (1).

<table>
<thead>
<tr>
<th>(\omega)</th>
<th>(c_1)</th>
<th>(c_2)</th>
<th>(N)</th>
<th>Mass (m) (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.72</td>
<td>1.58</td>
<td>0.98</td>
<td>49</td>
<td>2445</td>
</tr>
</tbody>
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Table 2: Best PSO parameters and fitness value found after performing the ATpC methodology to the 10-beams truss structure.
4 Conclusions and Contributions

This paper proposes a new methodology to automatize the tuning of PSO parameters applied to truss optimization problems. The optimal combination of PSO parameters is determined by a kriging process. Using this optimal combination of parameters, PSO is run with a classical Gbest topology configuration, in order to obtain the optimum of the considered problem. The ATpC methodology is applied to a 10-beams truss structure optimization problem. Results presented in this paper verify the validity of the proposed method. The average improvement of the ATpC methodology over the PSO parameter base-set is around 0.5% that represents a gain of mass of 12kg on the considered problem. From now, PSO parameters have to be calibrated for any optimization problem to solve. The investigation of the ATpC methodology with dynamic PSO parameters and other PSO topologies is an interesting and challenging direction to be developed in the future. Note that the proposed methodology may be applied to any other metaheuristic optimization algorithms that need parameters to be tuned, depending on the optimization problem to be solved.

References


